Experiments Games with Mixed Strategy Equilibrium (混合策略均衡實驗)

Joseph Tao-yi Wang (王道一)

EE-BGT, Lecture 5

Some Comments Regarding 20-Minute Presentations

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Comments Regarding 20-Minute Presentations

- Rule 1: Don't use more than 20 slides
  - It takes on average 1 minute to go over 1 slide
- Rule 2: Don't use font sizes below 28
  - Try looking at your slides from far behind
  - Font sizes $<$ 28 means: You DON'T want people to see it
- Rule 3: This is a teaser-trailer of the movie
  - Show the experiment + a snapshot of the results
- Write down a script and NOT memorize it

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Games with MSE 有混合策略均衡的賽局

- Zero-Sum Games (零和賽局)
  - Rock-Scissor-Paper (剪刀石頭布)
  - Sports (PK, tennis serves, basketball drives, etc.) (足球罰踢、網球發球、籃球切入或投籃)
  - Military attack (軍事行動如登陸諾曼地或加萊)
- Deter Undesired Behavior (嚇阻投機/不好希望發生的行為)
  - Searches of passengers after 9/11 (機場安檢、海關抓走私)
  - Randomizing across exam questions (老師隨機出題)
- But, there are folk theories about these games... (但總有一些有趣的「理論」)

玩家公開猜拳遊戲必勝絕招：先出剪刀 (中央社 2007-12-19)

- 媒體報導，大多數人都知道，在猜拳遊戲中，石頭贏剪刀，剪刀贏布，布勝拳頭，但很少有人知道，如何贏得這個相當普遍的遊戲。現在死忠玩家透露了必殺秘技：先出剪刀。 L0
- 英國「每日郵報」報導，研究顯示在這種快速擺出手部姿勢的猜拳遊戲中，石頭是三種猜拳手勢中玩家最喜歡出的一種。 L1
- 如果你的對手預期你會出石頭，他們就會選擇出布來贏過你，因此你要出剪刀才能贏，因為剪刀贏布。 L2

玩家公開猜拳遊戲必勝絕招：先出剪刀 (中央社 2007-12-19)

- 報導說，這套剪刀策略讓拍賣商佳士得前年成功贏得一千萬英鎊的生意。一名有錢的日本藝術品收藏家，無法決定要讓哪家拍賣公司來拍賣自己收藏的印象派畫作，於是他要求佳士得與蘇富比兩家公司猜拳決定。
- 佳士得向員工討教猜拳策略，最後在一名主管十一歲的女兒的建議下決定出剪刀。這名女孩現在還在讀書，經常玩猜拳，她推論「所有人都以為你會出石頭」。這代表蘇富比會出布，想要打敗石頭，因此佳士得應該選擇出剪刀。
- 一如預期，蘇富比最後出布，輸給了佳士得的剪刀，拱手將生意讓給對方。

Mixed-Strategy Equilibrium in Rock-Paper-Scissors

- How do you play Rock-paper-scissors (RPS)?
  - 如果你來玩剪刀石頭布，你會出什麼?
- What is the MSE here? (剪刀石頭布賽局的均衡為何?)
- Mix with probabilities (1/3, 1/3, 1/3) (三者隨機)
- Would you really play this MSE in RPS?
  - News article suggests a level-k model... (你真的會按均衡策略來玩嗎？新聞故事所反映的多層次思考模型預測為何？想知道更多請看課本第五章)
- Janken/RPS Robot with 100% winning rate:
  - v1: http://www.youtube.com/watch?v=3nxjjztQKtY
  - v2: https://www.youtube.com/watch?v=ZVNnoOcohaU

Advantages of Games with MSE (此種賽局的優點)

- Typically have unique equilibrium (有唯一均衡)
  - All games discussed have unique equilibrium
- Constant sum: No room for social preference
  - Not possible to help others without hurting self
    (總報酬為常數下通常無社會偏好，因為幫助別人一定傷到自己)
- Maximin leads to Nash in zero sum (避兇就是均衡)
  - Maximin is a simple rule: (對方就是要害我如何趨吉避兇)
  - "I want to maximize the worse case scenario..."
- A good place to test theory! (這是驗證理論的好地方)

