Level-k Reasoning (多層次思考)

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EE-BGT, Lecture 8

Outline

- Introduction: Initial Deviations from MSE
  - Hide-and-Seek: Crawford & Iriberri (AER07)
  - Initial Joker Effect: Re-assess O'Neil (1987)
- Simultaneous Dominant Solvable Games
  - Price Competition: Capra et al (IER 2002)
  - Traveler's Dilemma: Capra et al (AER 1999)
  - p-BC game: Nagel (AER 1995), CHW (AER 1998)
- Level-k Theory: Since Stahl-Wilson (GEB1995)
  - CGCB (ECMA2001), CGC (AER 2006)

Special Hide-and-Seek Games

- RTH: Rubinstein & Tversky (1993); Rubinstein, Tversky, & Heller (1996); Rubinstein (1998,1999)
- Your opponent has hidden a prize in one of four boxes arranged in a row.
- The boxes are marked as shown below: A, B, A, A. (Non-neutral Location Framing!)

A	B	A	A

Special Hide-and-Seek Games

- RTH (Continued):
- Your goal is, of course, to find the prize.
- His goal is that you will not find it.
- You are allowed to open only one box.
- Which box are you going to open?

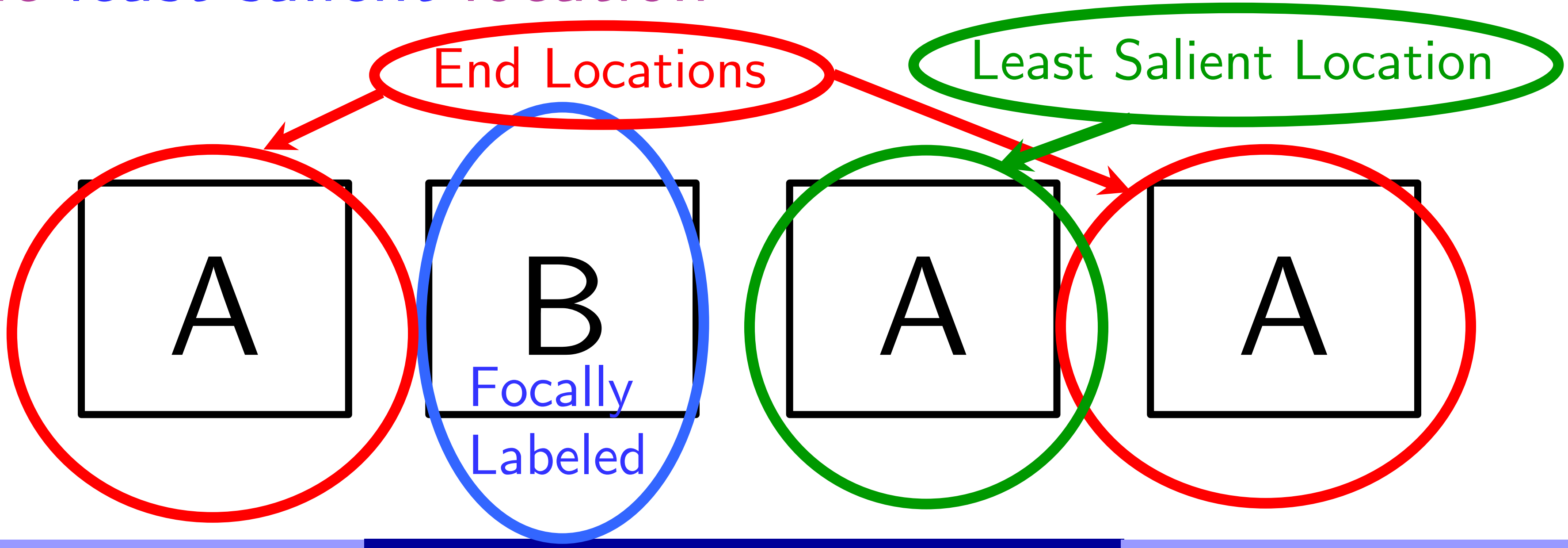
A	B	A	A

Special Hide-and-Seek Games

- Folk Theory: "...in Lake Wobegon, the correct answer is usually 'c'."
  - Garrison Keillor (1997) on multiple-choice tests
- Comment on the poisoning of the Ukrainian presidential candidate (later president):
- "Any government wanting to kill an opponent ...would not try it at a meeting with government officials."
  - Viktor Yushchenko, quoted in Chivers (2004)

Special Hide-and-Seek Games

- B is distinguished by its label
- The two end A may be inherently salient
  - This gives the central A location its own brand of uniqueness as the least salient location



Special Hide-and-Seek Games

- RTH's game has a unique equilibrium, in which both players randomize uniformly
- Expected payoffs: Hider 3/4, Seeker 1/4

Hider/Seeker	A	B	A	A
A	0, 1	1, 0	1, 0	1, 0
B	1, 0	0, 1	1, 0	1, 0
A	1, 0	1, 0	0, 1	1, 0
A	1, 0	1, 0	1, 0	0, 1

Special Hide-and-Seek Games

- All Treatments in RTH:
  - Baseline: ABAA (Treasure Treatment)
- Variants:
  - Left-Right Reverse: AABA
  - Labeling: 1234 (2 is like B, 3 is like central A)
- Mine Treatments
  - Hider hides a mine in 1 location, and Seeker wants to avoid the mine (payoffs reversed)
  - mine hiders = seekers, mine seekers = hiders

Hide-and-Seek Games: RTH Results

RTH-4	A	*B*	A	A
Hider (53)	9%	36%	40%	15%
Seeker (62)	13%	31%	45%	11%
RT-AABA-Treasure	A	A	*B*	A
Hider (189)	22%	35%	19%	25%
Seeker (85)	13%	51%	21%	15%
RT-AABA-Mine	A	A	*B*	A
Hider (132)	24%	39%	18%	18%
Seeker (73)	29%	36%	14%	22%
RT-1234-Treasure	1	*2*	3	4
Hider (187)	25%	22%	36%	18%
Seeker (84)	20%	18%	48%	14%
RT-1234-Mine	1	*2*	3	4
Hider (133)	18%	20%	44%	17%
Seeker (72)	19%	25%	36%	19%
R-ABAA	A	*B*	A	A
Hider (50)	16%	18%	44%	22%
Seeker (64)	16%	19%	54%	11%

Player roles reversed

Different locations for B

2 analogous to B

Hide-and-Seek Games: RTH Results

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Stylized facts

Hide-and-Seek Games: RTH Results

- Can pool data since no significant differences for Seekers ($p = 0.48$) or Hiders ($p = 0.16$)
  - Chi-square Test across 6 different Treatments

Role	A	B	A	A
Hiders (n=624)	21.63%	21.15%	36.54%	20.67%
Seekers (n=560)	18.21%	20.54%	45.89%	15.36%

Hide-and-Seek Games: Stylized Facts

- Central A/3 most prevalent for both Hiders & Seekers
- Central A/3 even more prevalent for Seekers (or Hiders in Mine treatments)
  - Hence, Seekers do better than in equilibrium!
- Shouldn't Hiders realize that Seekers will be just as tempted to look there?
  - RTH: "The finding that both choosers and guessers selected the least salient alternative suggests little or no strategic thinking."

Hide-and-Seek Games: Stylized Facts

- Can a strategic theory explain this?
  - Heterogeneous population with substantial frequencies of L2 and L3 as well as L1
  - Estimated 19% L1, 32% L2, 24% L3, 25% L4 reproduces the stylized facts
- More on Level-k later...
  - Let us first see more evidence in DS Games...

