

# Experimental Implementations and Robustness of Fully Revealing Equilibria (FRE) in Multidimensional Cheap Talk

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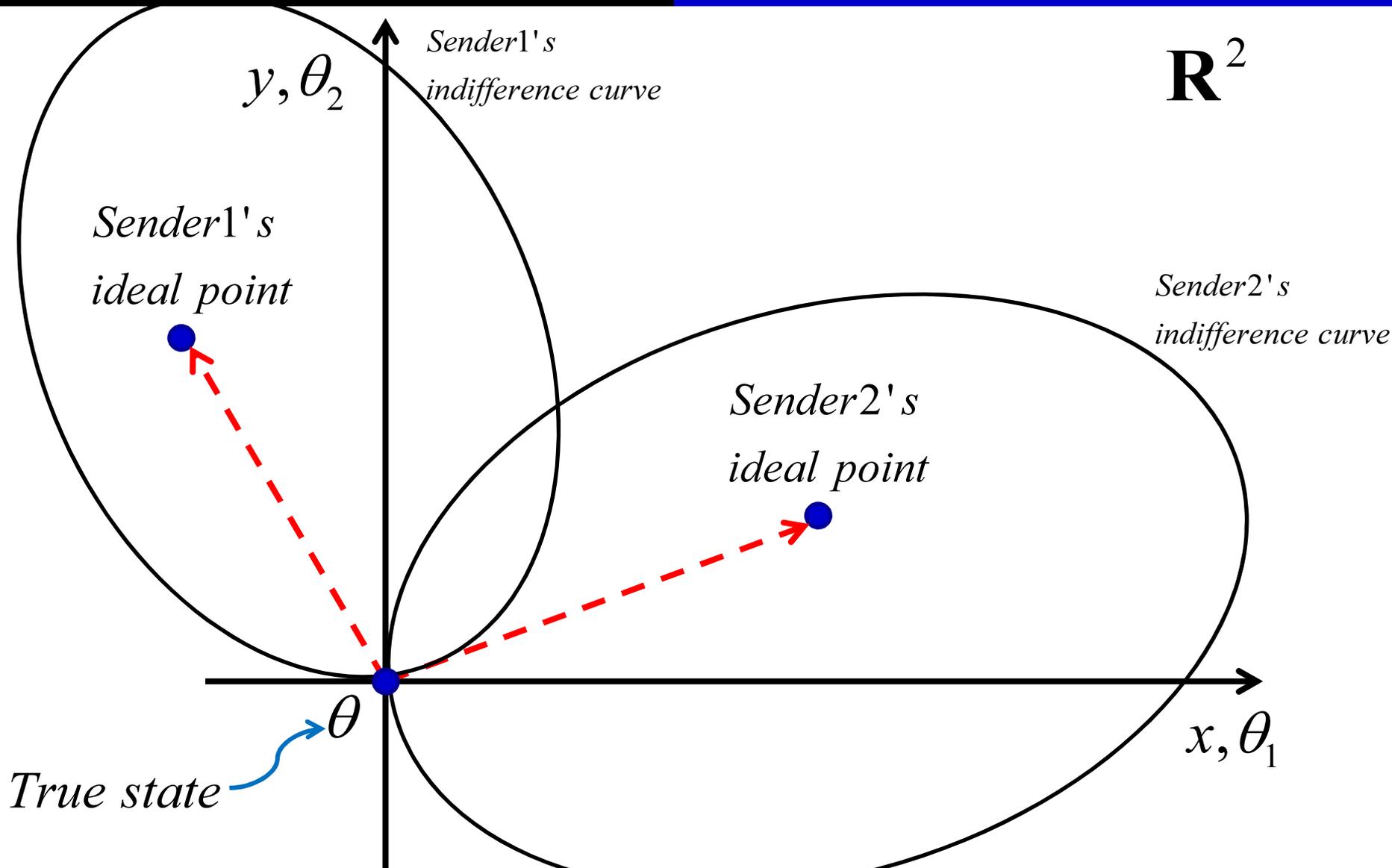
Joint with Ernest Lai (Lehigh) and Wooyoung Lim (HKUST)

# Fund Managers vs. You

- You ask two fund managers for advice:
  - Manager A's fund is heavy on stocks
  - Manager B's fund is heavy on bonds
- “Should I invest in stocks? In bonds?”
  - A: “Yes for stocks; Yes for bonds.”
  - B: “No for stocks; Yes for bonds.”
- Whose (which) advice should you listen to?
  - Who has an incentive to say buy stocks/bonds?

# Experts vs. Decision Maker

- Specialization in providing information:
  - Analysts-Investor
  - Experts-Government
  - Doctors-Patient
- Why ask for **Second Opinion**?
  1. Exploit **Differences in Conflict of Interest**
  2. Utilize Specialization in Information Collection?
- Battaglini (ECMA 2002)



# Experts vs. Decision Maker

- Battaglini (ECMA 2002)
  - FRE exists for 2 senders and multi-dim. Space
  - Completely overcomes strategic issue
- Is this robust?
  - Battaglini (2002): Yes (to noise)
  - Ambrus and Takahashi (2008): No (to continuity of out-of-equilibrium beliefs)
- Empirical evidence of FRE and robustness?

# Decision under 2D Uncertainty

- True State:

$$(H, V) \in \{L, R\} \times \{U, D\}$$

- Receiver Action:

$$(h, v) \in A = \{l, r\} \times \{u, d\}$$

$$(L, U) \rightarrow (l, u) \qquad (R, U) \rightarrow (r, u)$$

$$(L, D) \rightarrow (l, d) \qquad (R, D) \rightarrow (r, d)$$

回合

練習 2 / 3

剩餘時間 [秒]: 22

狀態: (L, U)

	(甲可獲得)	(乙可獲得)	(丙可獲得)	(甲可獲得)	(乙可獲得)	(丙可獲得)
若丙決定						
	左			右		
上	20	20	50	0	50	0
下	50	0	0	10	10	20

狀態: (L, D)

	(甲可獲得)	(乙可獲得)	(丙可獲得)	(甲可獲得)	(乙可獲得)	(丙可獲得)
若丙決定						
	左			右		
上	50	0	0	10	10	20
下	20	20	50	0	50	0

成員甲建議 左  
成員甲建議 下

狀態: (R, U)

	(甲可獲得)	(乙可獲得)	(丙可獲得)	(甲可獲得)	(乙可獲得)	(丙可獲得)
若丙決定						
	左			右		
上	0	50	0	20	20	50
下	10	10	20	50	0	0

狀態: (R, D)

	(甲可獲得)	(乙可獲得)	(丙可獲得)	(甲可獲得)	(乙可獲得)	(丙可獲得)
若丙決定						
	左			右		
上	10	10	20	50	0	0
下	0	50	0	20	20	50

成員乙建議 右  
成員乙建議 上

請決定:

左上

右上

左下

右下

回合

練習 2 / 3

剩餘時間 [秒]: 22

狀態: (L, U)