Maximin in Matching Pennies (黑白猜下避兇)

	H	T
H	1	-1
T	-1	1

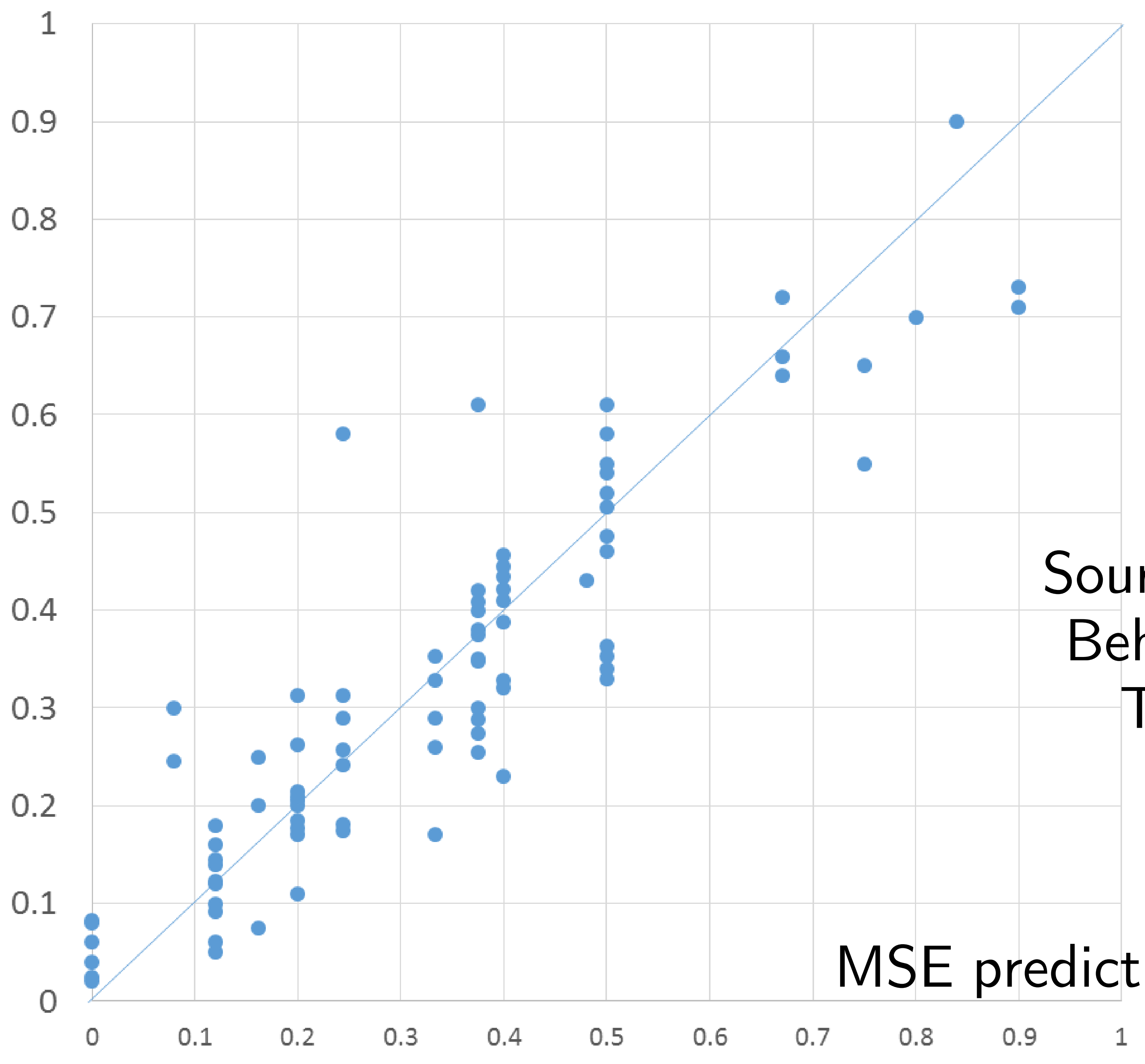
- Rowena thinks: (列子認為)
- Play H: Worse case = -1 (出正面最慘是對手選反面)
  - If Colin wants to destroy me
- Play T: Worse case = -1... (出反面最慘...)
- (1/2, 1/2): Worse case 0* (一半一半至少不賺不賠)
- Same for Colin (行家所見略同)
  - This is the MSE! (這正好是此賽局的混合策略均衡!!)