Simultaneous Dominant Solvable Games

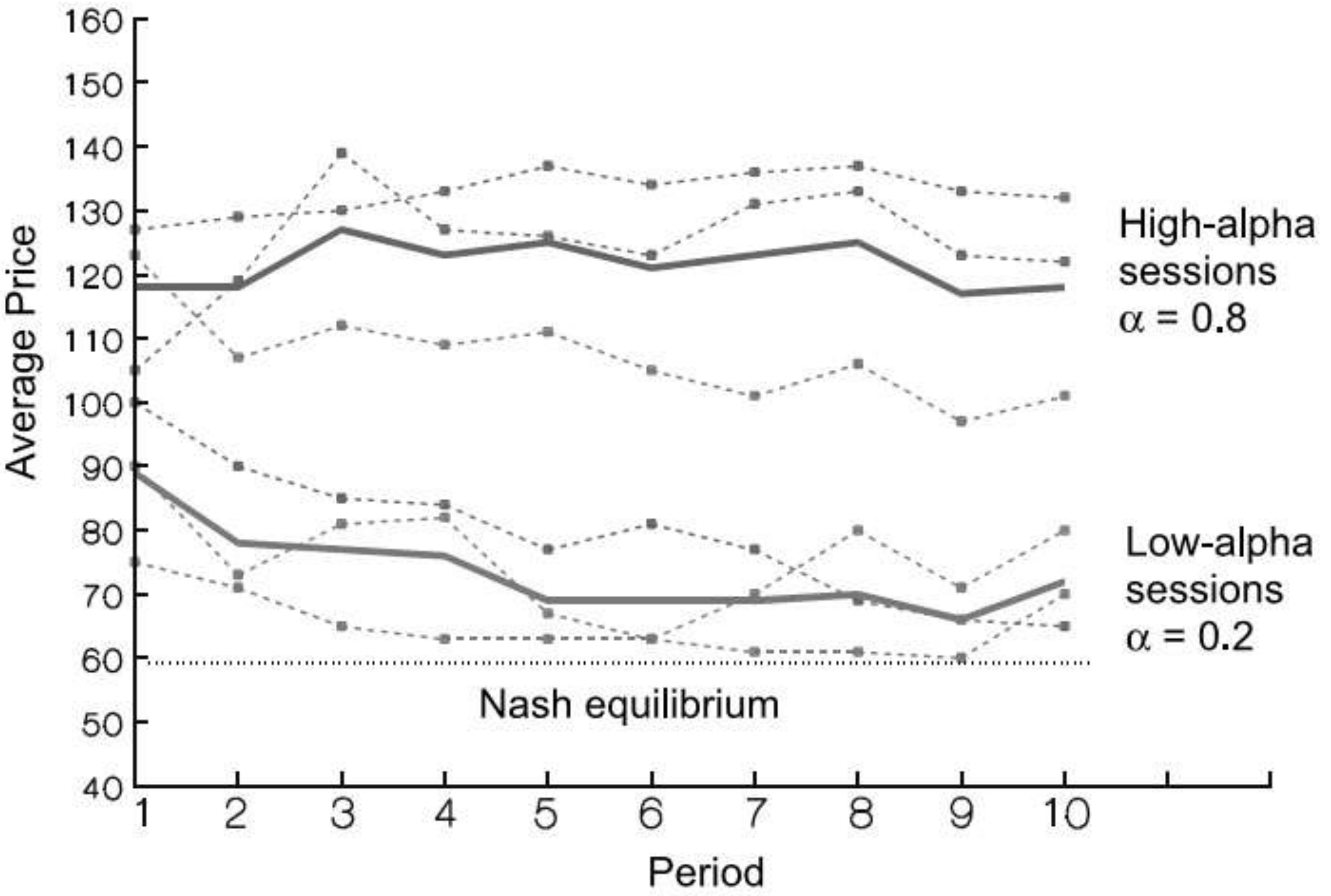
- Initial Response vs. Equilibration
- Price Competition
  - Capra, Goeree, Gomez and Holt (IER 2002)
- Traveler's Dilemma
  - Capra, Goeree, Gomez and Holt (AER 1999)
- p -Beauty Contest
  - Nagel (AER 1995)
  - Camerer, Ho, Weigelt (AER 1998)

Price Competition

- Capra, Goeree, Gomez and Holt (IER 2002)
  - Two firms pick prices p_1 and p_2 from \$0.60-\$1.60
- Split the market if $p_1 = p_2$: Both get $(1 + \alpha)$* p_1 / 2
- But if $p_1 < p_2$:
  - Low-price firm gets (1 x p_1), the other firm gets (α x p_1)
- α = Consumer Responsiveness to best price (=0.2/0.8)
  - $\alpha < 1$: Bertrand competition predicts lowest price
  - $\alpha \rightarrow 1$: Meet-or-release (low price guarantees)

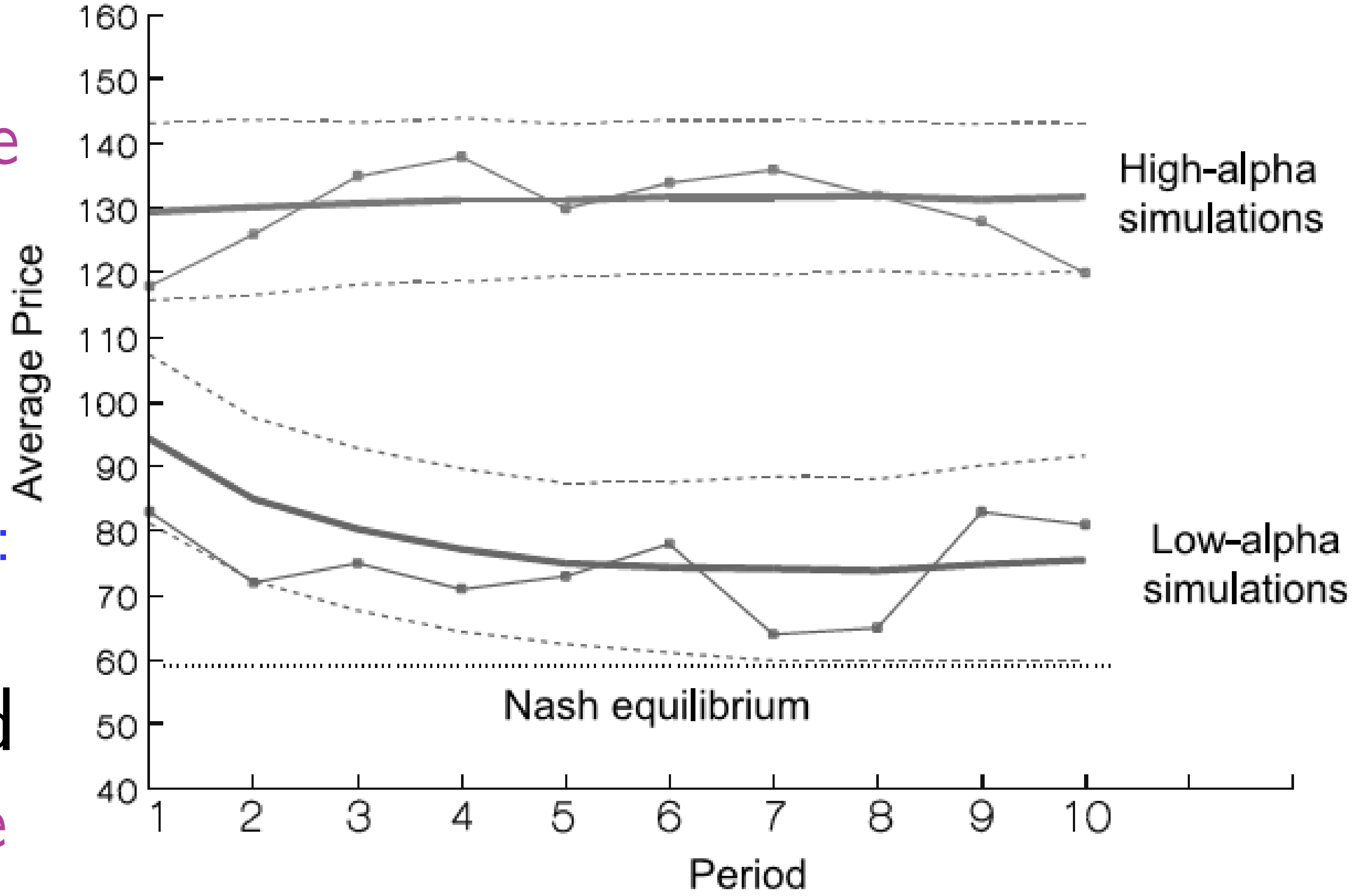
Price Competition: Average Prices in the Data

- α = 0.8: Collude!
  - Insensitive to price
    - Dashed Lines:
      - By Session
    - Dark Line:
      - By Treatment
- α = 0.2: Bertrand
  - React to low price



Price Competition: Average Prices of 1000 Simulations

- α = 0.8: Collude!
  - Insensitive to price
    - Dark Line:
      - 1000 Simulation
    - Dotted Lines:
      - +/- 2 std
    - Connect Squares:
      - A Typical Run
- α = 0.2: Bertrand
  - React to low price