	(甲可獲得)	(乙可獲得)	(丙可獲得)	(甲可獲得)	(乙可獲得)	(丙可獲得)
若丙決定						
	左			右		
上	20	20	50	0	50	0
下	50	0	0	10	10	20

狀態: (R, U)

	(甲可獲得)	(乙可獲得)	(丙可獲得)	(甲可獲得)	(乙可獲得)	(丙可獲得)
若丙決定						
	左			右		
上	0	50	0	20	20	50
下	10	10	20	50	0	0

狀態: (L, D)

	(甲可獲得)	(乙可獲得)	(丙可獲得)	(甲可獲得)	(乙可獲得)	(丙可獲得)
若丙決定						
	左			右		
上	50	0	0	10	10	20
下	20	20	50	0	50	0

狀態: (R, D)

	(甲可獲得)	(乙可獲得)	(丙可獲得)	(甲可獲得)	(乙可獲得)	(丙可獲得)
若丙決定						
	左			右		
上	10	10	20	50	0	0
下	0	50	0	20	20	50

成員甲建議 左  
成員甲建議 下

成員乙建議 右  
成員乙建議 上

請決定:

左上

右上

左下

右下

# Decision under 2D Uncertainty

- Knowledgeable Sender(s) send messages:  

$$m_i \in M_i = \{“(l, u),”“(r, u),”“(l, d),”“(r, d)”\}$$
- $u_i^S(h, v|H, V)$  &  $u^R(\cdot)$  generally misaligned, but:
- Sender 1 and receiver
  - Fix  $V$ , if  $H=L$ , both prefer  $(l, *)$  to  $(r, *)$
  - and if  $H=R$ ,  $(r, *)$  to  $(l, *)$
- Sender 2 and receiver
  - Fix  $H$ , if  $V=U$ , both prefer  $(*, u)$  to  $(*, d)$
  - and if  $V=D$ ,  $(*, d)$  to  $(*, u)$

回合

練習 2 / 3

狀態：(L, U)

	(甲可獲得) 若丙決定 $S_1$	(乙可獲得) $S_2$	(丙可獲得) $R$	(甲可獲得)	(乙可獲得)	(丙可獲得)	(甲可獲得)	(乙可獲得)
上	20	20	50	0	50	0	0	50
下	50	0	0	10	10	20	10	10

Truth-telling is Incentive Compatible!

狀態：(L, D)

	(甲可獲得)	(乙可獲得)	(丙可獲得)	(甲可獲得)	(乙可獲得)	(丙可獲得)	(甲可獲得)	(乙可獲得)
上	50	0	0	10	10	20	10	10
下	20	20	50	0	50	0	0	50

回合

練習 2 / 3

狀態：(L, U)

	(甲可獲得) 若丙決定 $S_1$	(乙可獲得) $S_2$	(丙可獲得) $R$
上	20	20	50
下	50	0	0



Special Case: Alignment is Dominant  
 (Regardless of other message)

狀態：(L, D)

	(甲可獲得) 若丙決定	(乙可獲得) 左	(丙可獲得)
上	50	0	0
下	20	20	50

	(甲可獲得)	(乙可獲得) 右	(丙可獲得)
上	10	10	20
下	0	50	0

	(甲可獲得) 若丙決定	(乙可獲得) 左
上	10	10
下	0	50

# Decision under 2D Uncertainty

- Suppose there is only one sender (**Game S**)
- And the true state is  $(L, U)$
- If you were Sender 1, what would you “say”?

$$m_i \in \{(l, u), (r, u), (l, d), (r, d)\}$$

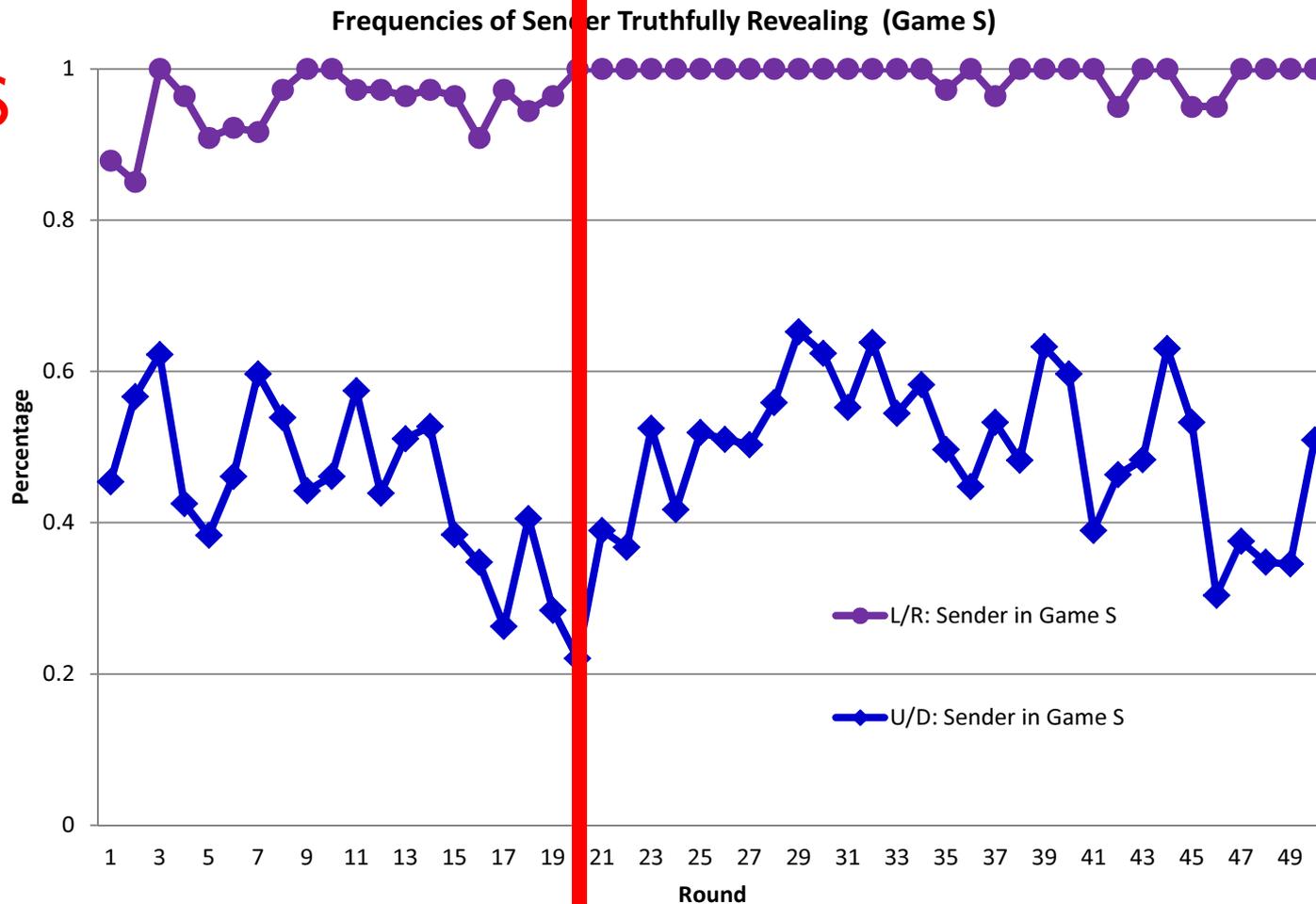
$$m_1 = (l, d)$$

- Wouldn't Receiver “figure this out”?
- How about mixing with...

