Experimental Economics I

Voter Turnout

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Voter Abstention

- In large elections, full participation may be a poor approximation to empirical reality.
- Typically, voter abstention is significant, and a satisfactory theory of electoral competition needs to admit this possiblity.
- Presently, formal models don't successfully address the question why people might choose to vote in large elections and account for observed levels and patterns of turnout. $B \cdot Pr[Piv|n] - K < 0$ (as $n \to \infty$) $\to 0$
- The canonical models grossly underpredict turnout rates in mass elections, which is sometimes dubbed "paradox of not voting."
- Our focus is on understanding the implications of permitting abstention for candidates' selection of electoral platforms, i.e., for the policy agenda offered to individuals through electoral competition.

Initial Assumptions

- If voters care only about policy and candidates offer identical platforms, then abstention is the best decision whenever there is any opportunity cost at all of casting a vote.
- Consequently, we begin by assuming the candidates' positions are fixed and distinct.
- We assume also that elections are decided by simple *plurality* voting, whereby the candidate receiving the most votes wins whether or not that candidate receives votes from more than half of the electorate.
  - If there is a tie, the outcome of the election is assumed to be determined by a fair lottery over the two candidates, so each wins with probability $1/2$.

Voter Payoffs

- Consider a single voter $i \in N$.
- WLOG normalize i's payoffs from the candidates' given platforms (a, b) to satisfy
$$u_i(a) = 1 > u_i(b) = 0.$$
- There is also a small opportunity cost to voting κ due, for example, to the time it takes to go to a voting booth and record a vote.
- Voting for b is strictly dominated by voting for a for i; consequently, if i chooses to vote at all then she surely votes for platform a.

Pivotal Events

- Treating the participation decisions of the remaining $n-1$ voters as given, there are four events relevant to i's decision on whether to vote or abstain.
- Without i's vote exactly one of the following occurs:

(LL) a loses by at least two votes;

(L) a loses by exactly one vote;

(T) a and b tie;

(W) a wins by at least one vote.

- Individual i cannot affect the outcome of the election in either event (LL) or (W).
  - In both cases i is strictly better off abstaining, so saving the cost of casting an irrelevant vote.
- On the other hand, i's vote surely matters in each of the remaining two events, (L) and (T).
  - Voter i is *pivotal* for the election in these (*pivotal*) events.

Expected Payoffs

- Given the electoral platforms are fixed and taking it as understood that no individual uses a strictly dominated strategy and votes for her least preferred candidate, a mixed vote strategy for i is a choice $v_i \in [0,1]$, where v_i is the probability that voter i votes for her most preferred alternative (here a).
  - If $v_i = 1$ then i votes for her preferred candidate surely; if $v_i = 0$ then i abstains surely.
- Let $p_e(n)$ denote the probability that event $e = LL, L, T, W$ occurs conditional on there being n eligible voters.
- Then i's expected payoff from voting for a is

$$\begin{aligned} E[u_i|v_i = 1] &= p_{LL}(n)u_i(b) + p_L(n)\frac{1}{2}[u_i(a) + u_i(b)] \\ &\quad + [p_T(n) + p_W(n)]u_i(a) - \kappa \\ &= p_L(n)\frac{1}{2} + [p_T(n) + p_W(n)] - \kappa \end{aligned}$$

Expected Payoffs

- Similarly, i's expected payoff from abstaining is

$$\begin{aligned} E[u_i|v_i = 0] &= [p_{LL}(n) + p_L(n)]u_i(b) + p_T(n)\frac{1}{2}[u_i(a) + u_i(b)] \\ &\quad + p_W(n)u_i(a) \\ &= p_T(n)\frac{1}{2} + p_W(n). \end{aligned}$$

- Hence i chooses to vote rather than abstain only if

$$E[u_i|v_i = 1] - E[u_i|v_i = 0] = [p_L(n) + p_T(n)]\frac{1}{2} - \kappa \geq 0.$$

Pivot Probability

- Let $Pr[piv|n] \equiv [p_L(n) + p_T(n)]$ be the probability of being pivotal, so i votes only if

$$Pr[piv|n]\frac{1}{2} - \kappa \geq 0. \qquad (*)$$

- From a purely decision-theoretic perspective, it is plausible and intuitive to suppose $Pr[piv|n]$ is strictly decreasing in n.
  - The larger is the electorate the less likely it is that any single vote tips the election one way or the other.
- Indeed, from a typical individual's perspective, we expect $Pr[piv|n] \to 0$ as $n \to \infty$; but then for any cost $\kappa > 0$ there exists a sufficiently large finite electorate for which $(*)$ fails so i rationally abstains and, if all individuals are reasoning similarly, the prediction seems to be that nobody votes.
- However, if all individuals are abstaining then the election must surely be tied in which case the probability of i being pivotal is one, implying $(*)$ holds with strict inequality (for sufficiently low κ) and it seems everyone should vote; and so on.