Traveler's Dilemma

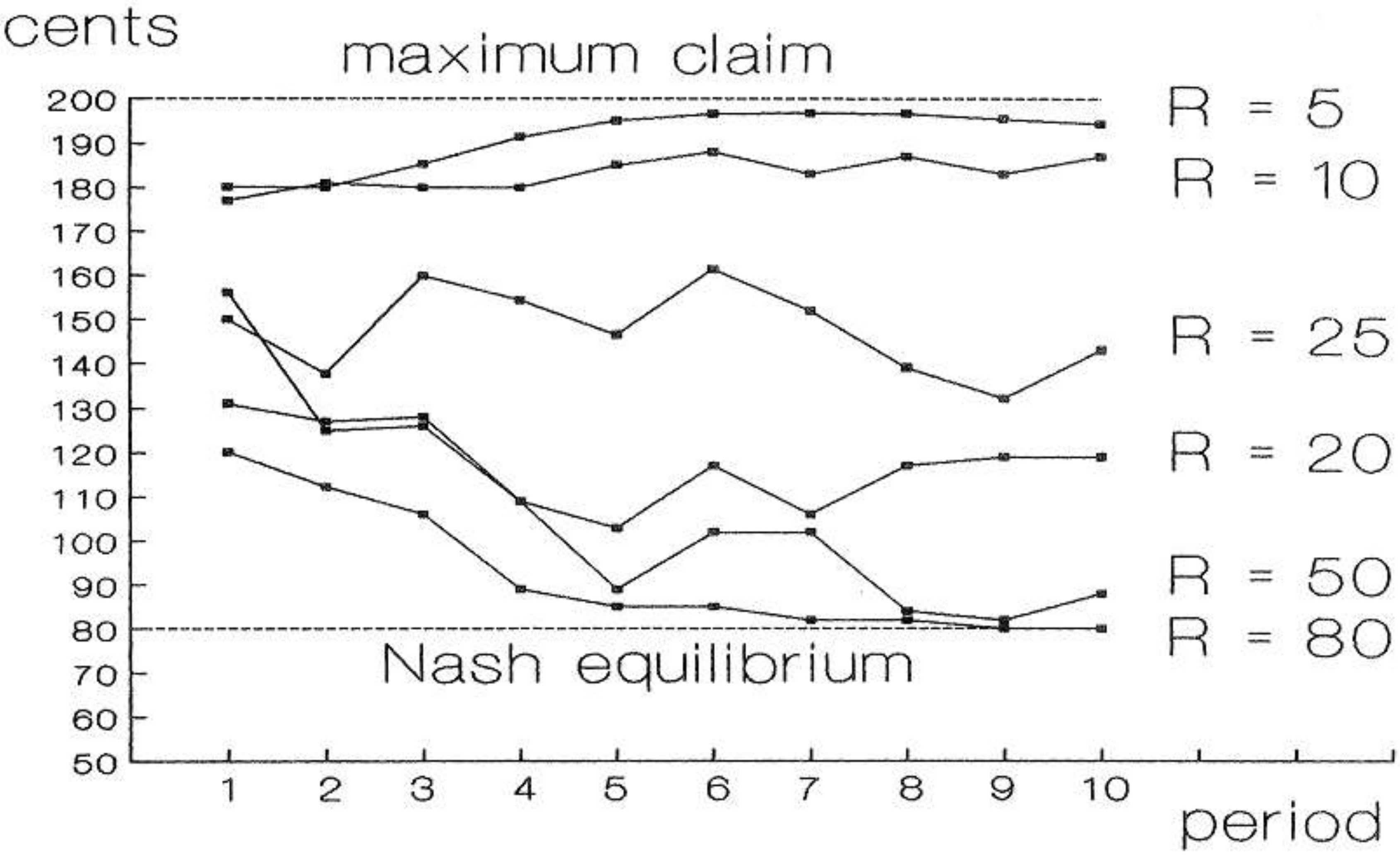
- Capra, Goeree, Gomez and Holt (AER 1999)
  - Two travelers state claim p_1 and p_2 : 80-200
  - Airline awards both the minimum claim, but
  - Reward R to the one who stated the lower claim
  - Penalize the other by R
- Unique NE: race to the bottom

  $\rightarrow$ lowest claim

  - Like price competition game or p-beauty contest

Traveler's Dilemma: Data

- Low R: Collude
  - Little reward to deviate
- High R: Nash
  - Large reward to deviate



p-Beauty Contest Games 選美結果預測實驗

- Each of N players choose x_i from $[0, 100]$
  - 每人選擇0到100之間的數字，希望最接近「所有數字平均乘以p倍」
- Target is $p \times (\text{average of } x_i)$
- Closest x_i wins fixed prize
- For $p = 2/3$,
- $(67, 100]$ violates 1st order dominance
  - 選擇67-100的人是選擇(一階的)劣勢策略
- $(45, \ 67]$ obeys 1 step of dominance (but not 2)
  - 選擇45-67的人是選擇除去一階劣勢策略後剩下的(二階)劣勢策略
- 1st Experiment (最早的實驗): Nagel (AER 1995)

Figure 1A of Nagel (AER 1995): $p = 1/2$

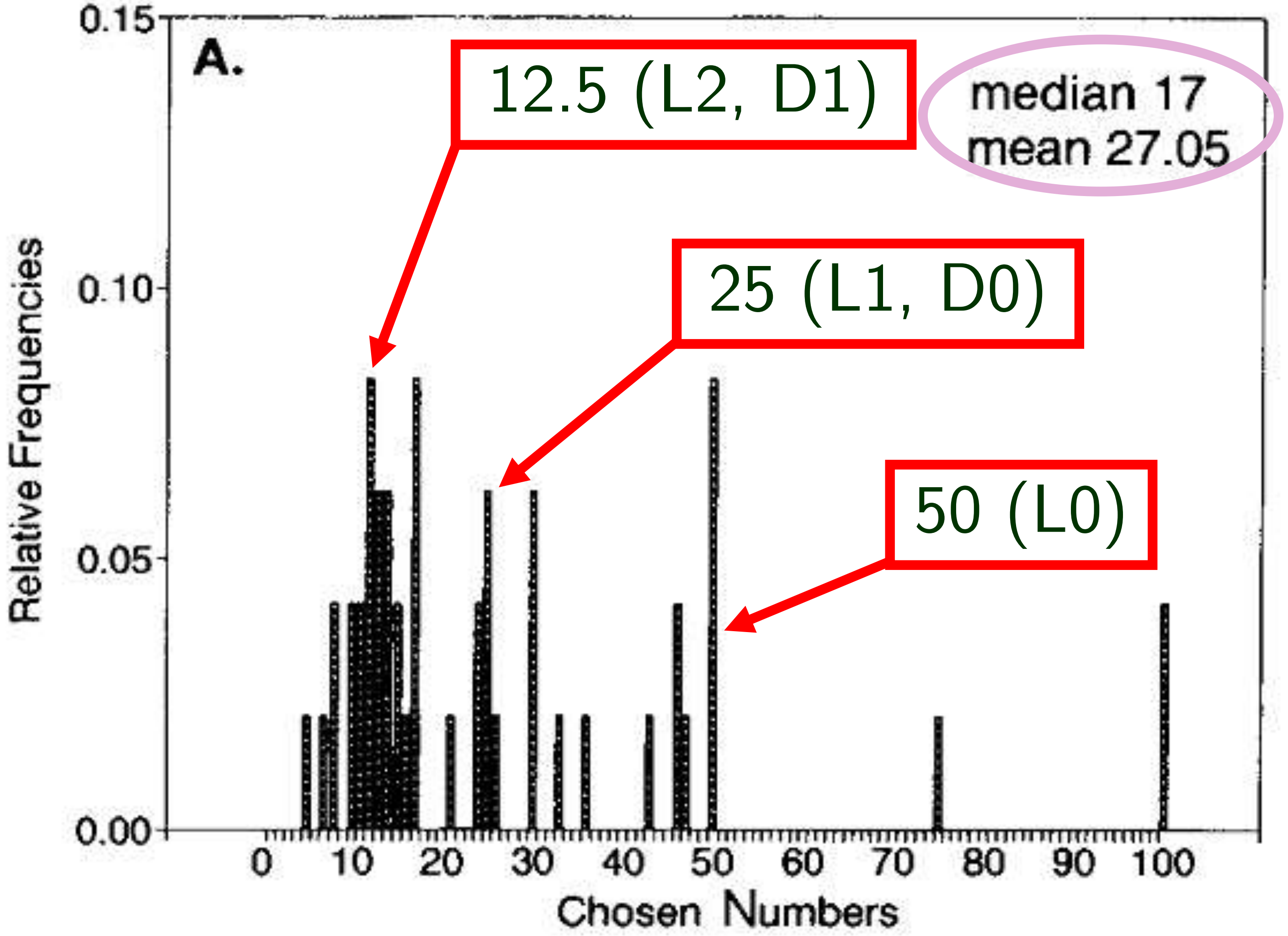
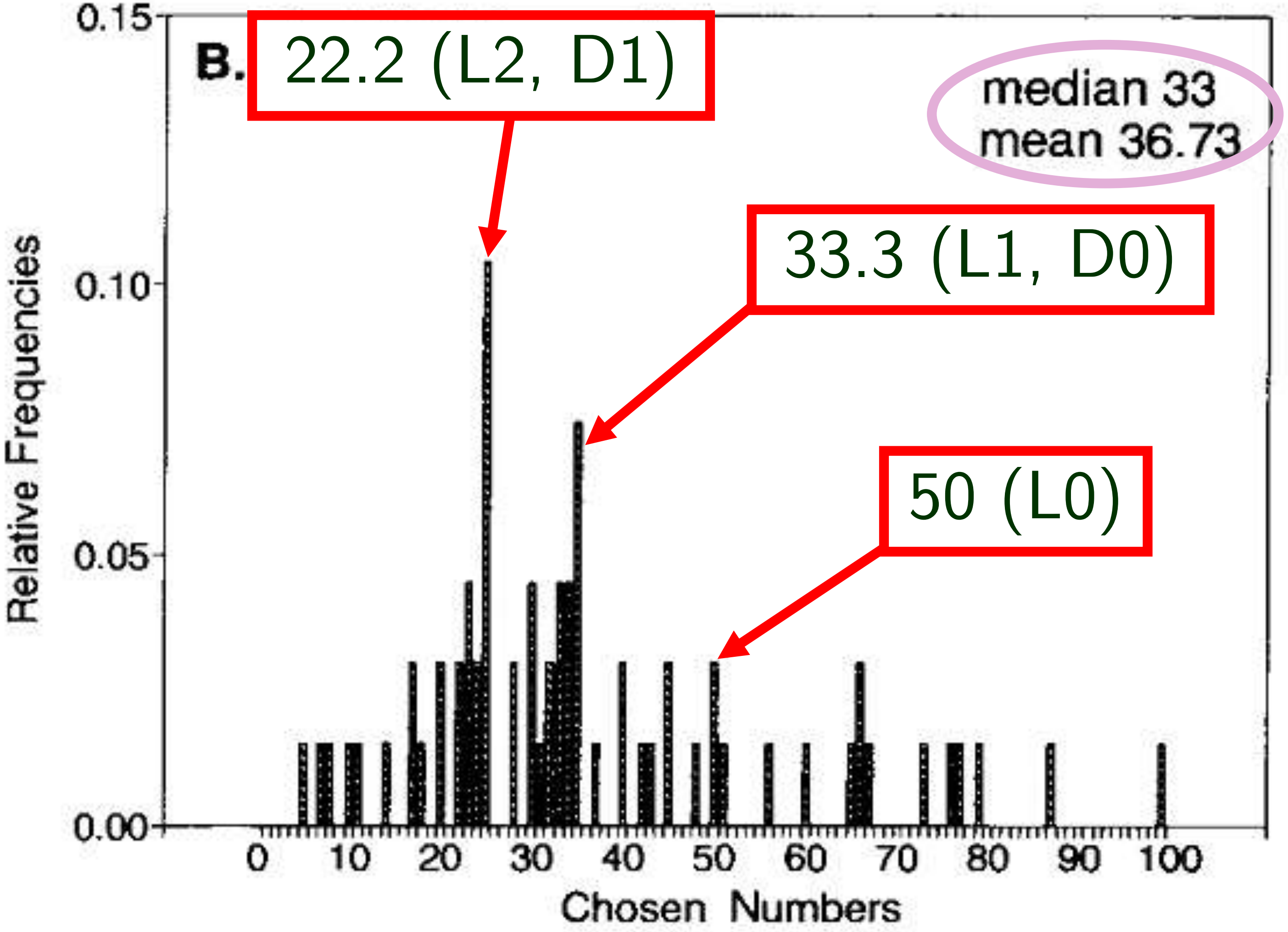