*We assume preferences satisfy axioms for EU... (假設偏好滿足期望效用公理)

Challenges of Games with MSE (對理論的挑戰)

- Epistemic Foundation (認知基礎：須清楚知道對手的策略)
  - Requires precise knowledge of opponent strategy
- Learning Dynamics may not work (動態學習不見得好)
  - Gradient processes spiral away (梯度逼近會螺旋脫離均衡)
  - No incentive to mix properly at MSE (均衡時亂做沒差)
- Randomization can be unnatural
  - Especially in repeated play (重複做的話，隨機亂選不太自然)
  - Purification (純化: 個體可做不同單純策略，整體看起來「混合」即可)
    - MSE can occur at population level, not individually

Actual Data
(實驗資料)



Source (資料來源):
Behavioral Game
Theory Ch 3

MSE predictions(均衡預測)

Joker Game: O'Neill (1987) (出鬼牌賽局)

- Earlier studies: Play between MSE and random
  - But had computerized opponents and/or low incentives, so hard to interpret the results... (早期實驗結果介於MSE和亂選之間，但通常對手是電腦且不見得有誘因)
- First "Modern" Studies: O'Neill (PNAS 1987)
- Good Design Trick: (很棒的實驗設計技巧！)
  - Risk aversion plays no role when there are only two possible outcomes (當實驗結果只有兩種可能時，風險偏好不會影響受試者的決定)

Joker Game: O'Neill (PNAS 1987) (出鬼牌賽局)

	1	2	3	J	MSE	Actual	QRE
1	-5	5	5	-5	0.2	0.221	0.213
2	5	-5	5	-5	0.2	0.215	0.213
3	5	5	-5	-5	0.2	0.203	0.213
J	-5	-5	-5	5	0.4	0.362	0.360
MSE	0.2	0.2	0.2	0.4			
Actual	0.226	0.179	0.169	0.426			
QRE	0.191	0.191	0.191	0.427			

- 實際的出牌頻率跟MSE預測很接近
- QRE的預測更接近，但無法解釋「不平均」

- Actual frequency quite close to MSE
- QRE better, but cannot get "imbalances"
  - With $\lambda = 1.313$

Quantal Response Equilibrium (QRE) (手滑反應均衡)

- QRE - McKelvey and Palfrey (1995)
- Better Response, not best response (更適/非最適)
- Logit payoff response function: (常用logit報酬反應函數)

$$P(s_i) = \frac{e^{\lambda \cdot \left[\sum_{s_{-i}} P(s_{-i}) u_i(s_i, s_{-i}) \right]}}{\sum_{s_k} e^{\lambda \cdot \left[\sum_{s_{-i}} P(s_{-i}) u_i(s_k, s_{-i}) \right]}}$$

Quantal Response Equilibrium (QRE)

- $\lambda = 0$: Noise (do not respond to payoffs) (對報酬無反應)
- $\lambda = \infty$: Nash (perfectly respond to payoffs) (完全反應)

$$P(s_i) = \frac{e^{\lambda \cdot \left[\sum_{s_{-i}} P(s_{-i}) u_i(s_i, s_{-i})\right]}}{\sum_{s_k} e^{\lambda \cdot \left[\sum_{s_{-i}} P(s_{-i}) u_i(s_k, s_{-i})\right]}}$$

Response to O'Neill (PNAS 1987) (後續討論)

- Brown and Rosenthal (ECMA 1990) criticize O'Neill:
  - Overly support MSE (太過支持混合策略均衡)
  - Aggregate tests not good enough (只有總體檢定不夠)
- They run (temporal dependence):

$$J_{t+1} = a_0 + a_1 J_t + a_2 J_{t-1}$$

(應該檢定跨期相關性)

$$b_0 J^*_{t+1} + b_1 J^*_t + b_2 J^*_{t-1}$$

$$c_1 J_t J^*_t + c_2 J_{t-1} J^*_{t-1} + \epsilon$$

$$J_t = \text{Own Choice};\ J^*_t = \text{Other's Choice};$$

- MSE implies only a_0 is not zero (均衡：只有 a_0 不是0)