Two Comments

- Even at the first step it is *not* true that "if all others vote, a rational individual should abstain"; rather, the conclusion from $(*)$ is that only those with negligible costs choose to vote ($\kappa \approx 0$ or $\kappa < 0$).
  - Whether the proportion of such individuals in any given society is large or small is entirely an empirical matter.
  - To assert on the basis of $(*)$ that rational choice predicts zero turnout in any sizeable election is simply unreasonable.
- There is no more of a problem with the "infinite regress" than there is with solving any pair of simultaneous equations: all the argument makes clear is that turnout is a function of the probability of being pivotal and the probability of being pivotal is a function of turnout.
  - In equilibrium, the pivot probability and expected turnout must be mutually consistent and mutual consistency almost always implies positive turnout.

Exemplary Model

- Suppose there are two candidates with distinct fixed platforms (a, b).
- Suppose further that the electorate is partitioned into two groups of identical individuals: all those in group L strictly prefer a to b and all those in group M strictly prefer b to a.
- Let $\#L = l$, $\#M = m$; WLOG assume $0 < l \leq m$ with $l + m = n$.
- Voter preferences are as described above: for all $i \in L$,

$$u_i(a) = 1 > u_i(b) = 0;$$

and for all $i \in M$,

$$u_i(b) = 1 > u_i(a) = 0.$$

- The cost to voting is $\kappa > 0$ for every individual.
- Individuals choose whether to bear the cost of voting and vote for their most preferred candidate, or at no direct cost, to abstain.
- The election is by plurality rule with ties broken by a fair coin toss.

Voter Strategy

- As above, $v_i \in [0,1]$ denotes the probability that voter i chooses to vote for i's most preferred candidate.
- We look for undominated Nash equilibria $v^* = (v_1^*, ..., v_n^*) \in [0,1]^n$.
- Following the logic supporting $(*)$ above and, for any $i \in N$ and $(n-1)$-strategy profile v_{-i}, writing $Pr[piv|v_{-i}]$ for the probability i is pivotal given the strategies of all other individuals, an individual i's best response decision criterion is

$$v_i \begin{cases} = 1 & \text{if } Pr[piv|v_{-i}^*]\frac{1}{2} > \kappa \\ \in [0,1] & \text{if } Pr[piv|v_{-i}^*]\frac{1}{2} = \kappa \\ = 0 & \text{if } Pr[piv|v_{-i}^*]\frac{1}{2} < \kappa. \end{cases}$$

Equilibrium Multiplicity

- The existence of equilibria is assured by the Nash existence theorem.
- Indeed, there are typically a great many equilibria.
- One apparent exception to the multiplicity is when the cost of voting exceeds $1/2$.
  - If $\kappa > 1/2$ then the expected payoff of voting is negative even in the case i is surely pivotal.
  - Thus $\kappa > 1/2$ implies there is a unique undominated Nash equilibrium: $v_i^* = 0$ for all $i \in N$, so all abstain.
- Hereafter, therefore, assume $\kappa < 1/2$; then we have multiple equilibria.

Pure Strategy Equilibria

- If $l = m$ there is a unique pure strategy equilibrium: $v_i^* = 1$ for all $i \in N$, so all vote.
  - Given that the two groups are of the same size, it is immediate that if $n - 1$ others vote then $p_L(n) = 1$.

    So your vote definitely matters, since not voting ⇒ Your group loses by one vote!
  - Therefore, if i abstains then i assures herself of a zero payoff but if i votes then i creates a tie yielding a payoff of $1/2 - \kappa > 0$.
- If $0 < l < m$ then there are no pure strategy equilibria in undominated strategies.
  - To check this, suppose to the contrary that v is a pure strategy equilibrium: $v_i \in \{0, 1\}$ for all $i \in N$.
  - Let l^v denote the number of group L individuals who vote in v and define m^v analogously for group M.

