Games with Incomplete Information
資訊不全賽局

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2010/10/8

(Lecture 6, Micro Theory I-2)

Games with Incomplete Information

- One or more players know preferences only probabilistically (cf. Harsanyi, 1976-77)
- Player i of Type $t_i \in \mathcal{T}_i = \{1, \cdots, T_i\}$
- Market Entry Game (of Section 9.2)
  - Entrant chooses Enter or Out
  - Incumbent chooses Fight or Share
- Both players choose before knowing how strong is player 1 (entrant)'s financial backing

Market Entry Game with Incomplete Information

If Entrant's backing is Weak

Agent 2: Incumbent

Agent 1: Entrant		Fight	Share
	Enter	-2, 4	3, 3
	Out	0, 6	0, 6

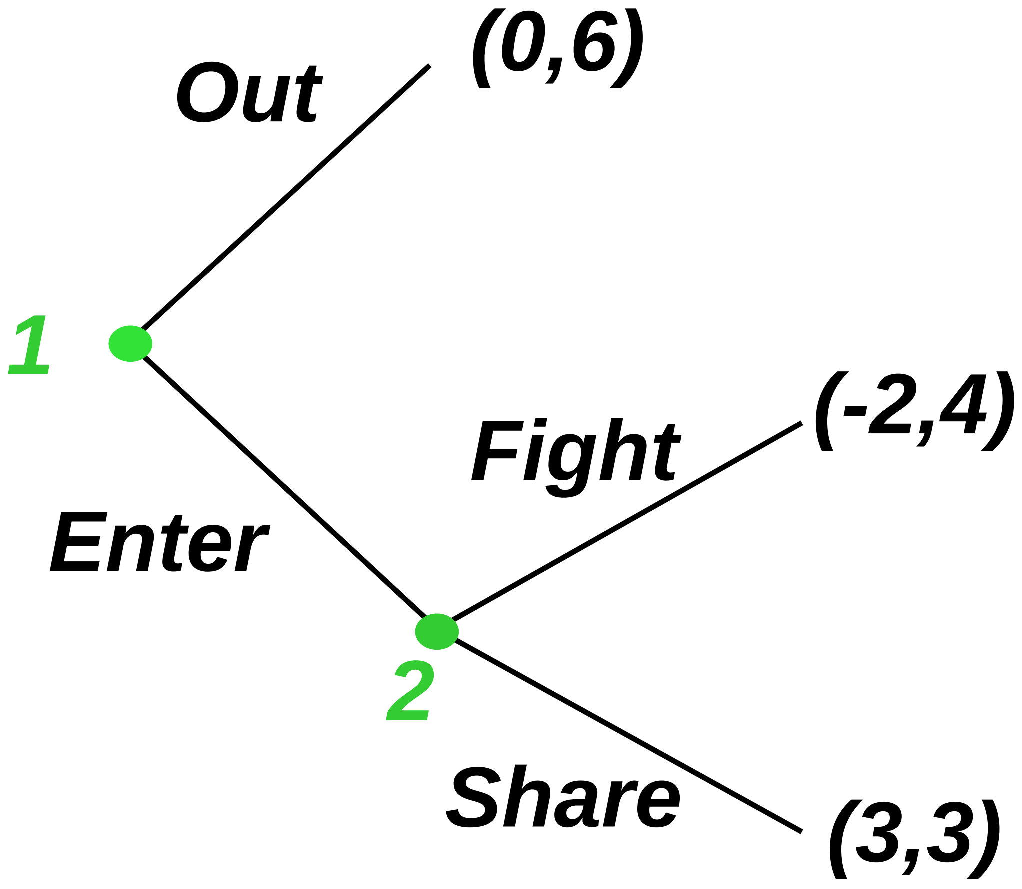
Market Entry Game with Incomplete Information

If Entrant's backing is Strong

Agent 2: Incumbent

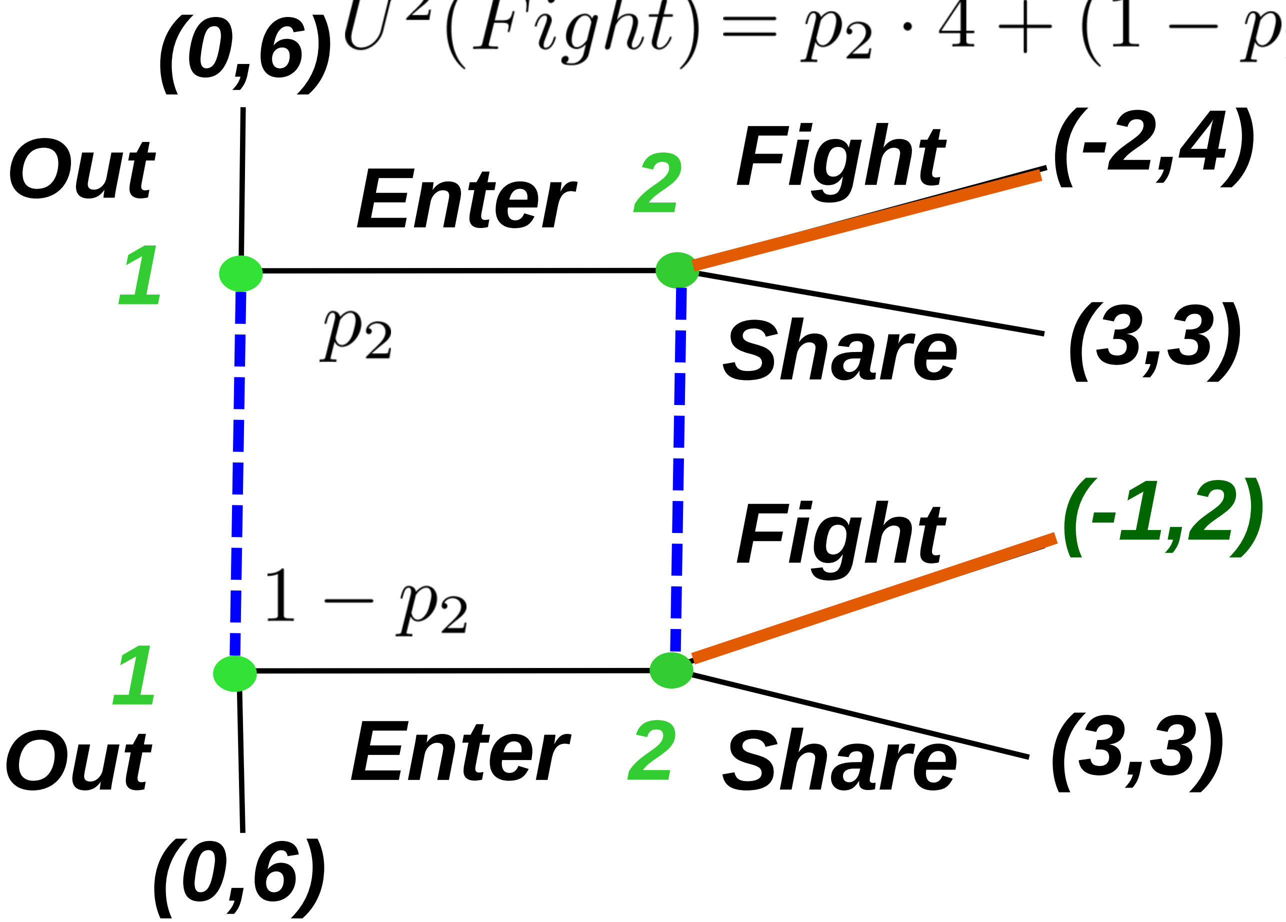
Agent 1: Entrant		Fight	Share
	Enter	**-1, 2**	3, 3
	Out	0 , 6	0 , 6