Brown and Rosenthal (ECMA 1990) Results

Effect	Coefficient	% Players w/ $p<0.05$
Guessing	b_0	8%
Previous Opponent Choices	b_1 , b_2	30%
Previous Outcomes	c_1 , c_2	38%
Previous Choices + Outcomes	b_1 , b_2 , c_1 , c_2	44%
Previous Own Choices	a_1 , a_2	48%
All Effects		62%

Source: Table 3.4, BGT.

Response to O'Neill (PNAS 1987) (後續討論)

- Run: 2 JJJJ 1 2 33 (連發太短)
  - Too Short runs: play J twice too rarely (鮮有連續J)
- Subjects react to what they see and do (對歷史有反應)
  - But most cannot use temporal dependence to guess opponent current action (無法用跨期相關性猜中對方這次行動)
- Equilibrium-in-beliefs somewhat supported (支持信念上的均衡)
  - Each player may deviate from MSE (每人各自可能偏離)
  - But these deviations cannot be detected (卻未被破解)

Response to O'Neill (PNAS 1987) (後續討論)

- Purification interpretation of MSE (純化的MSE)
  - Equilibrium in beliefs, not in mixtures (信念非策略)
- Other similar studies (相關延伸研究)
  - Rapoport and Boebel (1992): [BGT, Table 3.5]
  - Mookerjhee and Sopher (1997): [BGT, Table 3.6-3.7]
  - Tang (1996abc, 2001): [BGT, Table 3.8]
  - Binmore, Swierzbinski and Proulx (2001): [BGT, Table 3.9]

Rapoport and Boebel (GEB 1992): Experiment 1

	C	L	F	I	O	MSE	Actual	QRE
C	10	-6	-6	-6	-6	0.375	0.293	0.286
L	-6	-6	10	10	10	0.250	0.305	0.302
F	-6	10	-6	-6	10	0.125	0.123	0.138
I	-6	10	-6	10	-6	0.125	0.119	0.138
O	-6	10	10	-6	-6	0.125	0.160	0.138
MSE	0.375	0.250	0.125	0.125	0.125			
Actual	0.352	0.180	0.218	0.099	0.151			
QRE	0.412	0.169	0.140	0.140	0.140			

- 實際出牌頻率接近MSE預測(但85%個人拒絕)
- QRE的預測更接近，但無法解釋「不平均」

- Actual frequency close to MSE
  - 85% subjects reject χ^2 test
- QRE better,
  - $\lambda_1 = 0.248$
  - (Cannot get imbalances)

Rapoport and Boebel (GEB 1992): Experiment 2

	C	L	F	I	O	MSE	Actual	QRE
C	15	-1	-1	-1	-1	0.375	0.306	0.309
L	-1	-1	15	15	15	0.250	0.324	0.296
F	-1	15	-1	-1	15	0.125	0.100	0.132
I	-1	15	-1	15	-1	0.125	0.115	0.132
O	-1	15	15	-1	-1	0.125	0.155	0.132
MSE	0.375	0.250	0.125	0.125	0.125			
Actual	0.346	0.193	0.202	0.116	0.143			
QRE	0.410	0.184	0.135	0.135	0.135			

- 實驗二報酬和實驗一不同但MSE預測相同
- QRE的預測更接近
- 實驗結果和實驗一類似

- Same MSE
  - Different payoffs
- QRE better,
  - $\lambda_2 = 0.327$
- Actual result is similar to Experiment 1

Other Similar Studies (相關延伸研究)

- Stylized Facts: (整體實驗結果)
  1. Actual frequencies not far from MSE
     - (出牌頻衡很接近MSE)
  1. Deviations small but significant
     - (跟MSE差距小但統計上顯著)
  1. Temporal dependence at individual level
     - (個人有跨期相關性)
- Can a theory explain these?
  - (有何理論可以解釋這些實驗結果?)