Pure Strategy Equilibria

Case 1. $l^v > m^v + 1$ or $l^v + 1 < m^v$.

In both instances no individual is pivotal in which case, given the behavior of the $n-1$ others, every individual $i \in N$ has incentive to abstain. Hence v cannot be an equilibrium profile.

Case 2. $l^v = m^v$.

Because $l \neq m$ there exists at least one individual i who abstains in v. But $l^v = m^v$ implies $Pr[piv|v_{-i}] = 1$ in which case i's best response to v_{-i} is to vote $v_i^l = 1$, contradicting v being an equilibrium.

Pure Strategy Equilibria

Case 3. $l^v = m^v - 1$ or $l^v - 1 = m^v$.

Let i be a member of the group with smaller turnout. If $v_i = 0$ then $Pr[piv|v_{-i}] = 1$ and *i* can create a tie by voting; since $\kappa < 1/2$, this gives a higher payoff to i than does abstention.

Hence, if v is an equilibrium, it must be that all individuals of the group with smaller turnout are voting surely. But then no such individual is pivotal and that group's favored candidate loses surely; therefore abstention gives a better payoff than voting, again contradicting the claim that v is an equilibrium.

Comments on Pure Equilibrium

- Thus the existence of pure strategy equilibria with any sort of turnout at all is confined to the case of a perfectly evenly divided electorate.
- Although clearly special, should this circumstance occur then turnout is 100% irrespective of the size of the two groups or of the cost of voting so long as this cost is bounded above by one-half of the total benefit of winning.
- On the other hand, an evenly divided electorate seems to be an empirically unlikely scenario when candidates are commonly seen as being distinct.
- We therefore next consider mixed strategy equilibrium.

Quasi-symmetric Mixed Strategy Equilibria

- The term "symmetric" here refers to a restriction on mixed strategies; if any member i of a given group uses a nondegenerate mixed strategy $v_i \in (0,1)$, then every member of that group uses the same strategy.
  - If $v_i \in (0,1)$ then $v_i = v_j$ for all j in the same group as i.
- There is no substantive reason why all individuals in a group should adopt the same (nondegenerate) randomization, but the restriction is technically convenient
- However, as the qualifier "quasi-" suggests, it is not assumed that all voters use mixed strategies, or that individuals in different groups use the same nondegenerate mixed strategy, or that members of the group using pure strategies have to use the same pure strategy.

A Mixed Quasi-symmetric Equilibrium

- In this case we look for an equilibrium in which

(1) All individuals in group L use the same (nondegenerate) mixed strategy, $v_i = \hat{v} \in (0,1)$ for all $i \in L$, voting with probability $\hat{v}$ for a and abstaining with probability $1 - \hat{v}$; and

(2) Exactly l members of group M vote surely for b with the remaining $m - l \geq 0$ individuals abstaining.

- Let $v = (\hat{v}; m^v = l)$ denote this strategy profile where m^v is the *ex post* number of individuals from M who vote and write $\hat{v}_{-i}$ to denote the behavior of individuals in L other than i.

Condition for Voter $i \in L$

- If individual $i \in L$ is to use a mixed strategy $\hat{v}$ then i must be indifferent between voting for a and abstaining.
- And since exactly $l = \#L$ members of group M are presumed to vote surely, an individual $i \in L$ is pivotal only in the event that i's vote creates a tie.
- Therefore, in equilibrium we must have for all $i \in L$

$$\begin{aligned} Pr[piv|(\hat{v}_{-i}; m^v = l)] &= 2\kappa \Leftrightarrow \\ Pr[l^v = l-1|\hat{v}_{-i}] &= 2\kappa \Leftrightarrow \\ \hat{v}^{l-1} &= 2\kappa, \end{aligned}$$

$\Rightarrow \hat{v} = (2\kappa)^{\frac{1}{l-1}}$

where l^v is the *ex post* number of individuals from L who vote.

Condition for Voter $j \in M$ Who Votes

- On the other hand, given that $l-1$ other members of M are supposed to vote for b, an individual $j \in M$ can be pivotal either by creating or by breaking a tie.