Market Entry Game with Incomplete Information



Market Entry Game with Incomplete Information

$$U^2(Fight) = p_2 \cdot 4 + (1 - p_2) \cdot 2 = 2 + 2p_2$$



Market Entry Game with Incomplete Information

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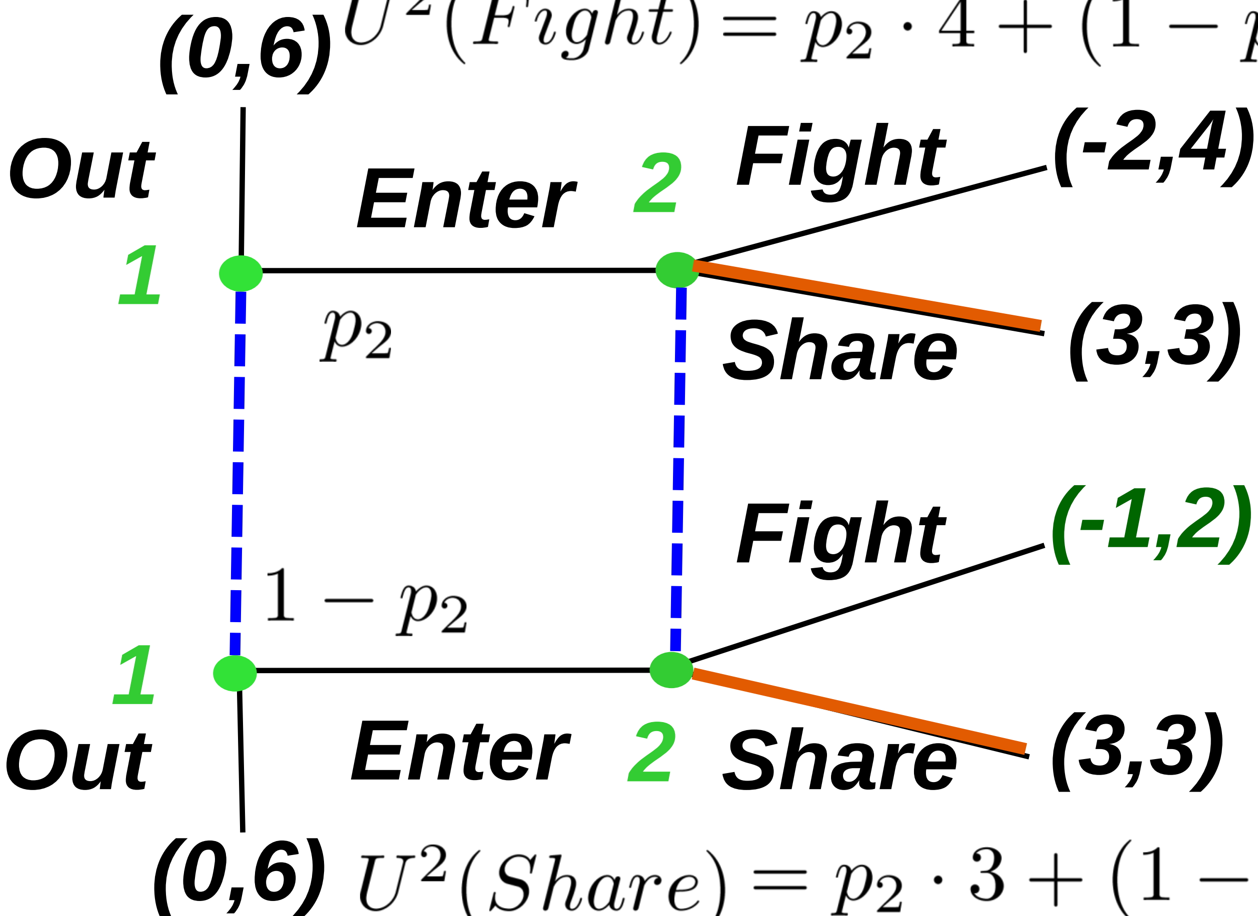


$Share$ if $p_2 < 1/2$

$Fight$ if $p_2 > 1/2$

$$U^2(Share) = p_2 \cdot 3 + (1 - p_2) \cdot 3 = 3$$

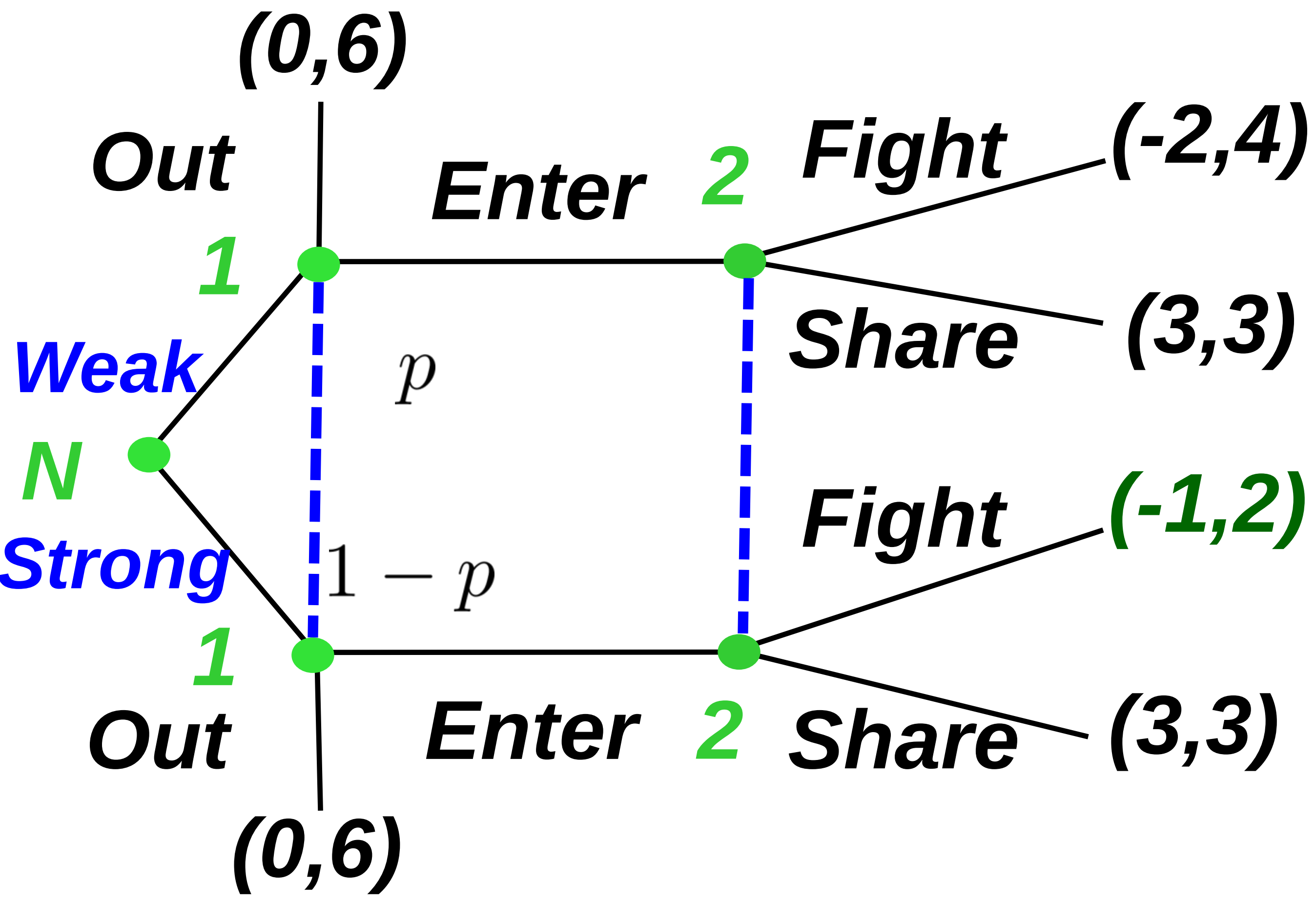
Bayesian Nash Equilibrium (BNE)