Response to O'Neill (PNAS 1987) (後續討論)

- Ask subjects generate random sequences (產生數列)
- Sequences resemble the underlying statistical process more closely than what short random sequences actually do (產生的比真正隨機還要更「隨機」)
    - Too balanced (太平衡)
    - Too few runs (連發太少)
    - Longest run is too short (最長的連發太短)
- Children don't learn this misconception until 5th grade
    - A learned mistake (這是一個後天學會的錯誤，小孩子在五年級之前沒有這個問題)

Game Play (賽局實驗) vs. Production (產生數列)

- Rapoport and Budescu (1992, 1994, 1997)
  - Compare sequences from a production task to strategies in a constant-sum game (R&B, 1992) (比較產生的數列和零和賽局實驗中的數列)
- Condition D: Matching pennies 150 times 1-by-1
  - 150次逐次黑白猜
- Condition S: Give sequence of 150 plays at once
  - 一次給150回合黑白猜的決定
- Condition R: Produce the outcome of tossing an unbiased coin 150 times (產生數列——丟銅板150次的結果)

Game Play (賽局實驗) vs. Production (產生數列)

- iid rejected for 40% (D), 65% (S), 80% (R) of the subjects in the three conditions
  - 三種分別有40%, 65% 和80%的受試者拒絕 iid 假設
  - Game play reduces deviations from randomness
    - 真的去玩會讓受試者比較隨機(降低偏離情形)
- Are subjects better motivated?
  - 這是因為受試者有更好的誘因,
- Or, are their working memory interfered and randomize "memory-lessly"?
  - 還是因為他們的腦部運作(工作記憶)受到干擾，以致於「忘記過去，努力面前」？

3-action Matching Pennies

	1	2	3
1	2	-1	-1
2	-1	2	-1
3	-1	-1	2

MSE
1/3
1/3
1/3

MSE	1/3	1/3	1/3

- Rapoport and Budescu (1994)

Runs in 3-act

Pattern	Game (%)	Production (%)	iid (%)
xx	26.9%	27.2%	33.3%
xxy	19.6%	20.9%	22.2%
xyy	19.6%	21.0%	22.2%
xxx	7.3%	6.3%	11.1%
xxxx	2.0%	1.8%	3.7%
xxxy	5.3%	4.5%	7.4%
yxxx	5.4%	4.5%	7.4%
xyxx	5.6%	3.5%	7.4%
xxyx	5.8%	3.7%	7.4%

Other Play in 3-action Matching Pennies

Pattern	Game (%)	Production (%)	iid (%)
xy	73.1%	72.8%	66.7%
xyx	23.7%	16.0%	22.2%
xyz	29.7%	35.9%	22.2%
yxzx	9.6%	7.8%	7.4%
xyxz	9.9%	7.9%	7.4%
xyzx	12.1%	17.3%	7.4%

Source: Table 3.10, BGT.

A Limited Memory Model (有限記憶模型)

- Subjects only remember last m elements
- Chose the $(m+1)$st to balance the number of H and T choices in the last $(m+1)$ flips (受試者只記得最後m回合，第$(m+1)$回合做決定來平衡正反面在$(m+1)$次中出現的次數。如果 m 很小就會正反變換太頻繁)
- If m is small, alternate choices too frequently
- Experimental Data: (Should all be 0.5 if iid)
  - $\mathrm{P(H|H)}=0.42$, $\mathrm{P(H|HH)}=0.32$, $\mathrm{P(H|HHH)}=0.21$
- Requires m=7 to generate this (Magic 7?)
  - 實驗結果：如果iid的話應該都是0.5，但需要m=7才能解釋實驗結果

Explicit Randomization (使用亂數產生器)

- Observe the randomization subjects want to play (觀察人們為亂數產生器設定何種機率)
  - Bloomfield (1994), Ochs (1995b), Shachat (2002)
- Explicit Randomization: (使用亂數產生器)

1. Allocate 100 choices to either strategies (決定100張牌兩邊各放幾張)
2. Choices are shuffled and computer selects one (讓電腦隨機打一張)
   - Deviations cannot be due to cognitive limit! (還偏離均衡就不是因為不能產生亂數)