Figure 1B of Nagel (AER 1995): $p = 2/3$



p-Beauty Contest Games (選美結果預測實驗)

- Named after Keynes, General Theory (1936)
- "...professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, (專業投資好比報紙上的選美比賽，要從上百張照片挑出最漂亮的六張)
- the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole..."
  - (目標是選擇最接近「平均參賽者會選到的照片」)

p-Beauty Contest Games (選美結果預測實驗)

- It is not a case of choosing those [faces] that, to the best of one's judgment, are really the prettiest,
  - 「這不是要挑每個人各自認為最漂亮的[臉蛋],
- nor even those that average opinion genuinely thinks the prettiest.
  - 更不是要挑大家公認最漂亮的。
- We have reached the third degree where we devote our intelligences to...
  - 我們已經想到第三層去,

p-Beauty Contest Games (選美結果預測實驗)

- Anticipating what average opinion expects the average opinion to be.
  - 努力預測一般人心目中認為大家公認最漂亮的會是誰。
- And there are some, I believe, who practice the fourth, fifth and higher degrees."
  - 而且我相信有些人還可以想到第四層、第五層或更高。」

  - Keynes (凱因斯, 1936, p.156)
- Follow-up Studies (後續研究)
  - Camerer, Ho and Weigelt (AER 1998)

Camerer, Ho & Weigelt (AER 1998): Design

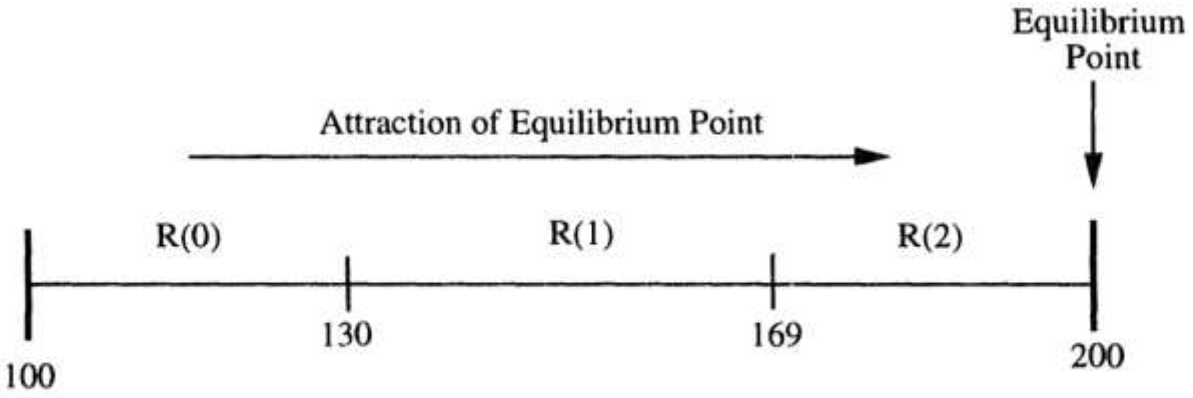


FIGURE 1A. A FINITE-THRESHOLD GAME, $FT(n) = ([100, 200], 1.3, n)$

3 rounds of IEDS

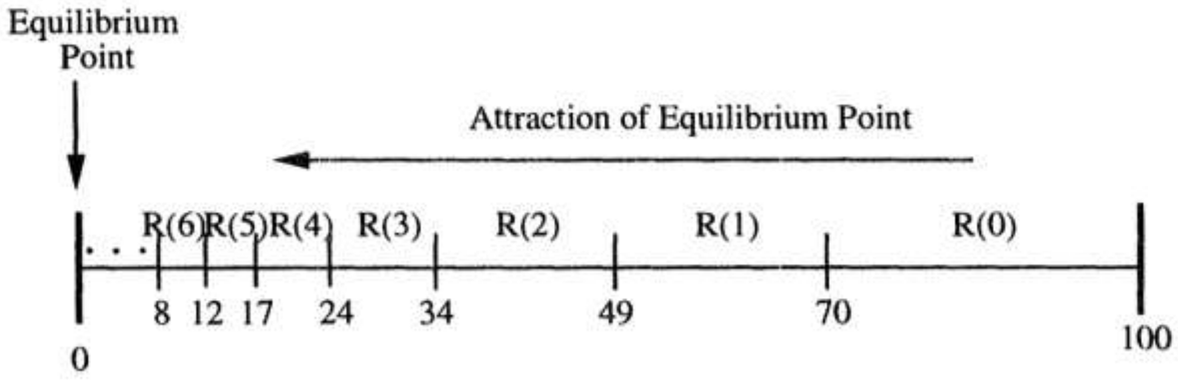


FIGURE 1B. AN INFINITE-THRESHOLD GAME, $IT(n) = ([0, 100], 0.7, n)$

∞ rounds of IEDS

Camerer, Ho & Weigelt (AER 1998): Design

TABLE 1—THE EXPERIMENTAL DESIGN

實驗設計

	Group size 3	每組人數: 3 vs. 7	Group size 7
先做有限次再做無限次(刪劣勢策略)	Finite → Infinite		
	FT(1.3, 3) → *IT*(0.7, 3) (7 groups)	1.3→0.7	*FT*(1.3, 7) → *IT*(0.7, 7) (7 groups)
	FT(1.1, 3) → *IT*(0.9, 3) (7 groups)	1.1→0.9	*FT*(1.1, 7) → *IT*(0.9, 7) (7 groups)
先做無限次再做有限次	Infinite → Finite		
	IT(0.7, 3) → *FT*(1.3, 3) (7 groups)	0.7→1.3	*IT*(0.7, 7) → *FT*(1.3, 7) (7 groups)
	IT(0.9, 3) → *FT*(1.1, 3) (6 groups)	0.9→1.1	*IT*(0.9, 7) → *FT*(1.1, 7) (7 groups)

Camerer, Ho and Weigelt (AER 1998)

Result 1:

- First-period choices are far from equilibrium, and centered near the interval midpoint.
- Choices converge toward the equilibrium point over time.

- Baseline: IT(0.9,7) and IT(0.7, 7)