- Let $U^i(s;t_i), s \in \mathcal{S}$ be the payoffs of player $i \in \mathcal{I}$
- If his type is $t_i \in \mathcal{T}_i = 1, \cdots, T_i$
- Let $f(t_1, \cdots, t_I)$ be the joint distribution over types, which common knowledge. Then, a
- strategy profile is a Bayesian Nash equilibrium
- If player *i*'s strategy is a BR at each decision node that is reached with positive probability
  - given the common knowledge beliefs

Bayesian Nash Equilibrium (BNE)

- As if Nature moves in stage 0 to choose
  - Player types $(t_1, \cdots, t_I) \in \mathcal{T}_1 \times \cdots \times \mathcal{T}_I$
- Nature's payoffs same for all outcomes
- It is a BR to play mixed strategy $f(t_1, \cdots, t_I)$
- BNE of the *I*-player game is NE of the (*I*+*1*)-player game (with Nature moving first)
  - All existence theorems apply…

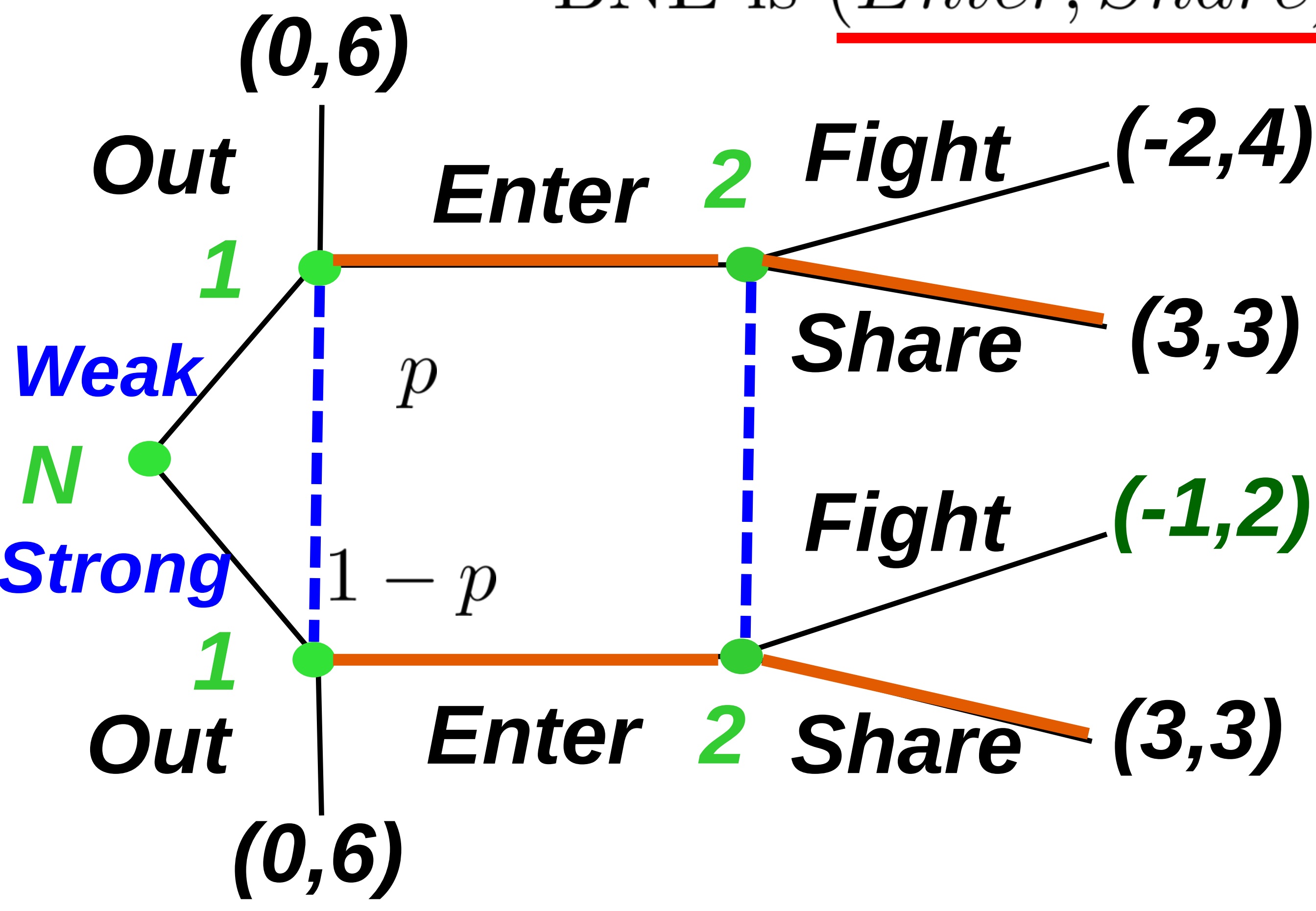
Market Entry Game with Incomplete Information