$$m_1 = (l, u)$$

# Exactly What Sender 1's Did!

% of  
M=S

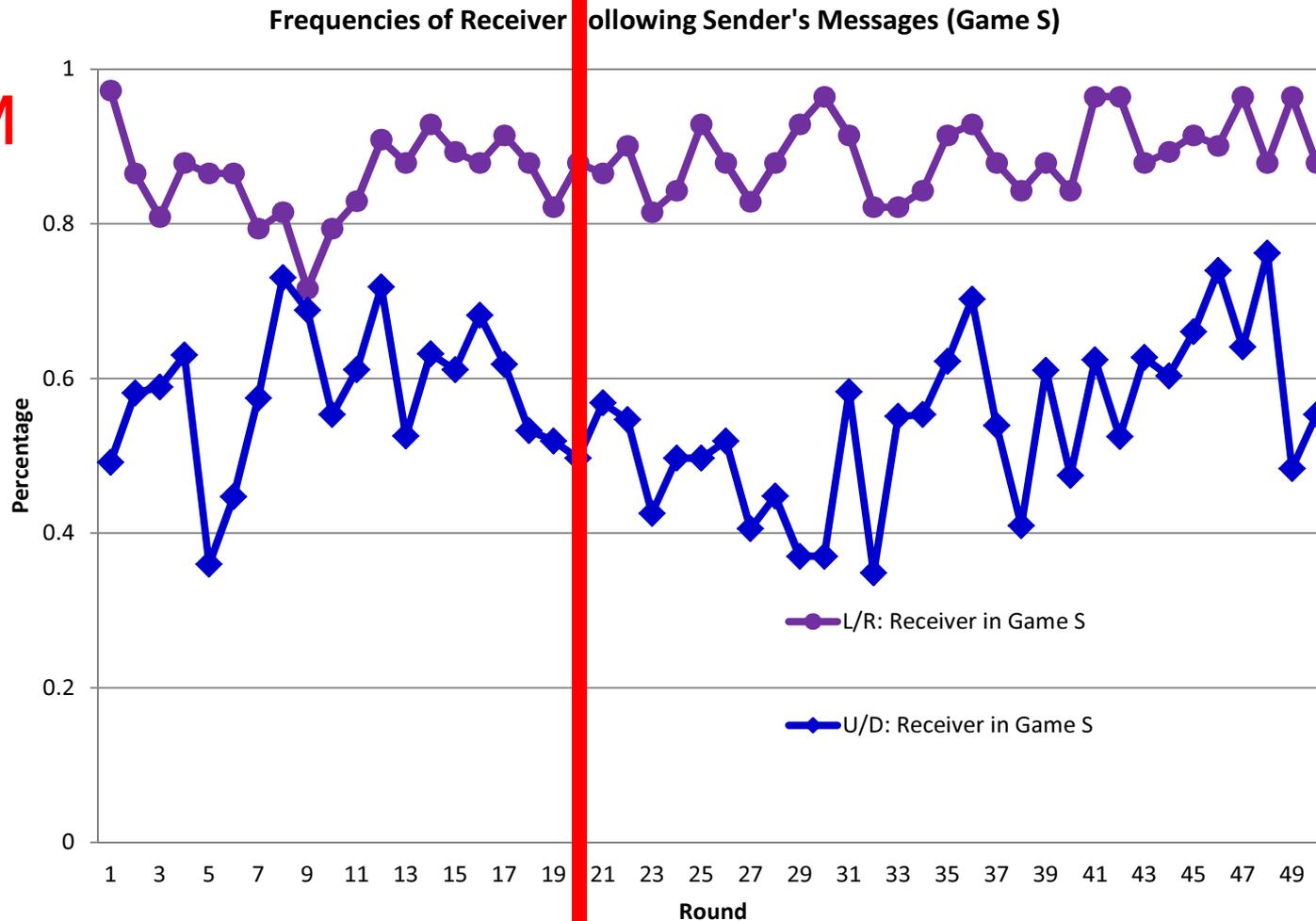


L/R  
(99%)

U/D  
(50%)