- Result: Deviations from MSE small but significant
- About 10% purists (偏離MSE很小但顯著。10%「單純的人」)

Explicit Randomization (使用亂數產生器)

- Ex: Ochs (1995b) - Matching Pennies (黑白猜)
  - Row player payoff of (H, H): $1 \to 9 \to 4$ (改列子報酬)
- MSE: Column MSE changes; row is same...
  - 行家的MSE會改變；列子的反而不會變
- Allocate 10 plays of H or T (分配十個選擇給正或反)
  - Becomes a 10-play sequence (變成「做十次的數列」)
- Note: Random draw without replacement
  - This is not exactly randomization of MSE...
  - 註：這是隨機抽取不放回，不是真的MSE...

Matching Pennies (Baseline)

	H	T
H	1, 0	0, 1
T	0, 1	1, 0

- MSE:
  - R: (0.500, 0.500)
  - C: (0.500, 0.500)
- Actual Frequency: (實際頻率)
  - R: (0.500, 0.500)
  - C: (0.480, 0.520)
- QRE:
  - R: (0.500, 0.500)
  - C: (0.500, 0.500)

Matching Pennies (Game 2)

	H	T
H	9, 0	0, 1
T	0, 1	1, 0

- MSE:
  - R: (0.500, 0.500)
  - C: (0.100, 0.900)
- Actual Frequency: (實際頻率)
  - R: (0.600, 0.400)
  - C: (0.300, 0.700)
- QRE:
  - R: (0.649, 0.351)
  - C: (0.254, 0.746)

Matching Pennies (Game 3)

	H	T
H	4, 0	0, 1
T	0, 1	1, 0

- MSE:
  - R: (0.500, 0.500)
  - C: (0.200, 0.800)
- Actual Frequency: (實際頻率)
  - R: (0.540, 0.460)
  - C: (0.340, 0.660)
- QRE:
  - R: (0.619, 0.381)
  - C: (0.331, 0.669)

MSE in Field Context (實際現場的MSE)

- Rapoport and Almadoss (2000)
- Patent Races Game (競相專利賽局)
  - Two firms with endowment e (兩家廠商，各有財產)
  - Invest 1, 2, ..., e (integer)
  - Win r if invest the most
- Unique MSE:
- Invest e with prob. 1-e/r, invest others with prob. $1/r$
  - Not obvious!

Patent Race Results (競相專利賽局實驗結果)