BNE with Player 2 Choosing *Share*: BR of Player 2

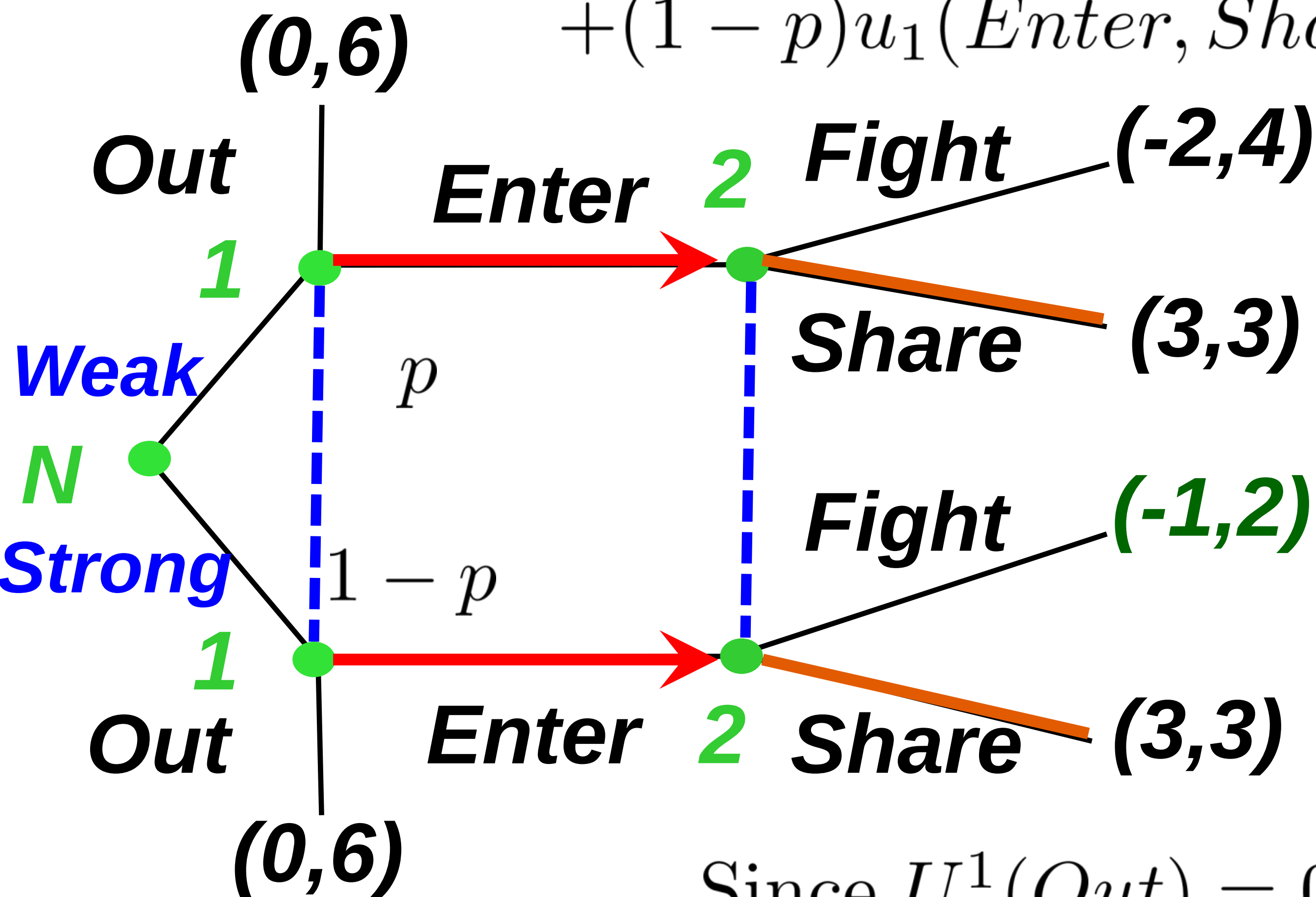
BNE is $(Enter, Share)$

$(\text{action of } 1,$
$\text{action if } Enter)$



BNE with Player 2 Choosing Share: BR of Player 1

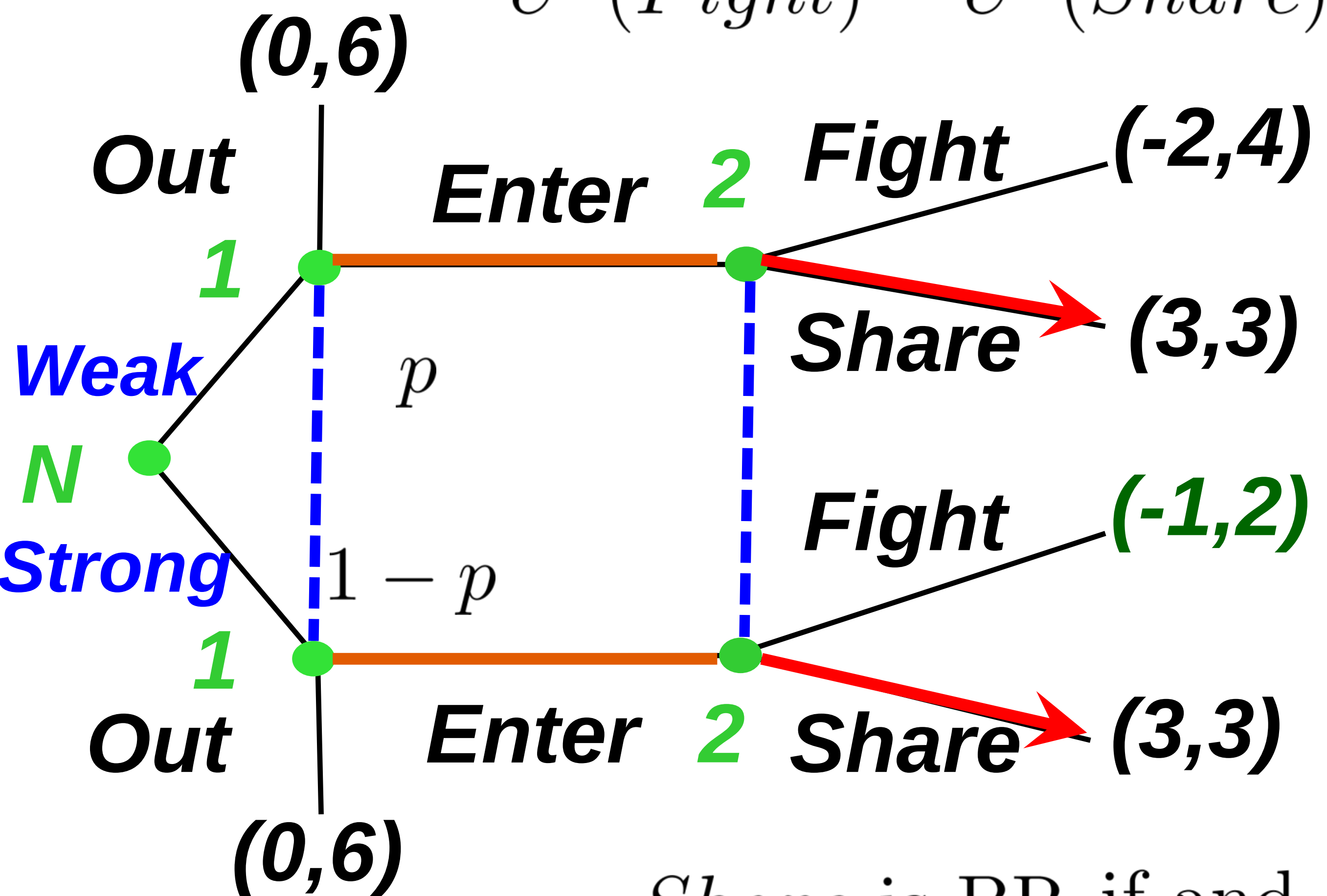
$$U^1(Enter) = pu_1(Enter, Share; Weak) + (1-p)u_1(Enter, Share; Strong) = 3$$