Camerer, Ho and Weigelt (AER 1998): p=0.9 vs. 0.7

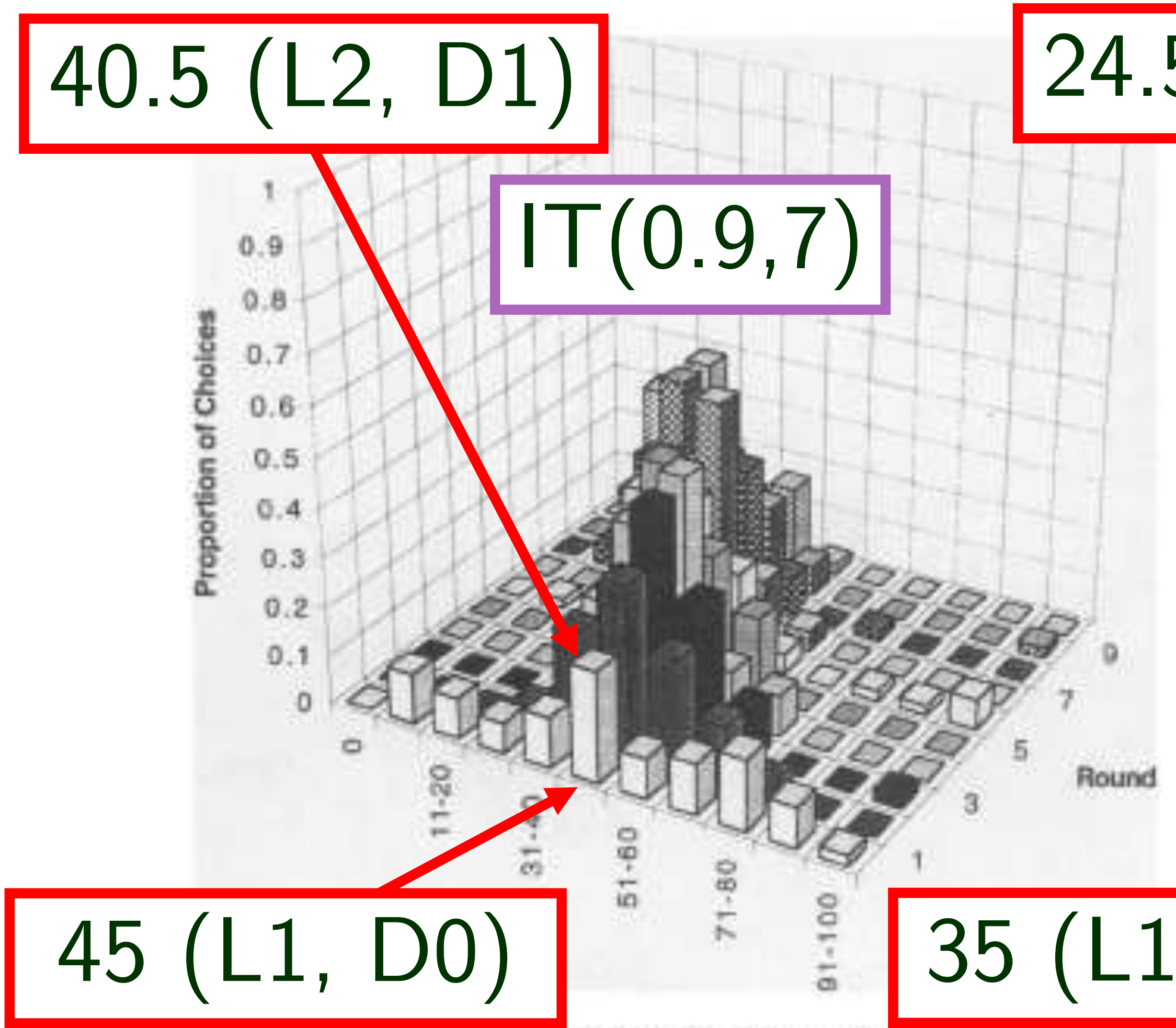


FIGURE 2C. INEXPERIENCED SUBJECTS' CHOICES OVER ROUND IN $IT(0.9, 7)$

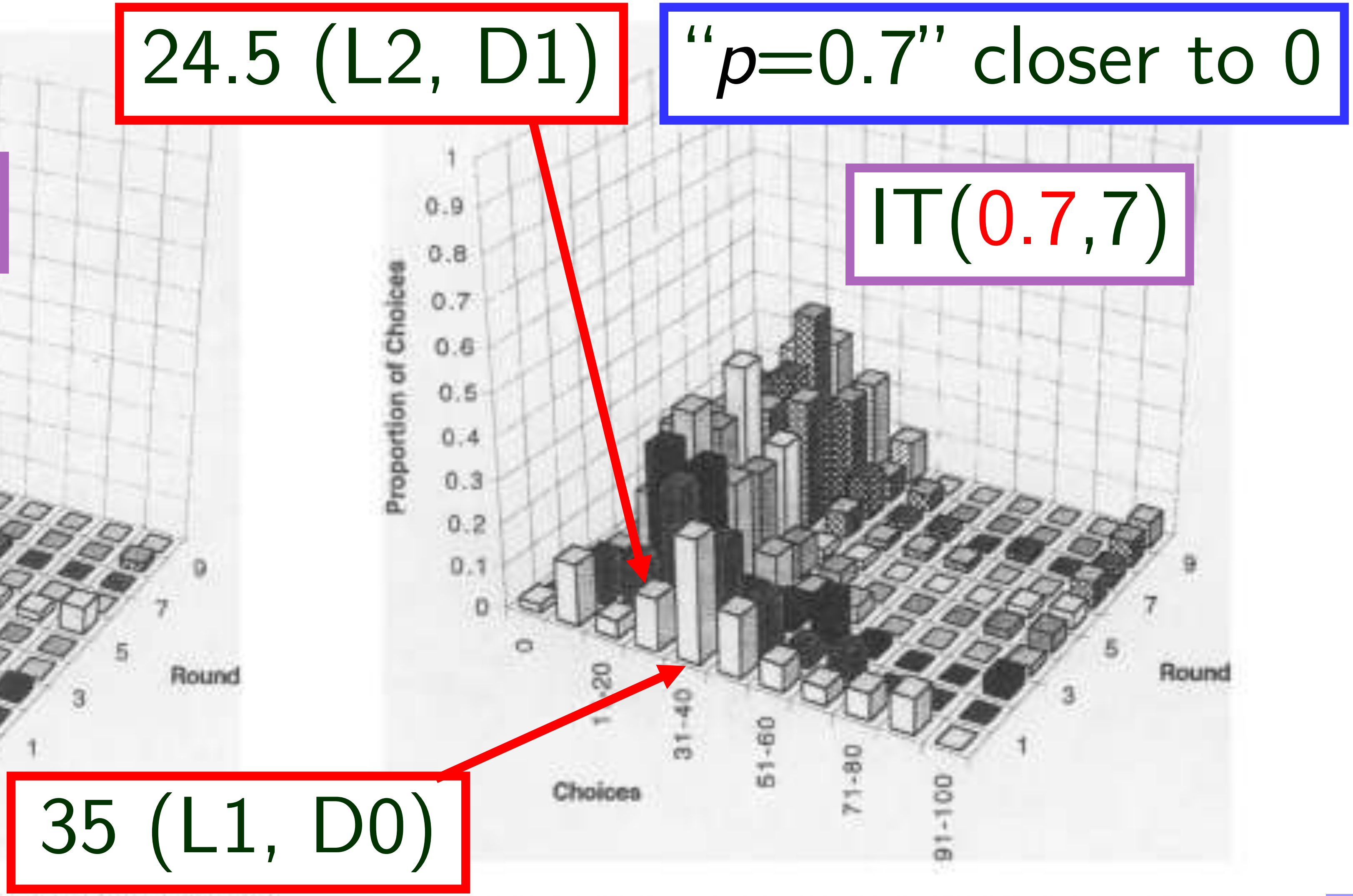


FIGURE 2A. INEXPERIENCED SUBJECTS' CHOICES OVER ROUND IN $IT(0.7, 7)$

Camerer, Ho and Weigelt (AER 1998)

Result 2:

- On average, choices are closer to equilibrium for
  - Games with finite thresholds, and
  - Games with p further from 1.

- Infinite vs. Finite...

Camerer, Ho and Weigelt (1998): FT vs. IT

FT closer to Equilibrium

FT(1.3,3)