# How Do Receivers Respond?

% of  
 A=M

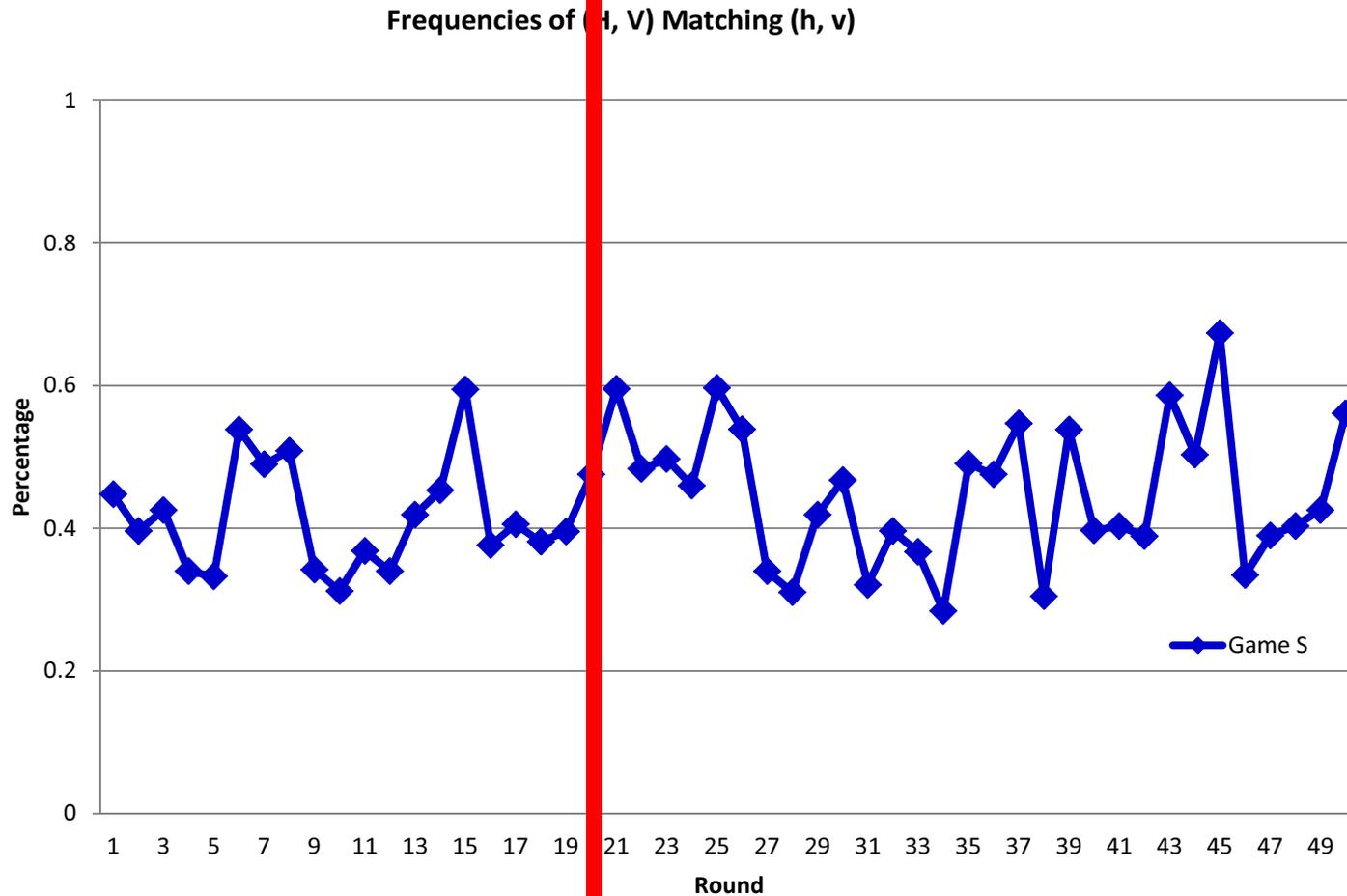


L/R  
 (89%)

U/D  
 (54%)

# Partially Revealing Equilibrium Outcome

% of  
 $A=S$



Action  
match  
State  
(45%)

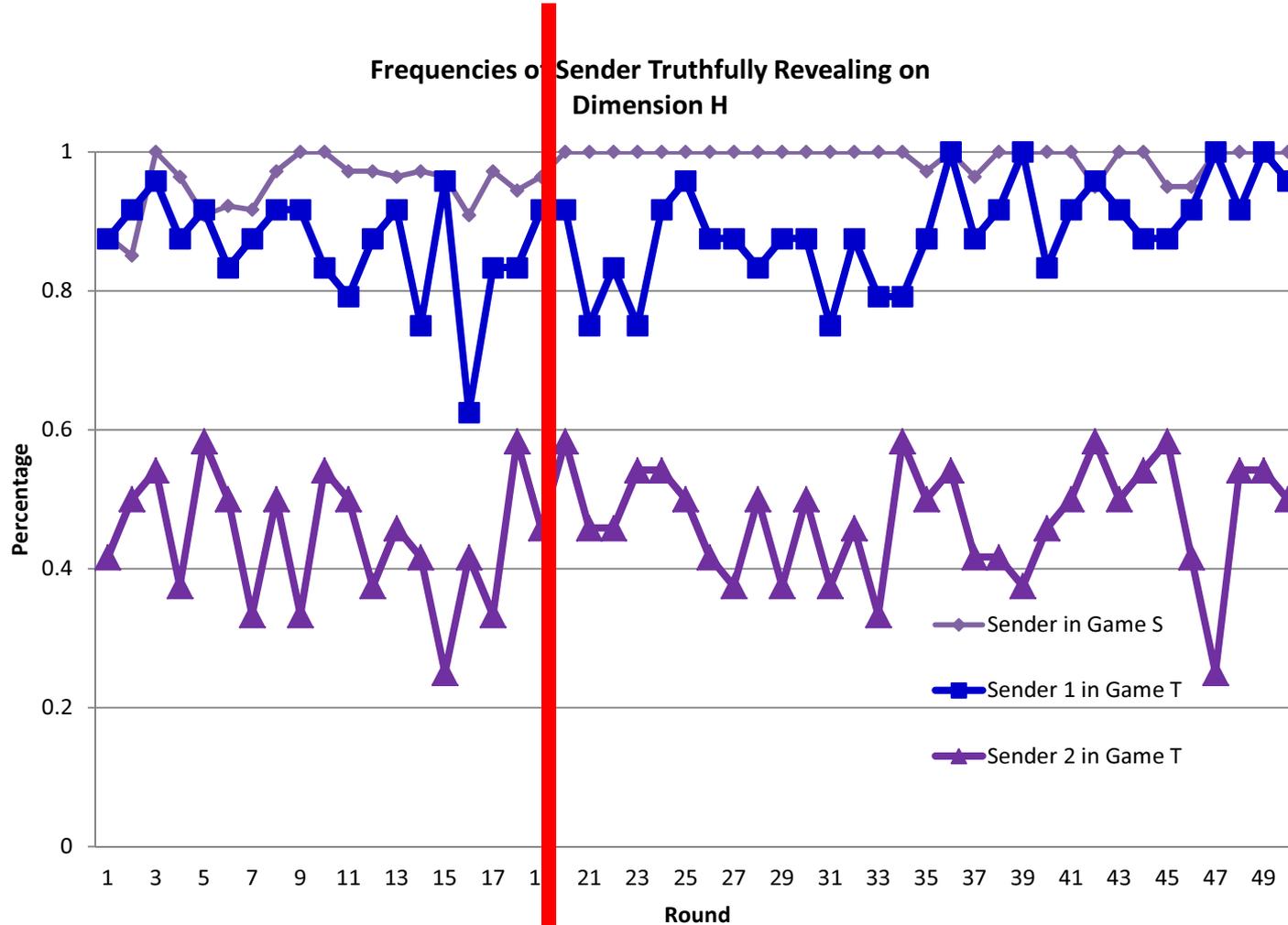
## Confirms: Proposition 1 (Game S)

- There exists a **partially revealing equilibrium** in Game S in which the single sender **truthfully reveals only on dimension  $H$** .
- Furthermore, the information partition  $\{\{(L, U) \mid \mathbf{L} \mid (L, D)\}, \{(R, U) \mid \mathbf{R} \mid (R, D)\}\}$
- that the receiver receives in the equilibrium is the **only** partition that is consistent with equilibrium.