Since $U^1(Out) = 0$, $Enter$ is BR

BNE with Player 2 Choosing *Share*: BR of Player 2

$$U^2(Fight) - U^2(Share) = (2+2p) - 3$$
$$= 2p - 1$$

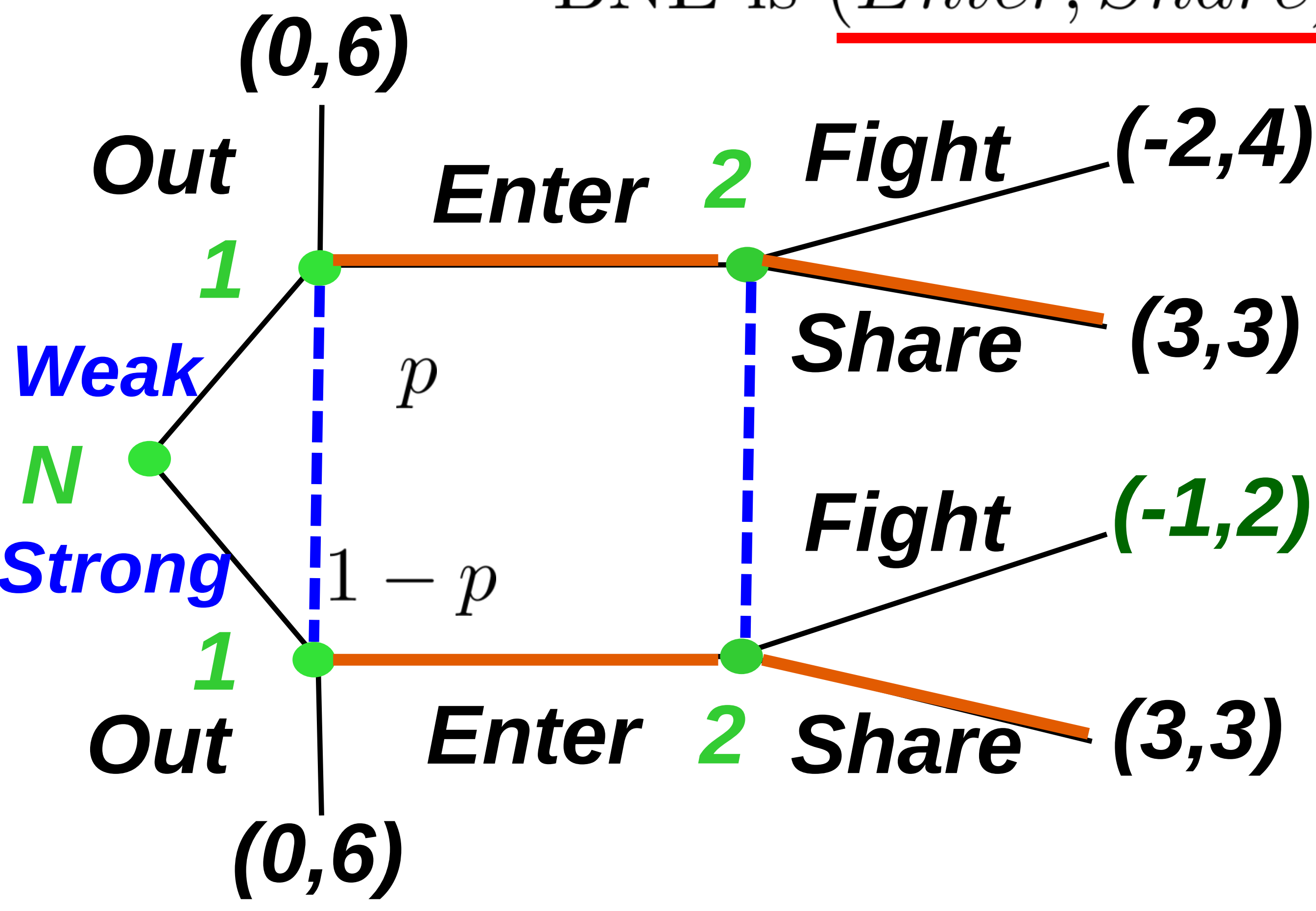


$Share$ is BR if and only if $p \leq 1/2$

BNE with Player 2 Choosing *Share*: BR of Player 2

BNE is $(Enter, Share)$