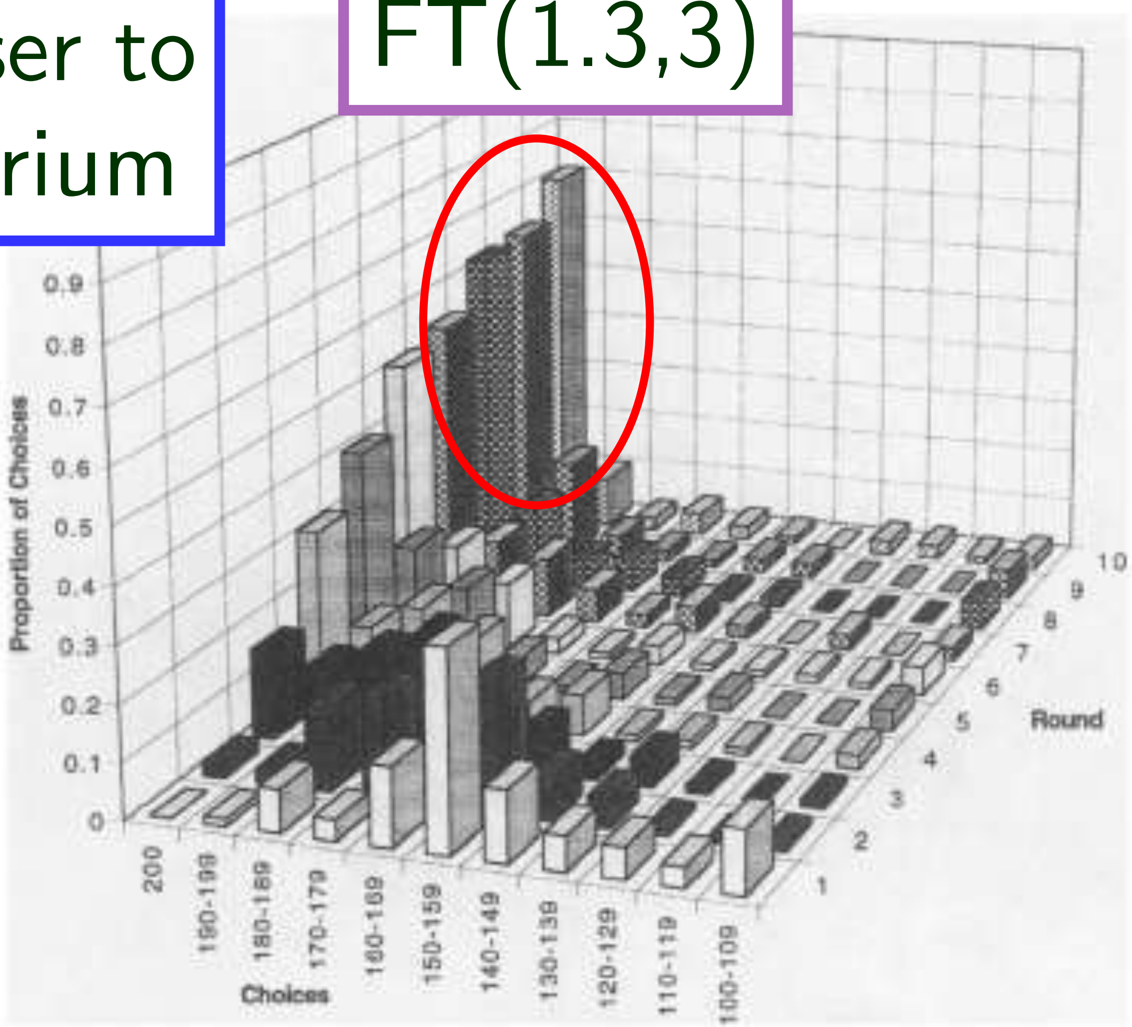
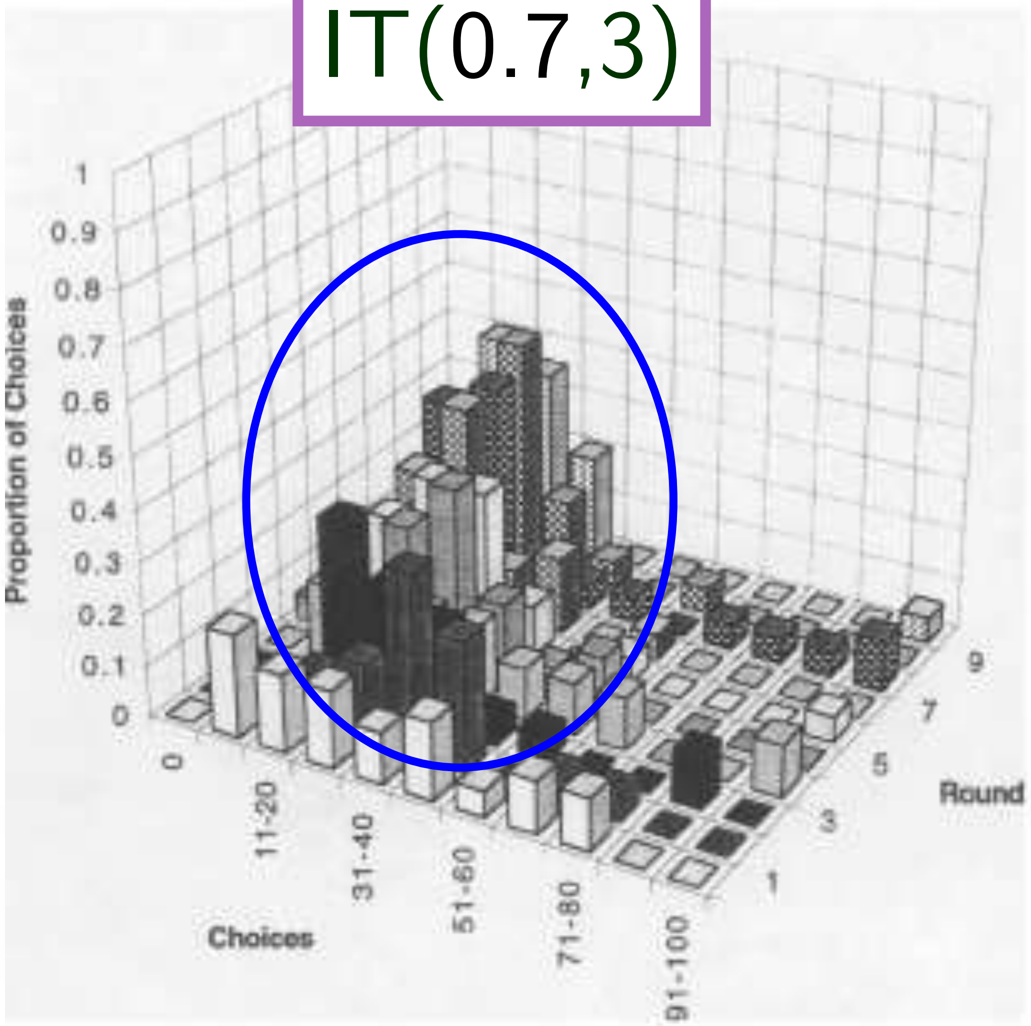


FIGURE 3A. CHOICES OVER ROUND IN *FT* GAMES PLAYED BY 3-PERSON GROUPS

IT(0.7,3)



URE 2E. INEXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.7, 3)

Result 3:

- Choices are closer to equilibrium
- for large (7-person) groups than for small (3-person) groups.
- 7-group vs. 3-group...

Camerer, Ho and Weigelt (1998): FT 3 vs. 7

FT(1.3,3)

FT(1.3,7)