**What If Two Senders?**

# Sender Behavior: Dimension H (L/R)

% of  
 $M=S$

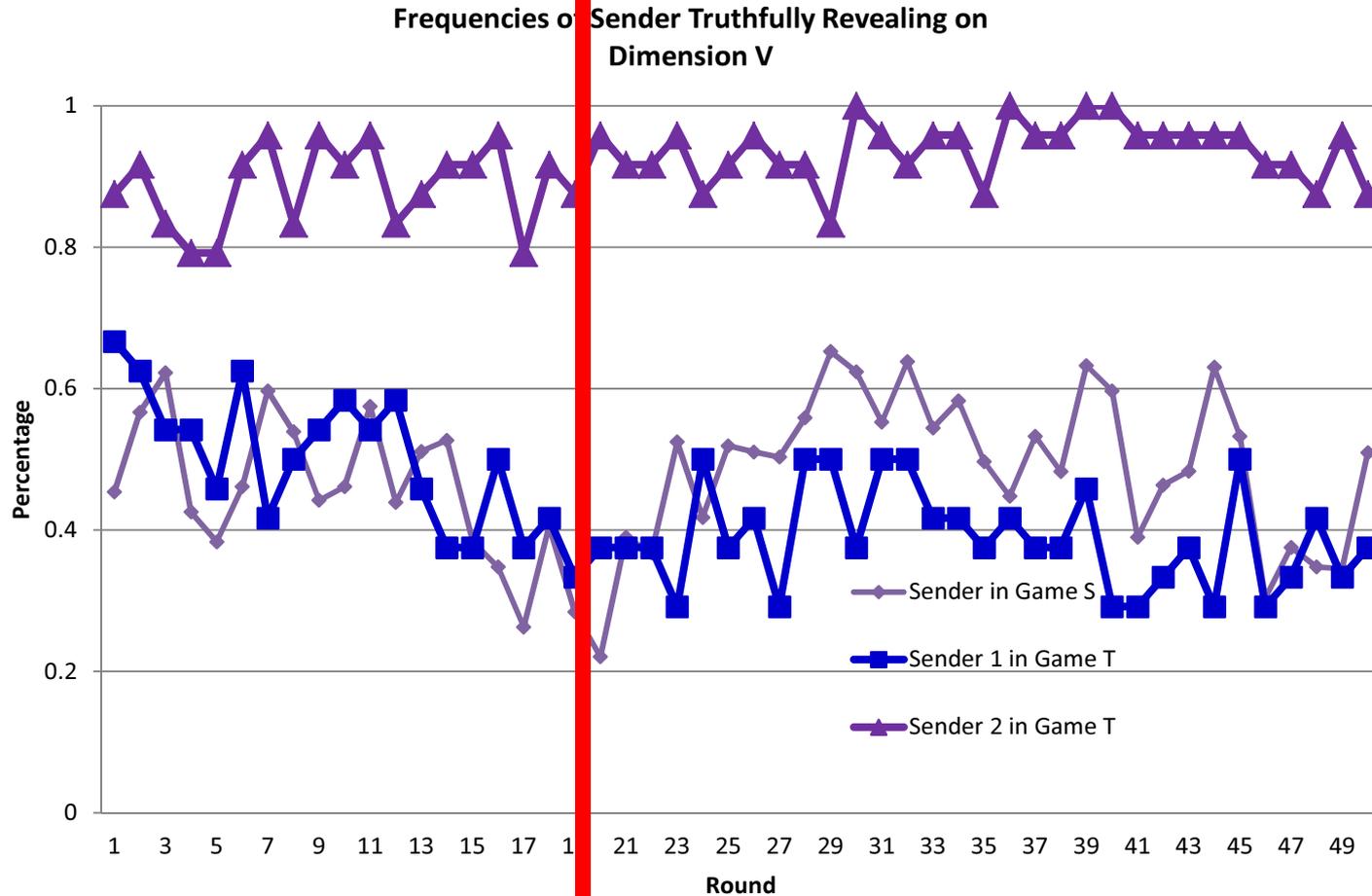


Sender 1  
(89%)

Sender 2  
(47%)

# Sender Behavior: Dimension V (U/D)

% of  
M=S

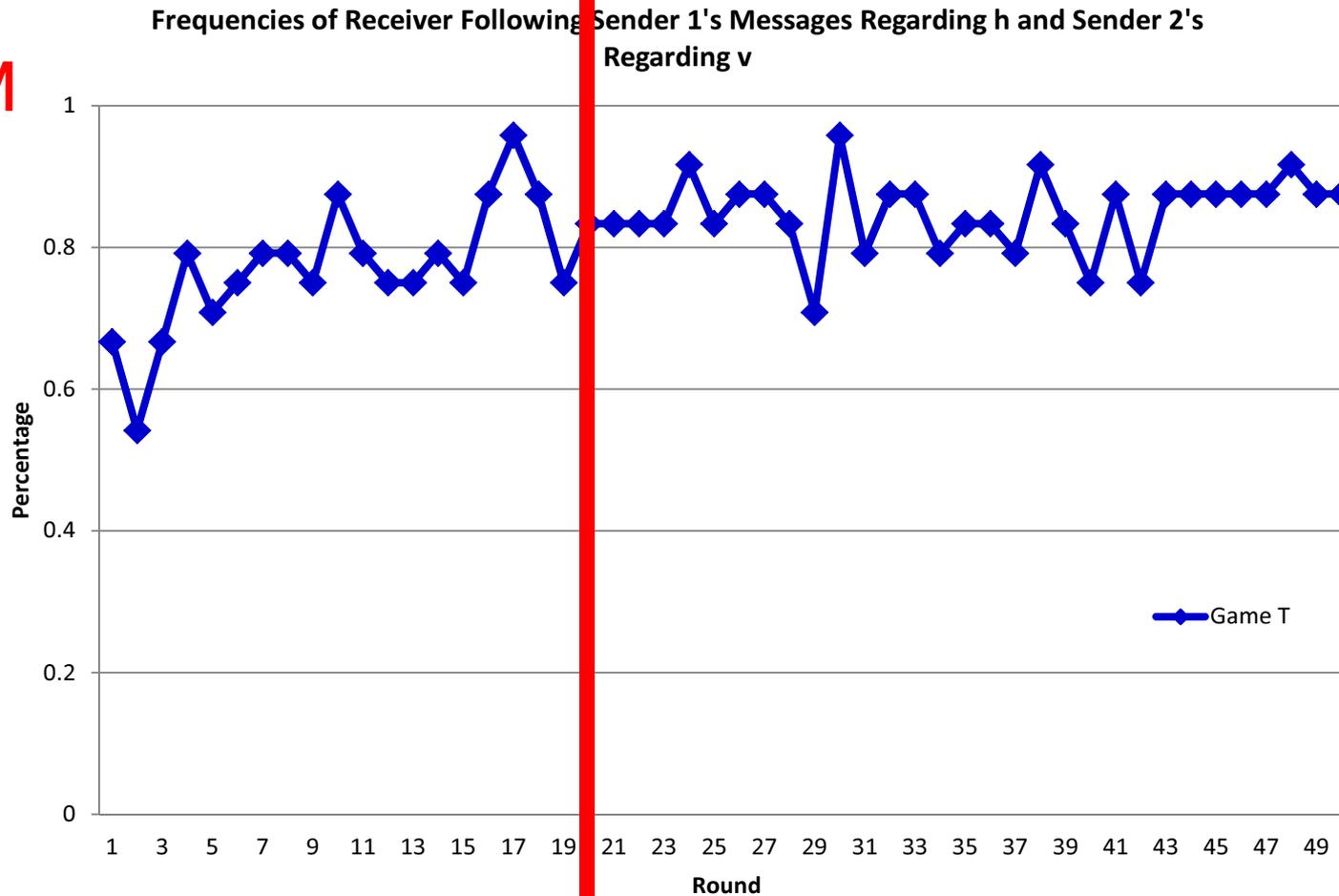


Sender 2  
(94%)

Sender 1  
(39%)