- Thus, if any individual $j \in M$ is expected to vote surely in equilibrium,

$$\begin{aligned} Pr[piv|(\hat{v}; m^v = l-1)] &\geq 2\kappa \Leftrightarrow \\ Pr[l^v = l|\hat{v}] + Pr[l^v = l-1|\hat{v}] &\geq 2\kappa \Leftrightarrow \\ \hat{v}^l + l\hat{v}^{l-1}(1-\hat{v}) &\geq 2\kappa. \end{aligned}$$

Condition for Voter $j \in M$ Who Abstains

- Similarly, if any individual $j \in M$ is expected to abstain surely in equilibrium, then necessarily

$$\begin{aligned} Pr[piv|(\hat{v}; m^v = l)] &\leq 2\kappa \Leftrightarrow \\ Pr[l^v = l|\hat{v}] &\leq 2\kappa \Leftrightarrow \\ \hat{v}^l &\leq 2\kappa. \end{aligned}$$

Increasing Turnout

- Clearly, if the equality constraint $\hat{v}^{l-1} = 2\kappa$ holds, then necessarily the latter two constraints do not bind.
- Hence there exists an equilibrium of this sort for every cost $\kappa < 1/2$.
- Solving for $\hat{v}$ gives $\hat{v} = (2\kappa)^{1/(l-1)}$.
- Therefore
$$\lim_{l \to \infty} \hat{v} = 1 \text{ and } \left.\frac{d\hat{v}}{d\kappa}\right|_{\kappa < 1/2} > 0.$$
- Hence this particular equilibrium of the game-theoretic model predicts expected turnout is *increasing*, both in the size of the electorate (with l and m growing at the same rate and $l \leq m$) and in the cost of voting.
- It follows that the probability of a tied result similarly increases with l and κ.

A Totally Mixed Quasi-symmetric Equilibrium

- We finally consider an equilibrium in which all individuals are using mixed strategies, with members of different groups adopting different randomizations.
- Suppose all $i \in L$ use a mixed strategy $\hat{v}$ and all $j \in M$ use a mixed strategy $\bar{v}$.
- Denote the specified strategy profile as $v = (\hat{v}; \bar{v})$ with $\hat{v}_{-i}$ (resp. $\bar{v}_{-i}$) denoting the behavior of all individuals in L (resp. M) other than i.
- The indifference condition supporting any equilibrium randomization for $i \in L$ is that

$$\begin{aligned} Pr[piv|(\hat{v}_{-i}; \bar{v})] &= 2\kappa \Leftrightarrow \\ Pr[l^v = m^v|\hat{v}_{-i}; \bar{v}] + Pr[l^v = m^v - 1|\hat{v}_{-i}; \bar{v}] &= 2\kappa. \end{aligned}$$

Pivot Probabilities

- Here the pivot probabilities are

$$Pr[l^v = m^v | \hat{v}_{-i}; \bar{v}]$$
$$= \sum_{t=0}^{min\{l-1,m\}} \binom{l-1}{t} \binom{m}{t} [\hat{v}^t (1-\hat{v})^{l-1-t}][\bar{v}^t (1-\bar{v})^{m-t}],$$
$$Pr[l^v = m^v - 1 | \hat{v}_{-i}; \bar{v}]$$
$$= \sum_{t=0}^{min\{l-1,m-1\}} \binom{l-1}{t} \binom{m}{t+1} [\hat{v}^t (1-\hat{v})^{l-1-t}][\bar{v}^{t+1} (1-\bar{v})^{m-1-t}]$$

- A similar condition holds for any individual $j \in M$, where we require $Pr[piv|(\hat{v}; \bar{v}_{-j})] = 2\kappa$.

Simplification with Complementary Mixed Strategy

- We restrict our attention to the case in which all $i \in L$ use $\hat{v}$ and all $j \in M$ use the complementary mixed strategy $\bar{v} = 1 - \hat{v}$.

- Then, with $l \leq m$, the two indifference conditions characterizing the equilibrium profile $(\hat{v}; \bar{v}) = (\hat{v}; 1 - \hat{v})$ simplify: for $i \in L$,

$$[\hat{v}^m (1 - \hat{v})^{l-1}]\Big[\sum_{t=0}^{l-1} \binom{l-1}{t}\binom{m}{t} + \frac{1-\hat{v}}{\hat{v}} \sum_{t=0}^{l-1} \binom{l-1}{t}\binom{m}{t+1}\Big] = 2\kappa$$

and, for $j \in M$,

$$[\hat{v}^m (1 - \hat{v})^{l-1}]\Big[\sum_{t=0}^{l-1} \binom{l}{t+1}\binom{m-1}{t} + \frac{1-\hat{v}}{\hat{v}} \sum_{t=0}^{min\{l,m-1\}} \binom{l}{t}\binom{m-1}{t}\Big] = 2\kappa.$$