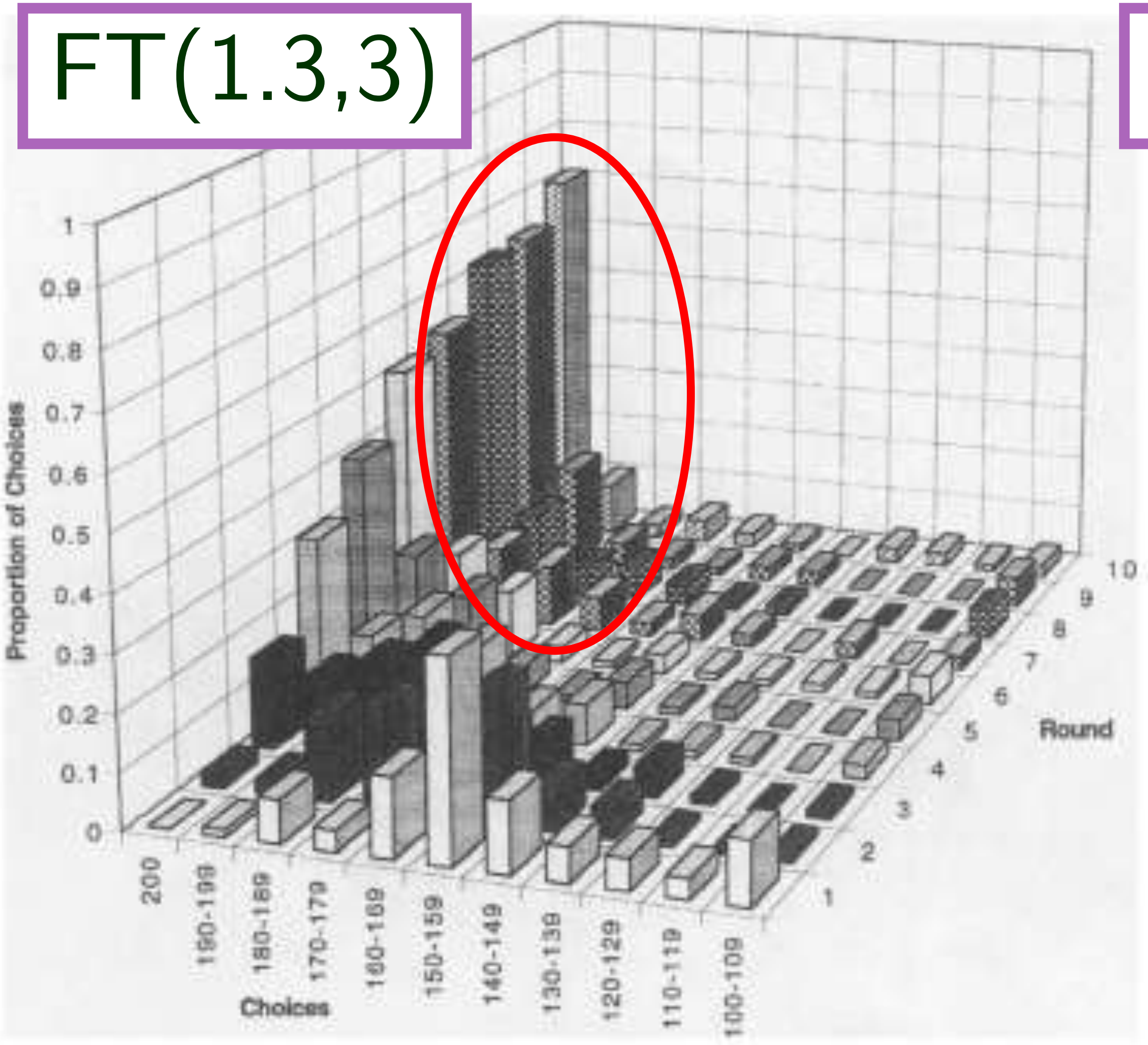


FIGURE 3A. CHOICES OVER ROUND IN *FT* GAMES PLAYED BY 3-PERSON GROUPS

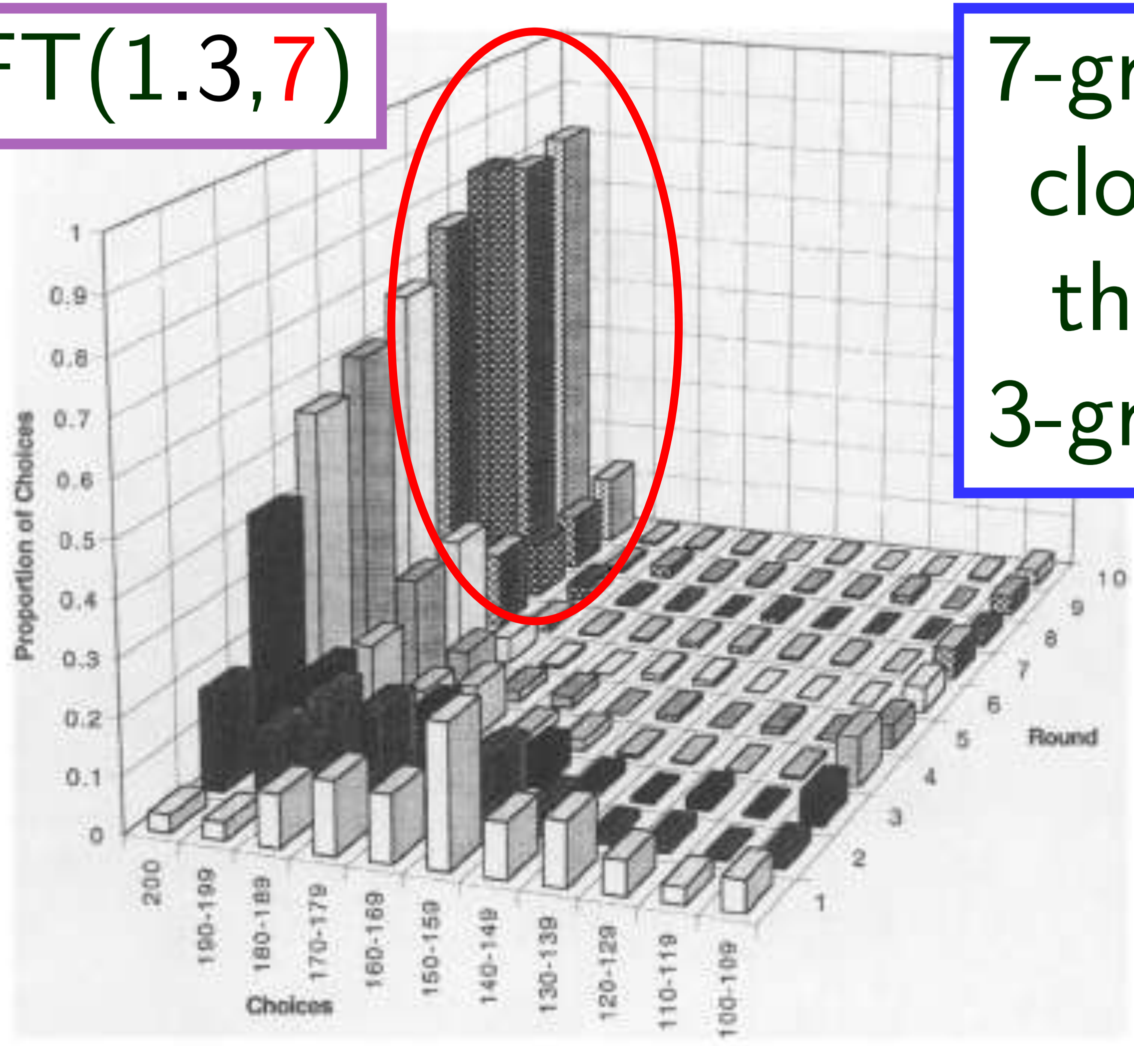


FIGURE 3B. CHOICES OVER ROUND IN *FT* GAMES PLAYED BY 7-PERSON GROUPS

7-group closer than 3-group

Camerer, Ho and Weigelt (1998): IT 3 vs. 7 (0.7)

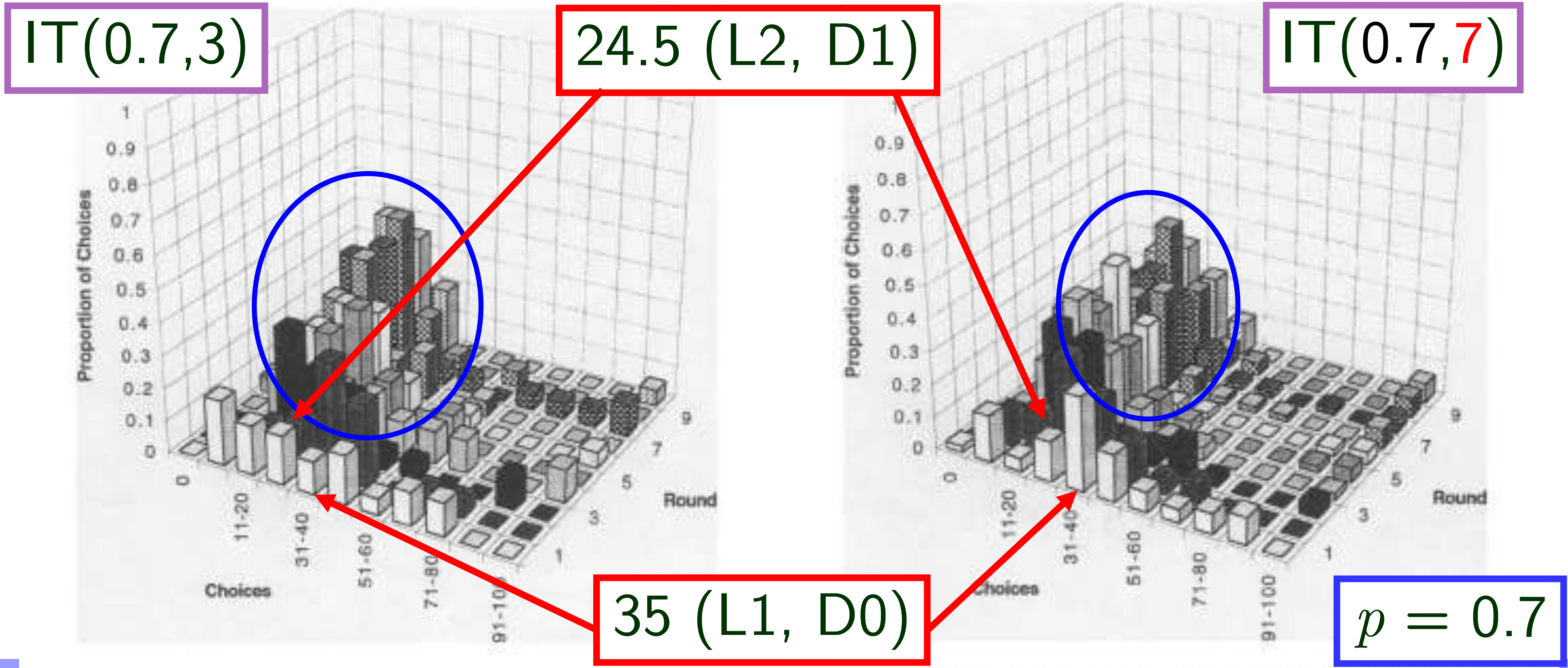


FIGURE 2E. INEXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.7, 3)

FIGURE 2A. INEXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.7, 7)

Camerer, Ho and Weigelt (1998): IT 7 vs. 3 (0.9)