# Receivers Follow the “Right” Dimensions

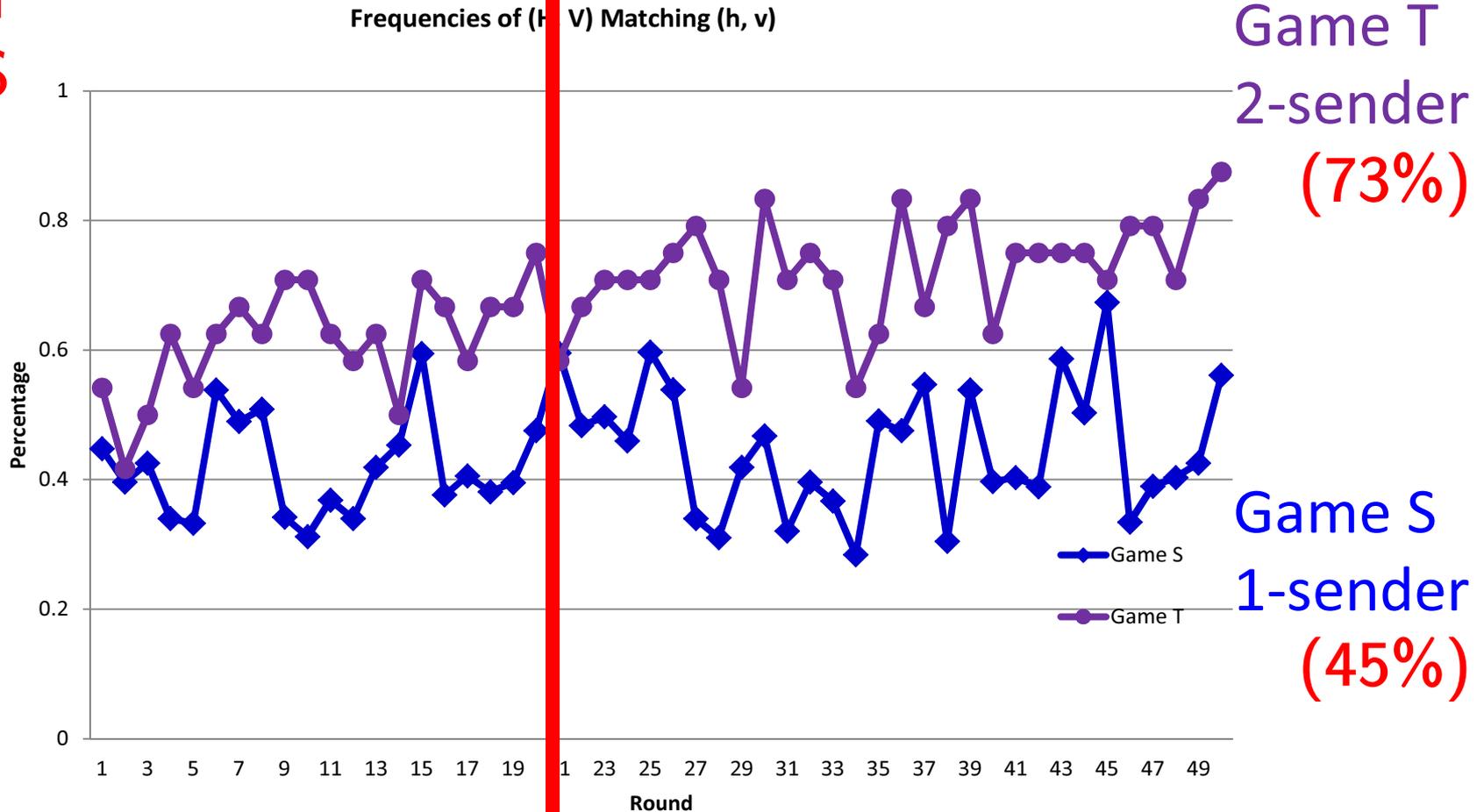
% of  
 $A=M$



Follow  
 $S1@H$   
 $S2@V$

# FRE: Frequency of State=Action

% of  
 $A=S$



## Confirms: Proposition 2 (FRE in Game T)

- There exists a fully revealing equilibrium (FRE) in Game T in which each sender truthfully reveals on at least one dimension.
- Two major classes of senders' strategy profiles that constitute a FRE are:
  - Both Sender 1 and Sender 2 truthfully reveal on both dimensions  $H$  and  $V$
  - Sender 1 truthfully reveals only on dimension  $H$  and Sender 2 only on dimension  $V$  (both babble by means of randomization on the other dimension)

# Summary: Laboratory Experiments

- **Four Treatments** (Between-Subject Design)
  - Baseline: Game S vs. Game T
  - Robustness: Game T' vs. Game R
- **Subjects:** 260 NTU students
  - @ Taiwan Social Science Experimental Laboratory (TASSEL) at National Taiwan University
- **Four Sessions Each:** 5-7 groups (or 9-group)
  - 3 practice + 50 real rounds, fixed role, random match
  - Averaged US\$26.91; ranging [\$12.43, \$45.50]

# Robustness of FRE

- What if we only allow Sender 1 to say “L/R” and Sender 2 to say “U/D”?
- Two Effects:
  - Receivers can no longer cross-check
  - Focus on FRE dimension
- **Game T'**: Restrict to FRE-dim messages
- **No “out-of-equilibrium” belief!**
- Also, Slightly Perturb Payoffs (to kill Dominance)

回合 練習 2 / 3 剩餘時間 [秒]: 22

狀態：(L, U)

		(甲可獲得)			(乙可獲得)			(丙可獲得)		
		左			右					
若丙決定	上	20	20	50	0	50	0			
	下	50	0	0	10	10	20			

狀態：(R, U)

		(甲可獲得)			(乙可獲得)			(丙可獲得)		
		左			右					
若丙決定	上	0	15	0	20	20	50			
	下	10	30	20	50	0	0			

狀態：(L, D)

		(甲可獲得)			(乙可獲得)			(丙可獲得)		
		左			右					
若丙決定	上	15	0	0	30	10	20			
	下	20	20	50	0	50	0			

狀態：(R, D)

		(甲可獲得)			(乙可獲得)			(丙可獲得)		
		左			右					
若丙決定	上	10	10	20	50	0	0			
	下	0	50	0	20	20	50			

成員甲建議 左  
 成員甲建議 下

成員乙建議 右  
 成員乙建議 上

請決定：

Similar results, but sharper... (skip)

左上

右上

左下

右下

# What If FRE Require Crazy Out-of-Eq Beliefs?

回合 練習 1 / 3 剩餘時間 [秒]: 0

狀態: (L, U)

		若丙決定 左			若丙決定 右		
		(甲可獲得)	(乙可獲得)	(丙可獲得)	(甲可獲得)	(乙可獲得)	(丙可獲得)
上		20	20	50	0	50	0
下		50	0	0	10	10	20

狀態: (R, U)

		若丙決定 左			若丙決定 右		
		(甲可獲得)	(乙可獲得)	(丙可獲得)	(甲可獲得)	(乙可獲得)	(丙可獲得)
上		0	15	0	20	20	50
下		10	30	20	50	0	0

狀態: (L, D)