Further Simplification with Combinatorial Identity

- A standard identity in combinatorics states

$$\sum_{t=0}^{T} \binom{T}{t}\binom{S}{t+s} = \binom{T+S}{T+s}.$$

- Let $T = l-1$ and $S = m$; then setting $s = 1$ and applying the above identity yields

$$\sum_{t=0}^{l-1} \binom{l-1}{t}\binom{m}{t+1} = \binom{l+m-1}{l}.$$

- Now set $s = 0$ to obtain

$$\sum_{t=0}^{l-1} \binom{l-1}{t}\binom{m}{t} = \binom{l+m-1}{l-1}.$$

- Substituting into the indifference condition for $i \in L$ then gives a further simplification to

$$[\hat{v}^m(1-\hat{v})^{l-1}]\binom{l+m-1}{l-1} + [\hat{v}^{m-1}(1-\hat{v})^{l}]\binom{l+m-1}{l} = 2\kappa. \qquad (**)$$

- A similar argument for $j \in M$ yields exactly the same indifference condition.

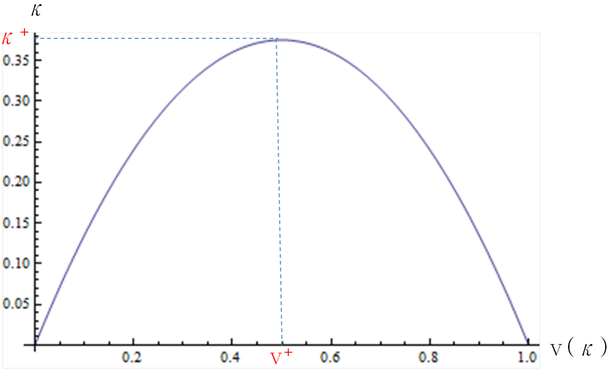
Equilibrium Characterization

- $(**)$ describes the equilibrium (given $\kappa < 1/2$).
- We treat the cost of voting as the dependent variable and solve for the value of κ, say $\kappa(\hat{v})$, that rationalizes a mixed strategy $\hat{v}$.
- In particular, $\kappa(0) = \kappa(1) = 0$ and

$$\frac{d\kappa}{d\hat{v}} \gtreqless 0 \;\Leftrightarrow\; \hat{v} \lesseqgtr \frac{\sqrt{m(m-1)}}{\sqrt{l(l-1)} + \sqrt{m(m-1)}} \equiv \hat{v}^{+}.$$

- Thus the relationship between expected turnout and the cost of voting is not monotonic.
- Equilibria of the sort being considered exist only for voting costs $\kappa \leq \kappa(\hat{v}^{+})$.
- There can be multiple equilibria for a given cost κ.

Expected Turnout and Cost of Voting



Expected Turnout and Cost of Voting

- Letting $l = m = 2$, for $\kappa < \kappa^+ \equiv \kappa(\hat{v}^+) = 0.375$ ($\hat{v}^+ = 0.5$) in the figure, there are two equilibria of the $(\hat{v}; 1 - \hat{v})$ sort (*high turnout* and *low turnout* equilibrium).
- When $\kappa > \kappa^+$, however, there is no $(\hat{v}; 1 - \hat{v})$ equilibrium.
- Assuming $\hat{v}$ does change continuously in voting cost, expected turnout in this example with $l = m$ is constant in κ: $[\hat{v}l + (1 - \hat{v})m] = n/2$.
- If a similar figure applies to asymmetric electorates with $l < m$, then aggregate expected turnout is increasing in κ for the continuous selection $\hat{v}(\kappa) > \hat{v}^+$ and decreasing in κ for the selection $\hat{v}(\kappa) < \hat{v}^+$.

Remarks

- The decision-theoretic intuitions derived from $(*)$ may not apply in a strategic setting.
- On the one hand, individuals face the familiar collective action free-riding problem whereby a single vote is deemed sufficiently unlikely to matter in the aggregate.
- On the other hand, there is an incentive to coordinate behavior within a given group to promote the group's electoral interests.
- However, this result of positive (and possibly high) turnout is not robust to the following two extensions; a (continuous) cost distribution (rather than a fixed cost) and strategic choices of platforms (rather than fixed platforms).

\- Palfrey and Rosenthal: Public Choice (1983) vs. APSR (1985).