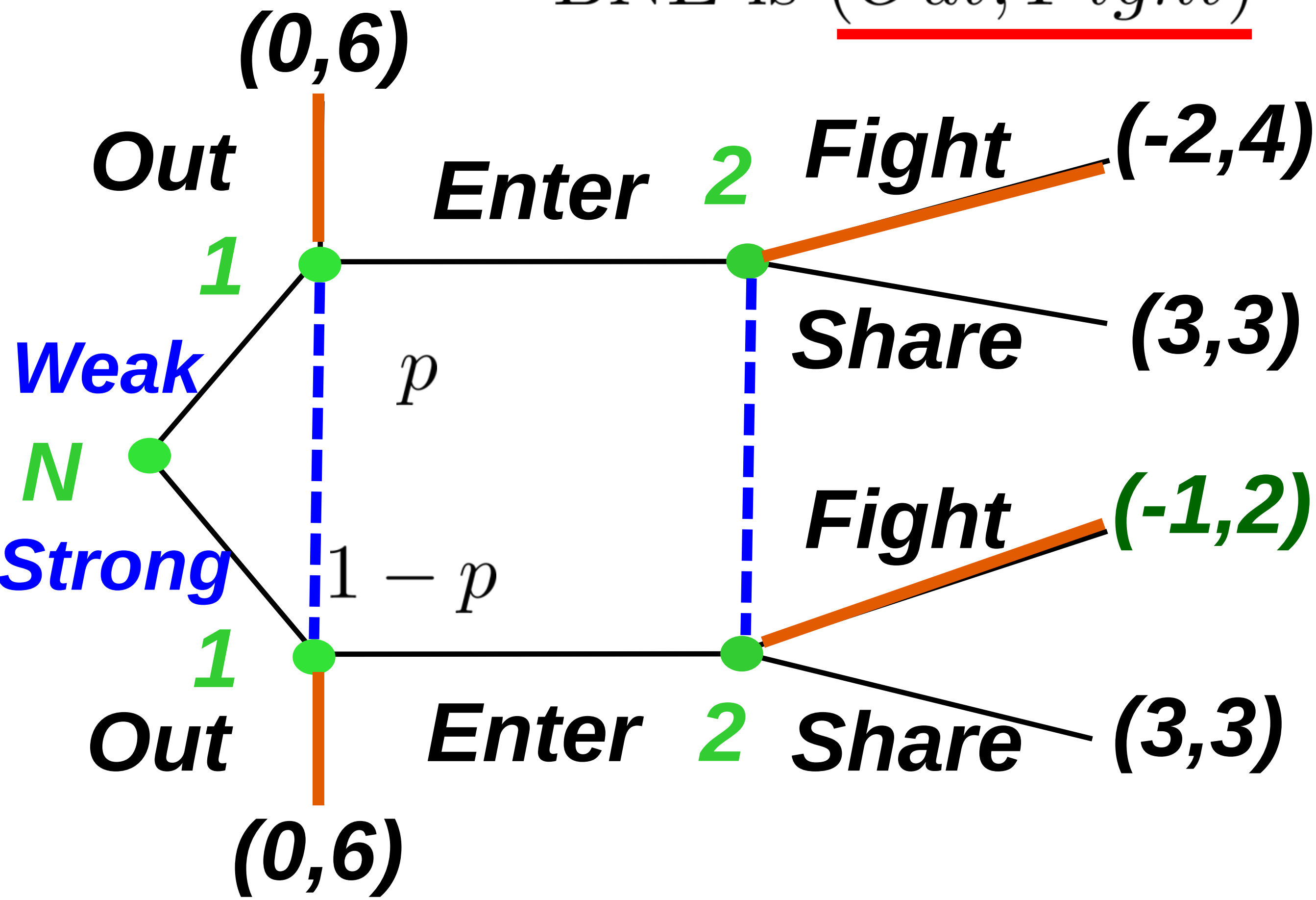
$($action of 1, action if $Enter)$



BNE with Player 2 Choosing *Fight*: BR of Player 1

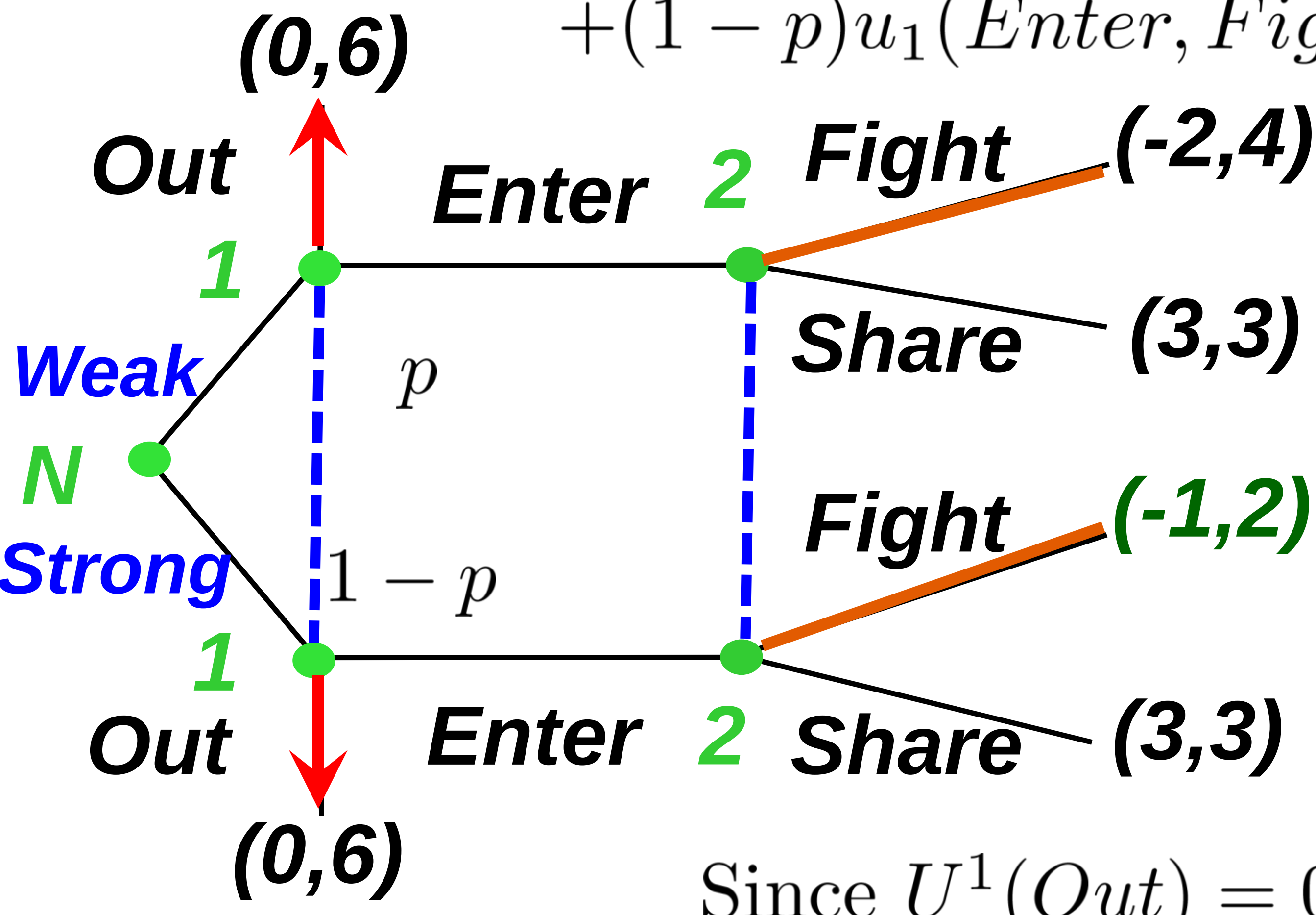
BNE is $(Out, Fight)$

(action of 1, action if $Enter$)



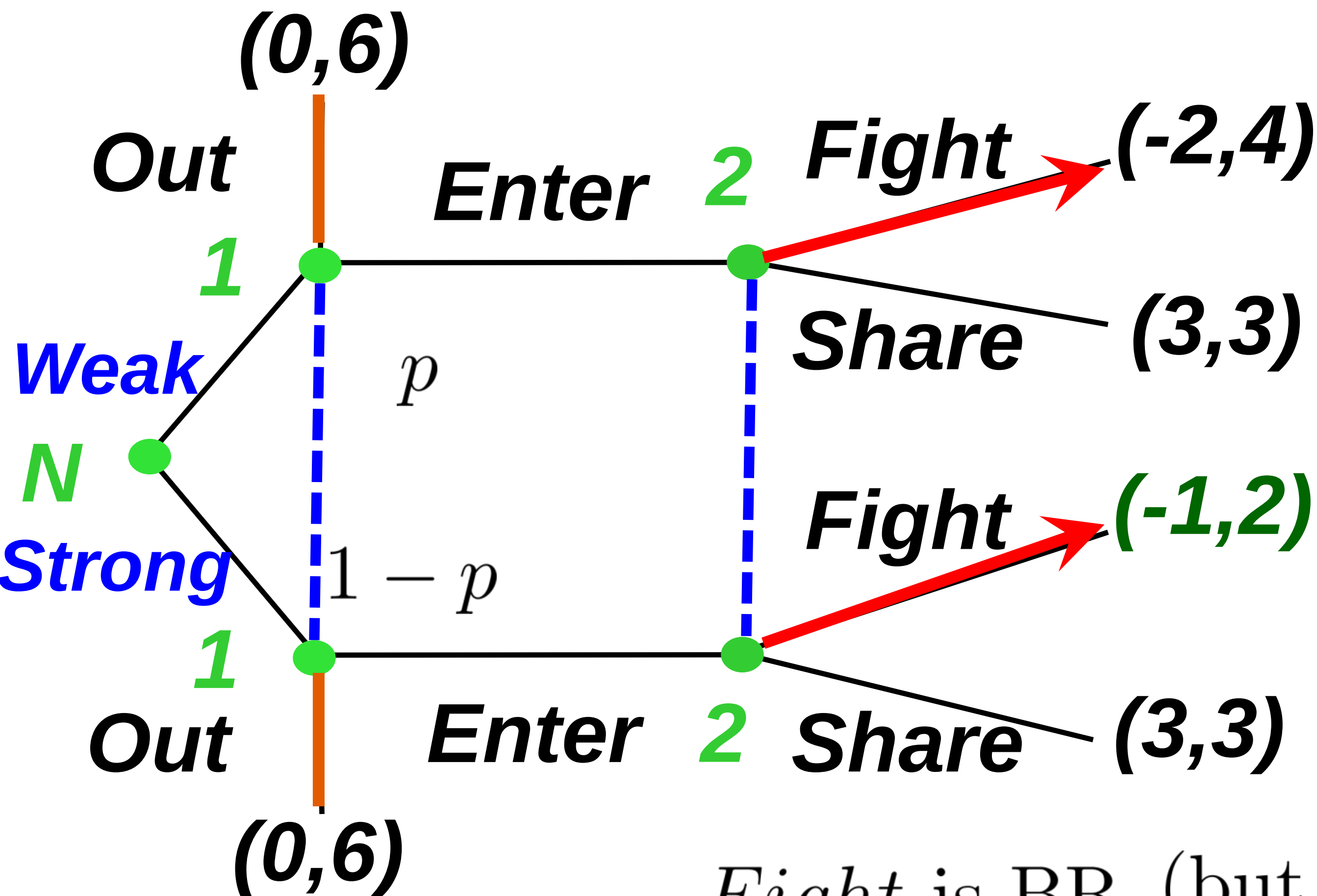
BNE with Player 2 Choosing *Fight*: BR of Player 1