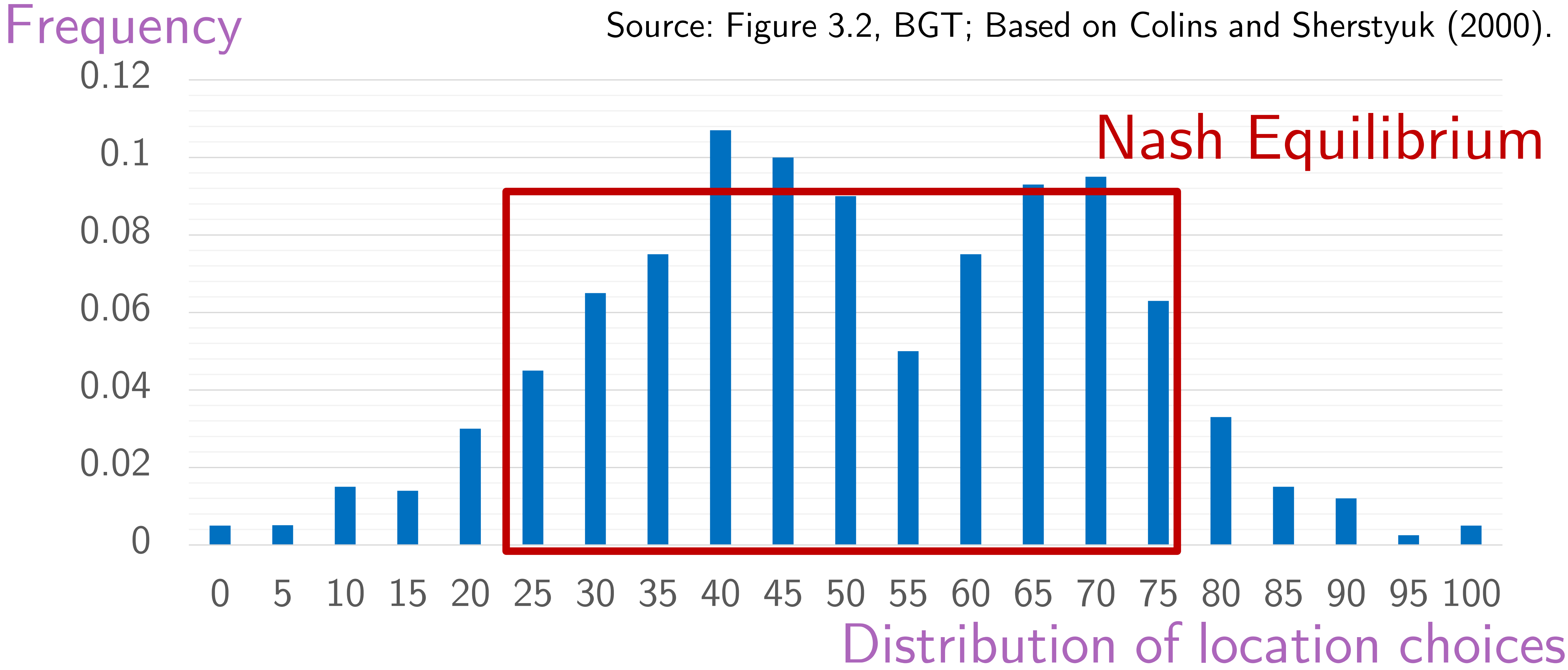
(Table 3.14)	Game L ($e = 5,\ r = 8$)		Game H ($e = 5,\ r = 20$)	
Investment	MSE	Actual	MSE	Actual
0	12.5%	16.9%	5.0%	14.1%
1	12.5%	11.6%	5.0%	5.5%
2	12.5%	8.8%	5.0%	5.3%
3	12.5%	11.8%	5.0%	5.3%
4	12.5%	9.0%	5.0%	6.9%
5	37.5%	41.8%	75.0%	62.8%

MSE in Field Context (實際現場的MSE)

- 3 Firm Hotelling: Collins and Sherstyuk (2000)
  - 2-Firm: Brown-Kruse, Cronshaw & Schenk (1993)
  - 4-Firm: Huck, Muller and Vreiend (2002)
- Location Games (3 Firm Hotelling Model)
  - Three firms simultaneously choose [0,100]
  - Consumers go to nearest firm
  - Profits proportional to units sold
- Unique MSE: Randomize uniformly [25,75]

MSE in Field Context (實際現場的MSE)

Source: Figure 3.2, BGT; Based on Colins and Sherstyuk (2000).



Two Field Studies

- Walker and Wooders (2001)
  - Serve decisions (L or R) of tennis players in 10 Grand Slam matches
- Result:
  - Win rates across two different directions are not statistically different ($p<0.10$ for only $2/40$)
  - Players still exhibit some over-alteration in serve choices through temporal dependence ($p<0.10$ for $8/40$)
    - Weaker than lab subjects