		若丙決定 左			若丙決定 右		
		(甲可獲得)	(乙可獲得)	(丙可獲得)	(甲可獲得)	(乙可獲得)	(丙可獲得)
上		15	0	0	30	10	20
下		20	20	50	0	50	0

你是 成員甲  
 本回合狀態 (L, U)

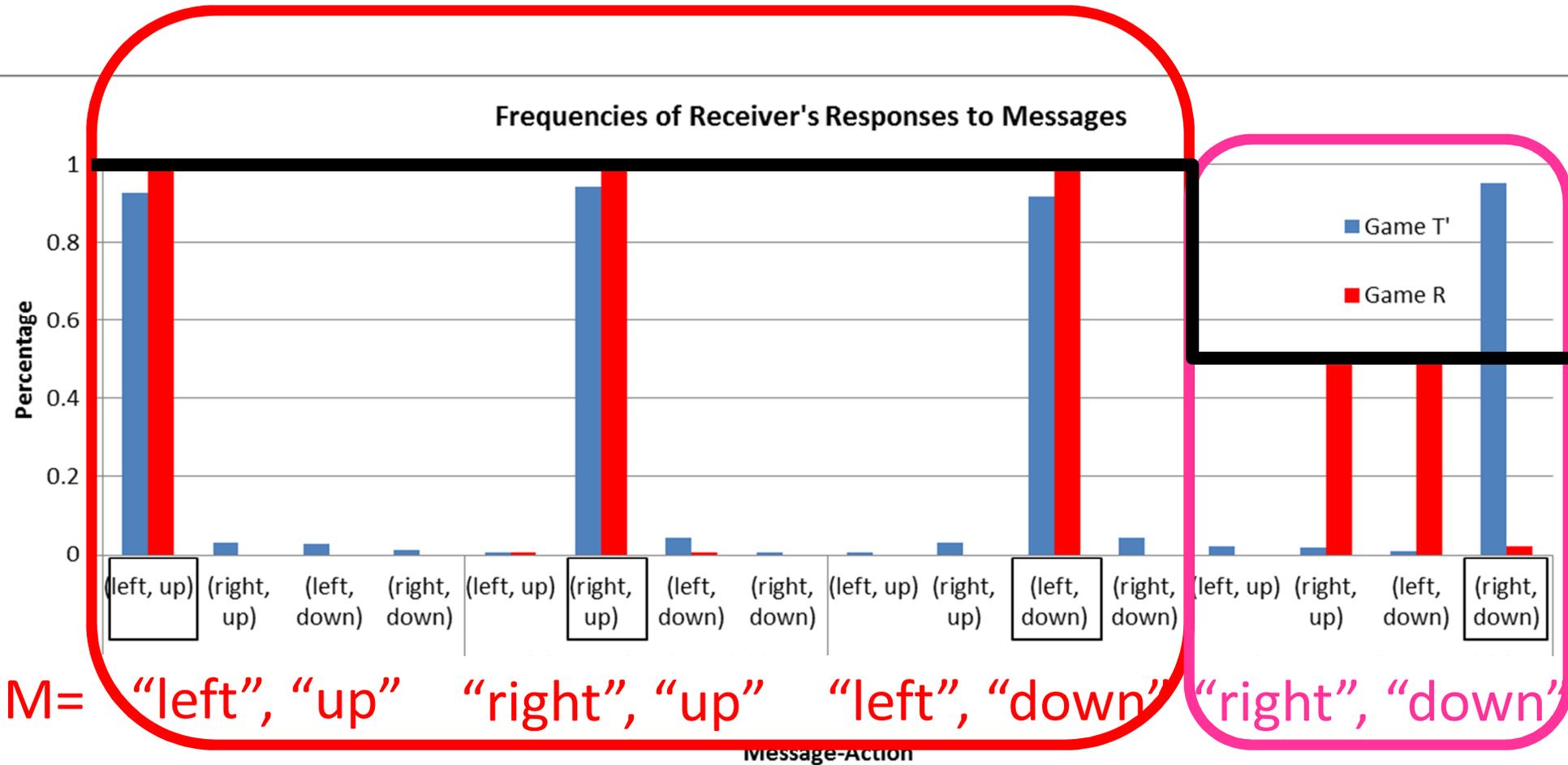
State (R,D)  
 doesn't exist!

FRE supported by believing  
 message (r,d) "means" (L,U)

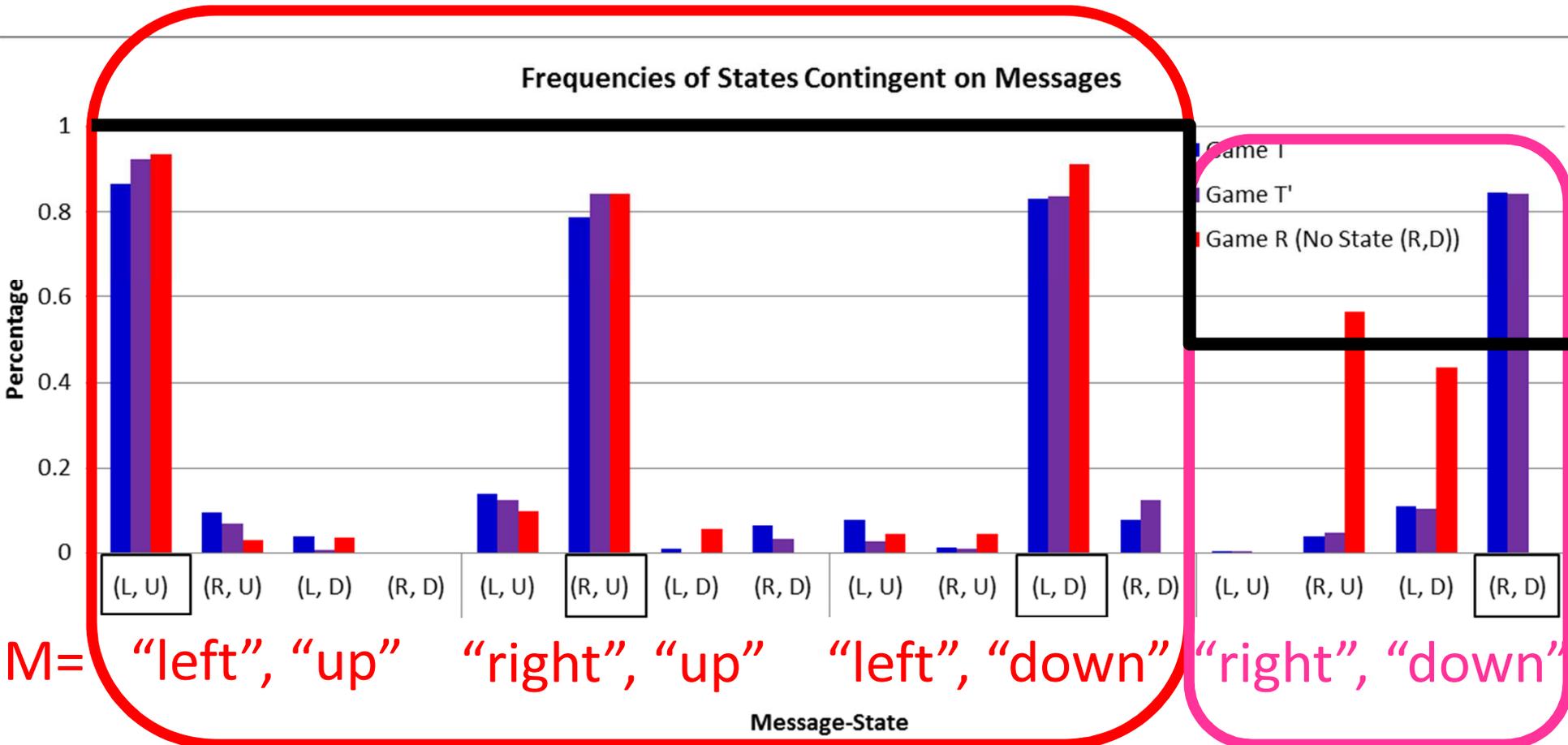
## Proposition 4 (Game R)