$$U^1(Enter) = pu_1(Enter, Fight; Weak) + (1-p)u_1(Enter, Fight; Strong) < 0$$



Since $U^1(Out) = 0$, Out is BR

BNE with Player 2 Choosing *Fight*: BR of Player 2

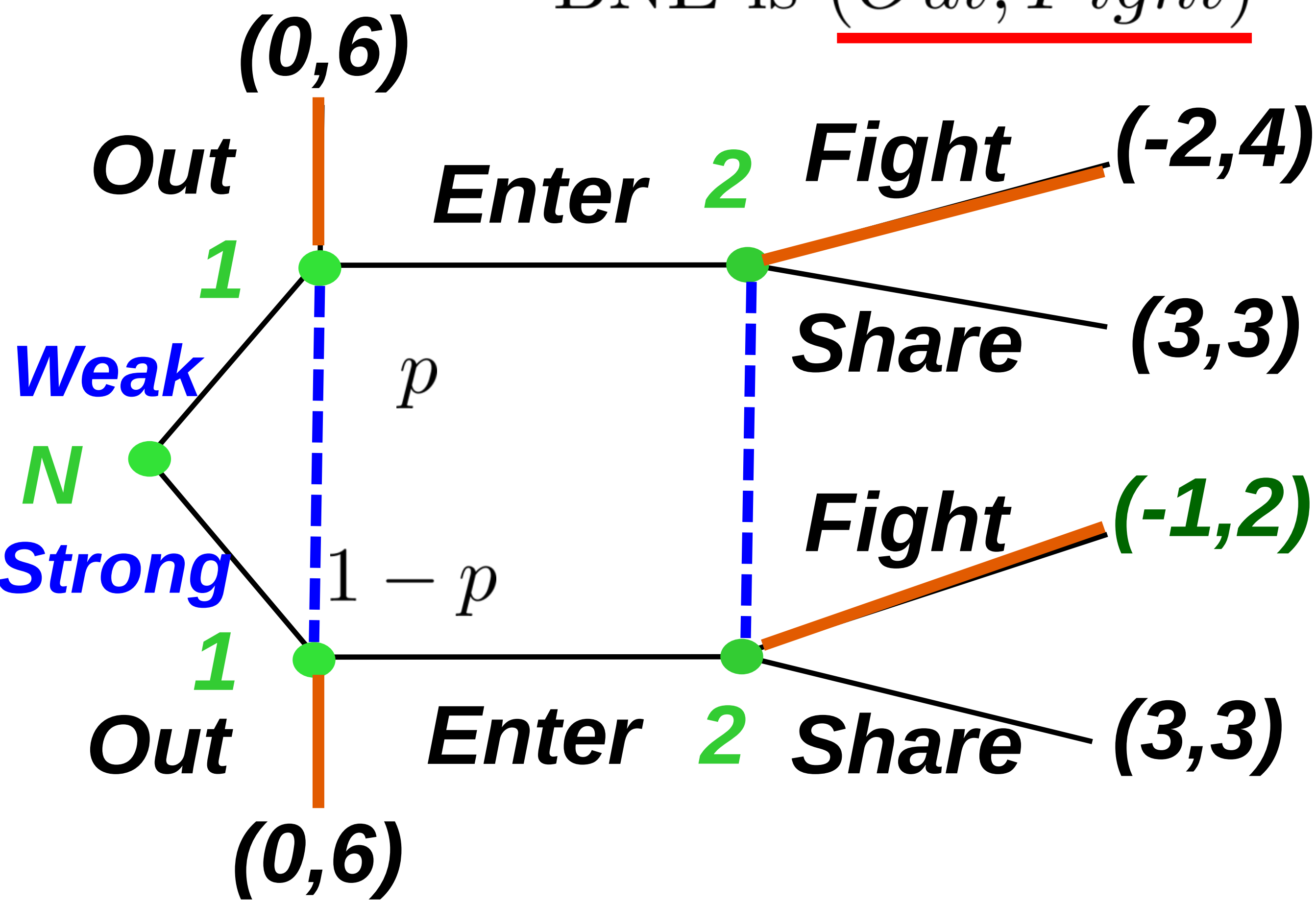


$Fight$ is BR (but never "tested")

BNE with Player 2 Choosing *Fight*: BR of Player 1

BNE is $(Out, Fight)$

(action of 1, action if $Enter$)



Empty Threats Off the Equilibrium Path

- If $p \leq 1/2$, Incumbent would not want to *Fight*
- Not a "Sensible" Equilibrium…
- Problem due to "crazy" beliefs that are:
- Off the Equilibrium Path: nodes that are not reached in equilibrium
  - Not reached = Zero probability? Yes here, but not true with continuous types… Comparison:
- On the Equilibrium Path: nodes that are reached in equilibrium

Trembling-Hand Perfect Equilibrium

- To rule out "crazy" equilibrium, can perturb the BNE by making them completely mixed:
  - Consider a game with T stages
- Set of feasible actions at stage t is A_t (finite)
- For the BNE $\overline{\pi} = (\overline{\pi}_1, \cdots, \overline{\pi}_T)$
- Consider a sequence of completely mixed strategies $\{\pi^k\}_{k=1}^{\infty} \to \overline{\pi}$ (trembles)
  - All nodes are reached (and tested in the BNE)
  - No more "crazy" beliefs off the equilibrium path…