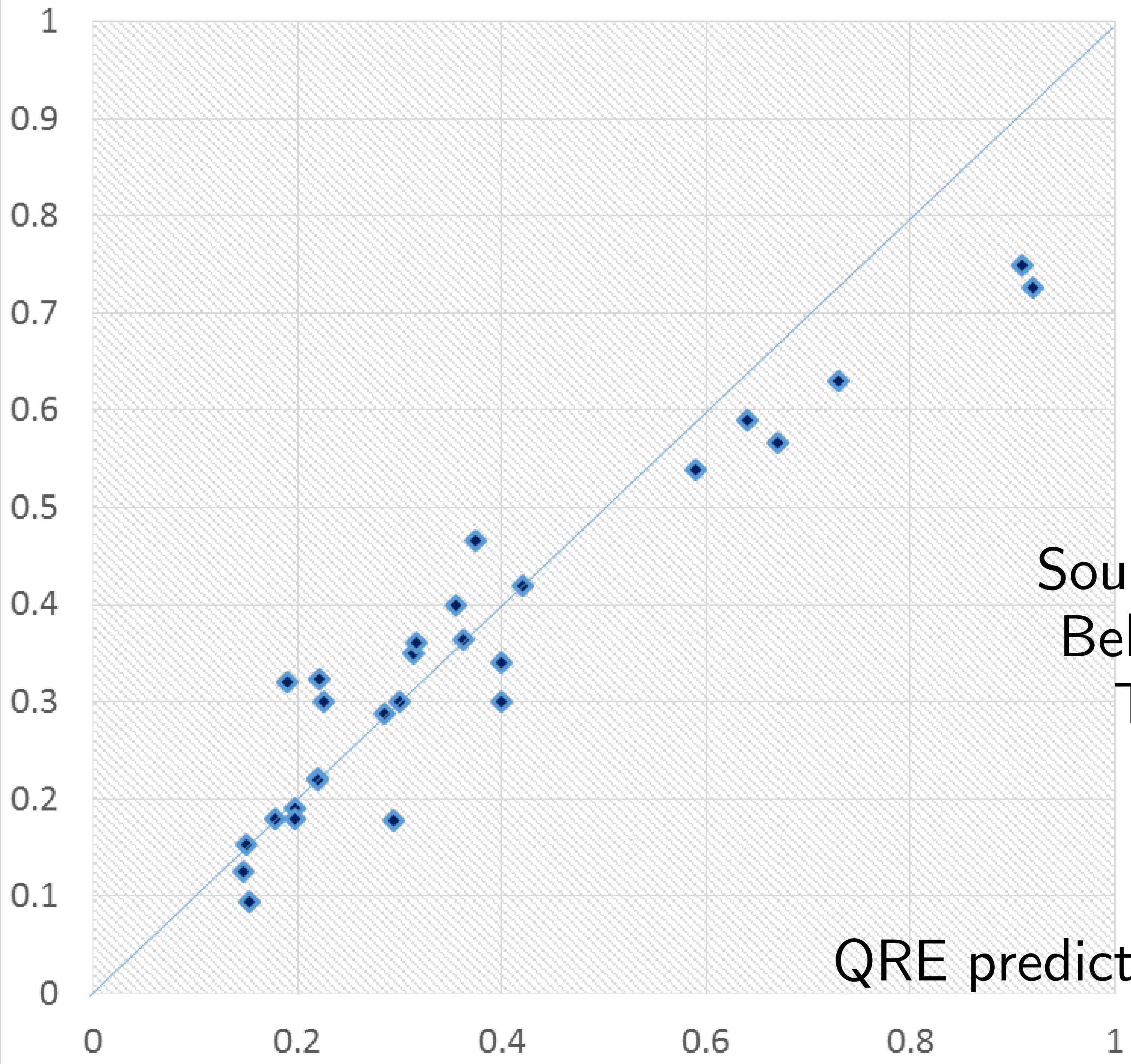
Two Field Studies

- Palacios-Huerta (2001): soccer penalty kicks
  - Code both kicker and goalie's choices
  - No selection bias (look at all games)
- Win rates are equal; no serial dependence
  - Not surprising since penalty kicks are few and are often done by different players
- Recent: Huang, Hsu, and Tang (AER 2007)
  - Chen-Ying Huang (here at NTU)

Conclusion

- Take-Home Message:
- Aggregate frequencies of play are close to MSE but the deviations are statistically significant.
- QRE seems to fit behaviors well
- Temporal dependence frequently observed

Actual Data
(實驗資料)



Source (資料來源): Behavioral Game Theory Ch 3

QRE predictions(均衡預測)

Conclusion

- With explicit randomization, the existence of purists hint on equilibrium in beliefs
  - Players cannot guess what opponents are doing
  - Beliefs about opponents are correct on average
  - But, they may not be randomizing themselves
- Field-Lab-Theory: Ostling, Wang, Chou and Camerer (2011), "Testing Game Theory in the Field: Evidence from Swedish Poisson LUPI Lottery Games," *American Economic Journal: Microeconomics*, 3(3), 1-33.