- A **FRE** exists in Game R if and only if it is supported by out-of-equilibrium beliefs that induce the receiver to
- **take action (left, up) with probability  $\geq \frac{1}{2}$**
- after receiving irreconcilable message pair  $(r,d)$ 
  - “Believing” message  $(r,d)$  “means”  $(L,U)$
- Is this FRE “robust”?
- Can people really play this weird FRE?

# Game R Receivers: Follow M Except (R,D)



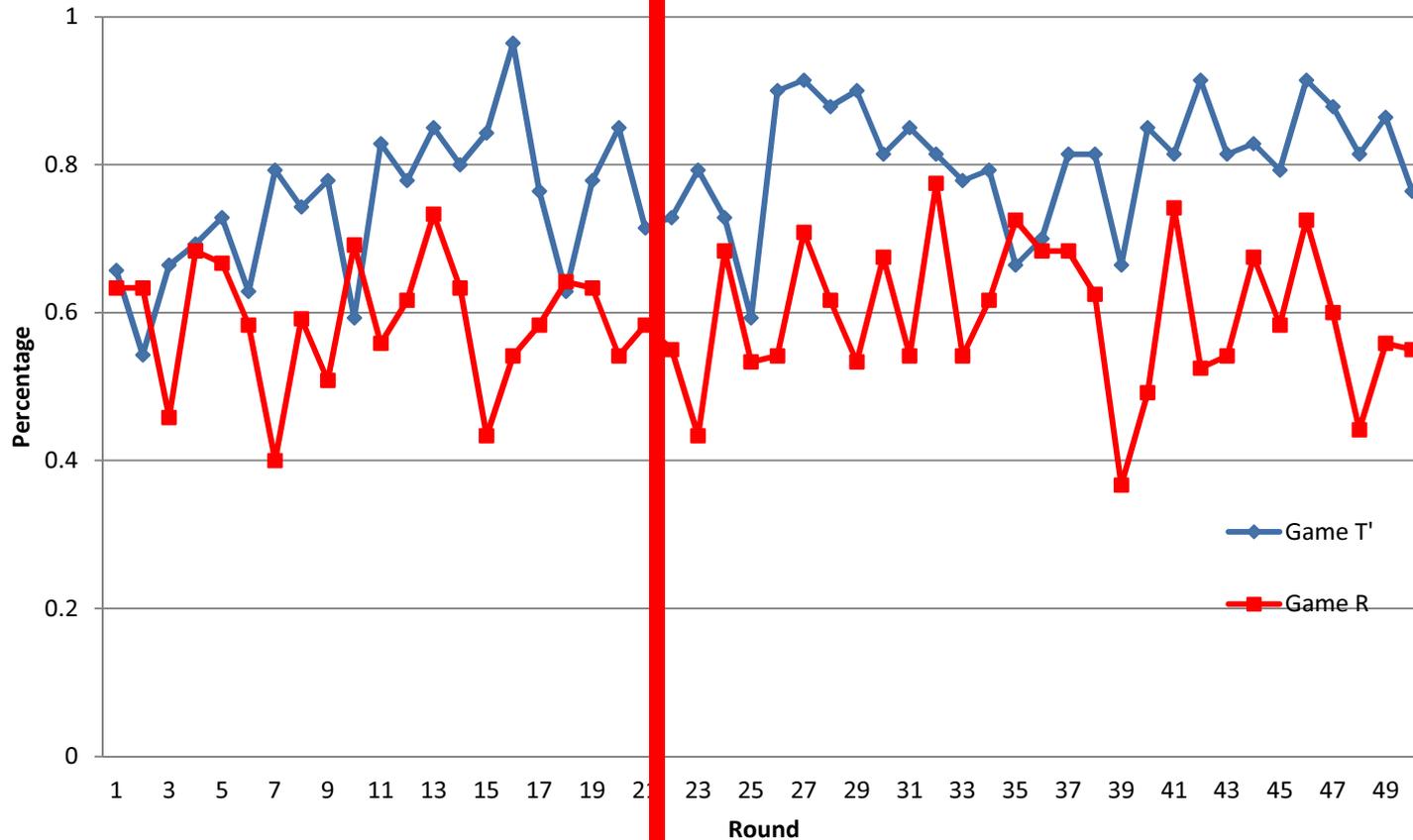
# Game R: Beliefs about (R,D) are Correct!



# “Don't Play” FRE: Frequency of $A=M=S$

% of  $A=M=S$

Frequencies of (H, V) Matching ("h", "v")  
and ("h", "v") Matching (h, v)



Game T'  
(80%)  
Game R  
(60%)

# Individual Subject Data

- Honest senders always tell the truth
  - 43% senders in the experiment are honest
- Strategic Sender 1
  - State (L,U) - “left”      State (R,U) - “right”
  - State (L,D) - “right”
- Strategic Sender 2
  - State (L,U) - “up”      State (R,U) - “down”
  - State (L,D) - “down”

## Proposition 5 (Game R)

- Sender **nonstrategic honest** with probability  $\lambda$ 
  - Common knowledge
- there exists a **robust partially revealing equilibrium** in which the receiver follows recommendations unless it is **irreconcilable**
  - Uniformly randomizes between actions (**right, up**) & (**left, down**) if she receives the irreconcilable message pair (“right”, “down”).

## Also: Proposition 6 (Game S, T, T')

- Sender is **nonstrategic truthful** with prob.  $\lambda$ 
  - Common knowledge
- The unique informative equilibrium outcome of **Game S** in the original model:
  - Sender truthfully reveals only on dimension **H**
- still survives for any  $\lambda \leq \frac{1}{2}$
- The FRE in **Game T and T'** survives for any  $\lambda$

# Conclusion

- Can we implement FRE in 2-sender, 2D state space setting?
- Qualified YES!
  - Simplest case, 50 rounds of learning, etc.
- Crazy out-of-equilibrium beliefs don't "work"
- Behavioral Model with some **nonstrategic, honest Senders** explains this...
- Next Step: What about other FREs?