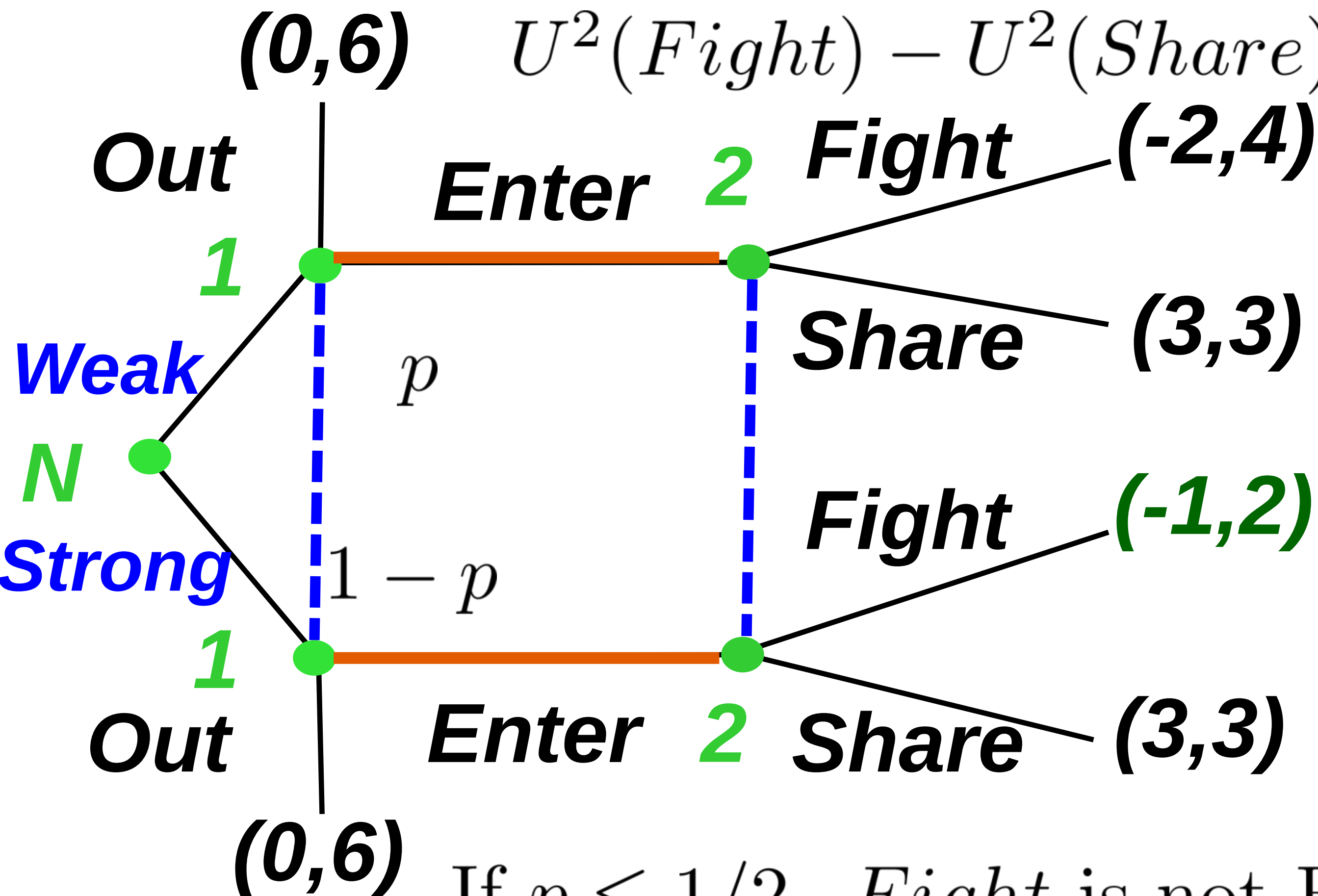
Trembling-Hand Perfect Equilibrium

- A BNE is Trembling-Hand Perfect (THP) if
- There exists some sequence of completely mixed strategy profiles $\{\pi^k\}_{k=1}^{\infty}$
- Converging to the equilibrium strategies, s. t.
- For all sufficiently large k, the equilibrium strategies are BR
  - Note: If a sequence of Logit QRE converges to a BNE, would the BNE automatically be THP?
  - QRE solves this by construct since it is completely mixed already…

BNE with Player 2 Choosing *Fight*: Not THP

For all $\{\pi^k\}_{k=1}^{\infty}$, $Enter$ with error probability ϵ_1^k

$$U^2(Fight) - U^2(Share) = (2+2p) - 3$$
$$= 2p - 1$$

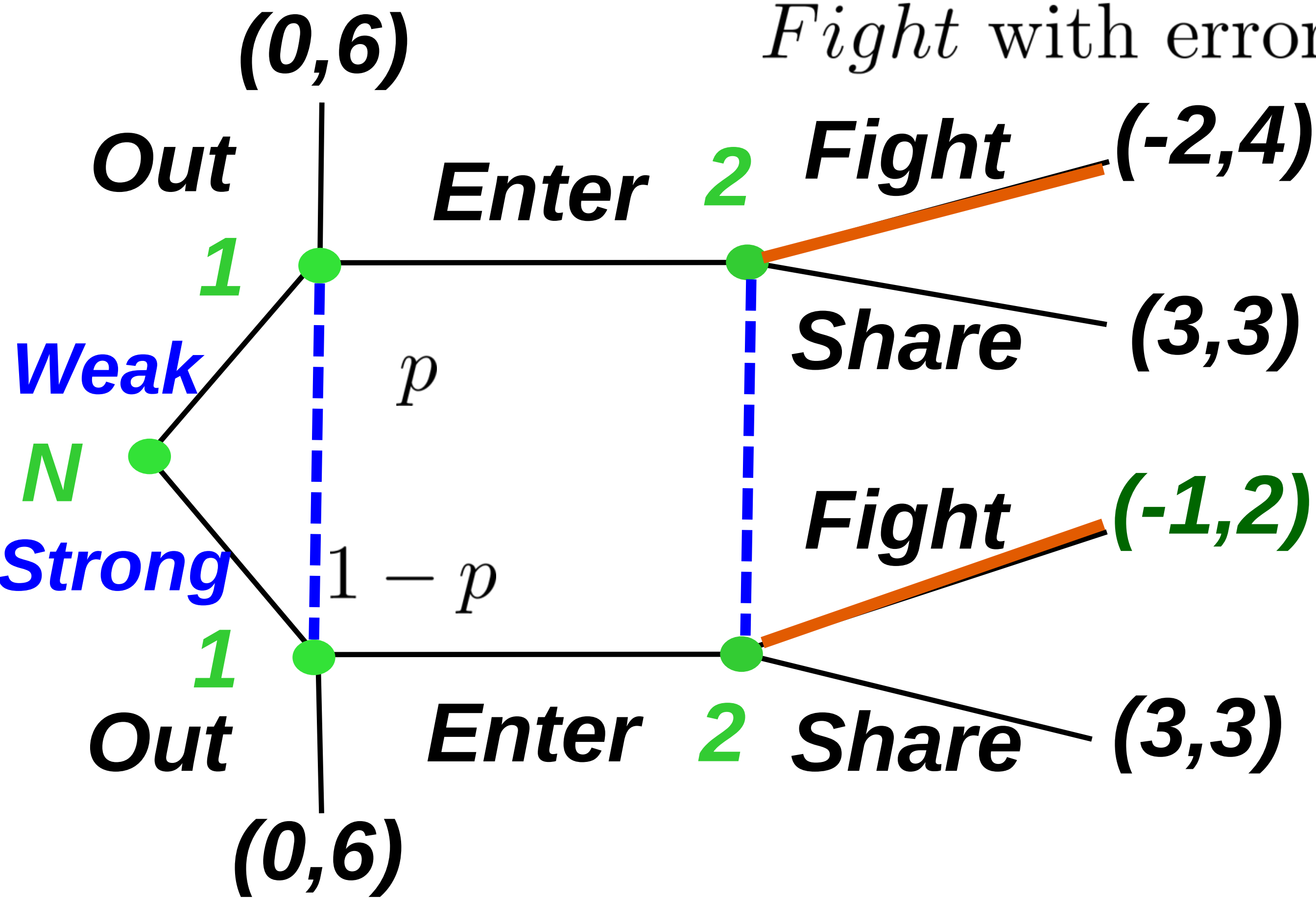


If $p \leq 1/2$, <u>$Fight$ is not BR when $Enter$</u>

BNE with Player 2 Choosing *Share*: Indeed THP

For all $\{\pi^k\}_{k=1}^{\infty}$,

$Fight$ with error probability ϵ_2^k



$$U^1(Enter) = 3(1-\epsilon_2^k) - \epsilon_2^k - p\epsilon_2^k > 0$$

for small ϵ_2^k

$Enter$ is BR

Sequential Equilibrium

- The BNE profile $(s_1, \cdots, s_n)$ of the n players in a game is a sequential equilibrium if
- Each strategy is a BR at each node
- When beliefs at each node are the limits of beliefs associated with trembles as the probability of trembles → 0
- Note: THP → SE

Summary of 9.6

- Bayesian Games
  - Incomplete Information as “Types”

- Bayesian Nash Equilibrium
- Trembling-hand Perfect Equilibrium
- Sequential Equilibrium

- HW 9.6: Riley – 9.6-1~3