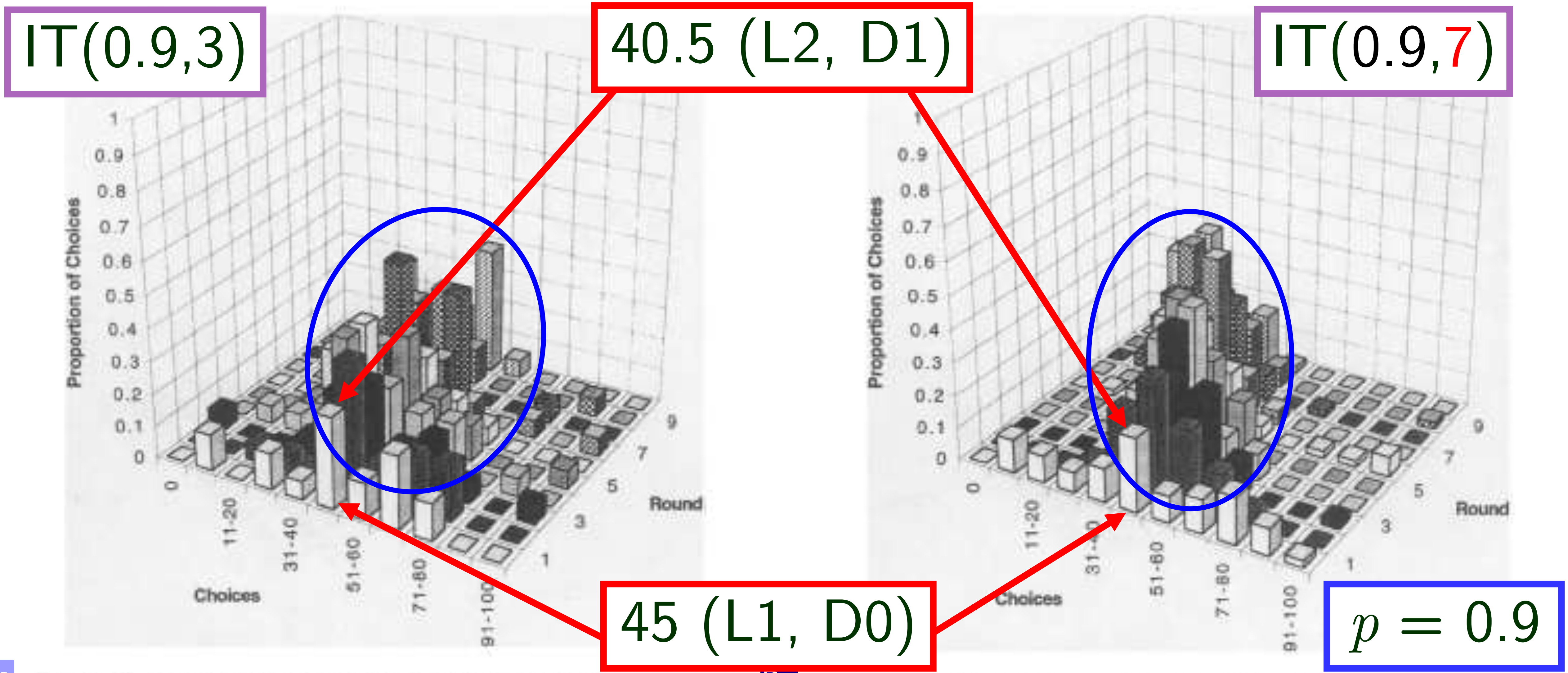


FIGURE 2G. INEXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.9, 3)

FIGURE 2C. INEXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.9, 7)

Camerer, Ho and Weigelt (AER 1998)

Result 4:

- Choices by [cross-game] experienced subjects are
- No different than choices by inexperienced subjects in the first round,
- But converge faster to equilibrium.
  - IT(0.7,7)-first vs. IT(0.7,7)-later
  - IT(0.9,7)-first vs. IT(0.9,7)-later
  - IT(0.7,3)-first vs. IT(0.7,3)-later
  - IT(0.9,3)-first vs. IT(0.9,3)-later

Camerer, Ho and Weigelt (1998): Experience

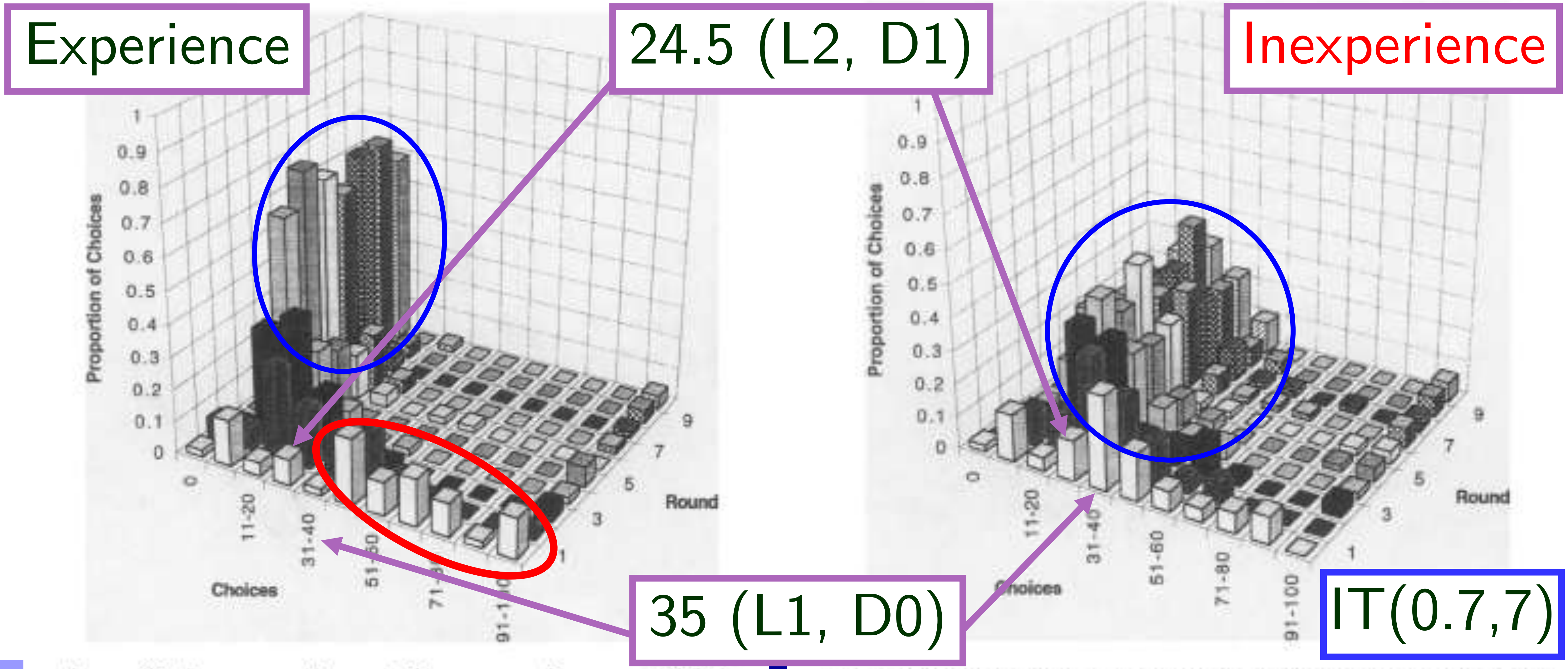


Figure 2B. Experienced Subjects' Choices over Round in IT(0.7, 7)

Figure 2A. Inexperienced Subjects' Choices over Round in IT(0.7, 7)

Camerer, Ho and Weigelt (1998): Experience

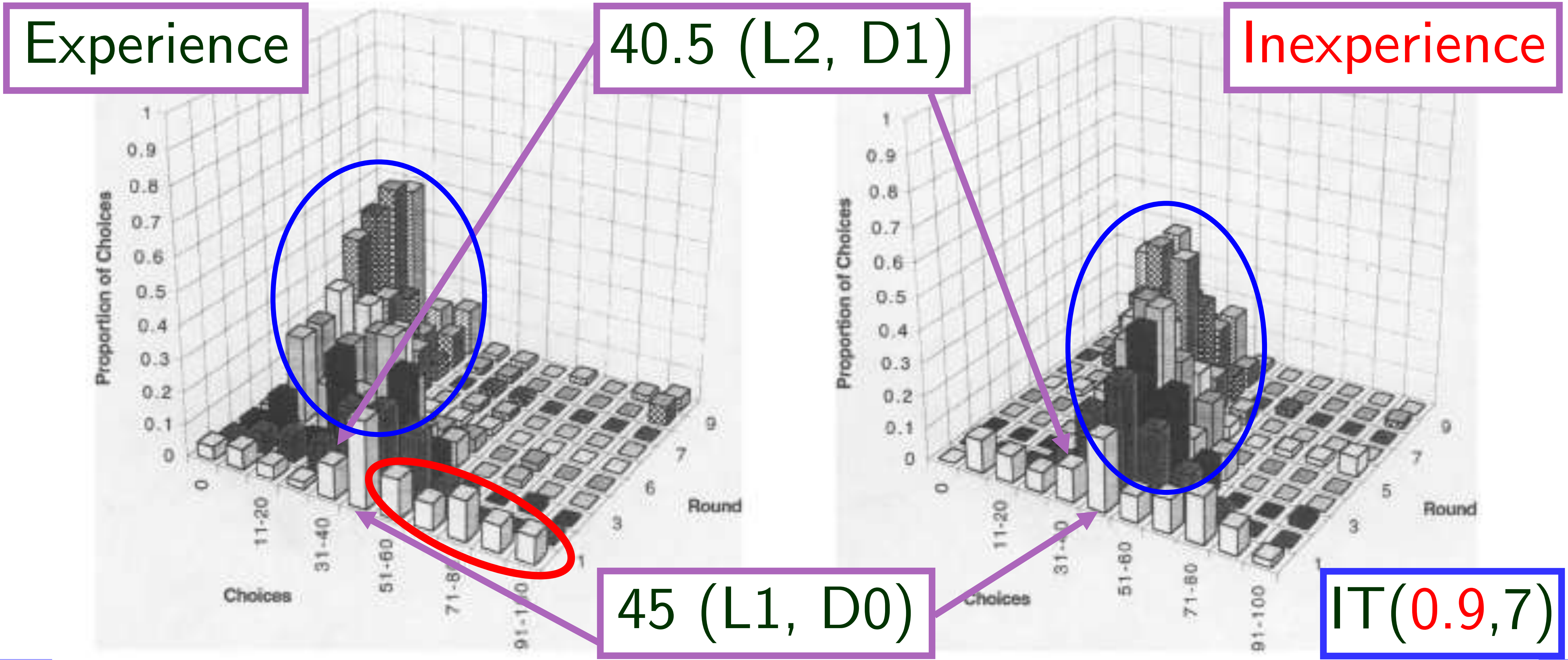


FIGURE 2D. EXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.9, 7)

FIGURE 2C. INEXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.9, 7)

Camerer, Ho and Weigelt (1998): Experience

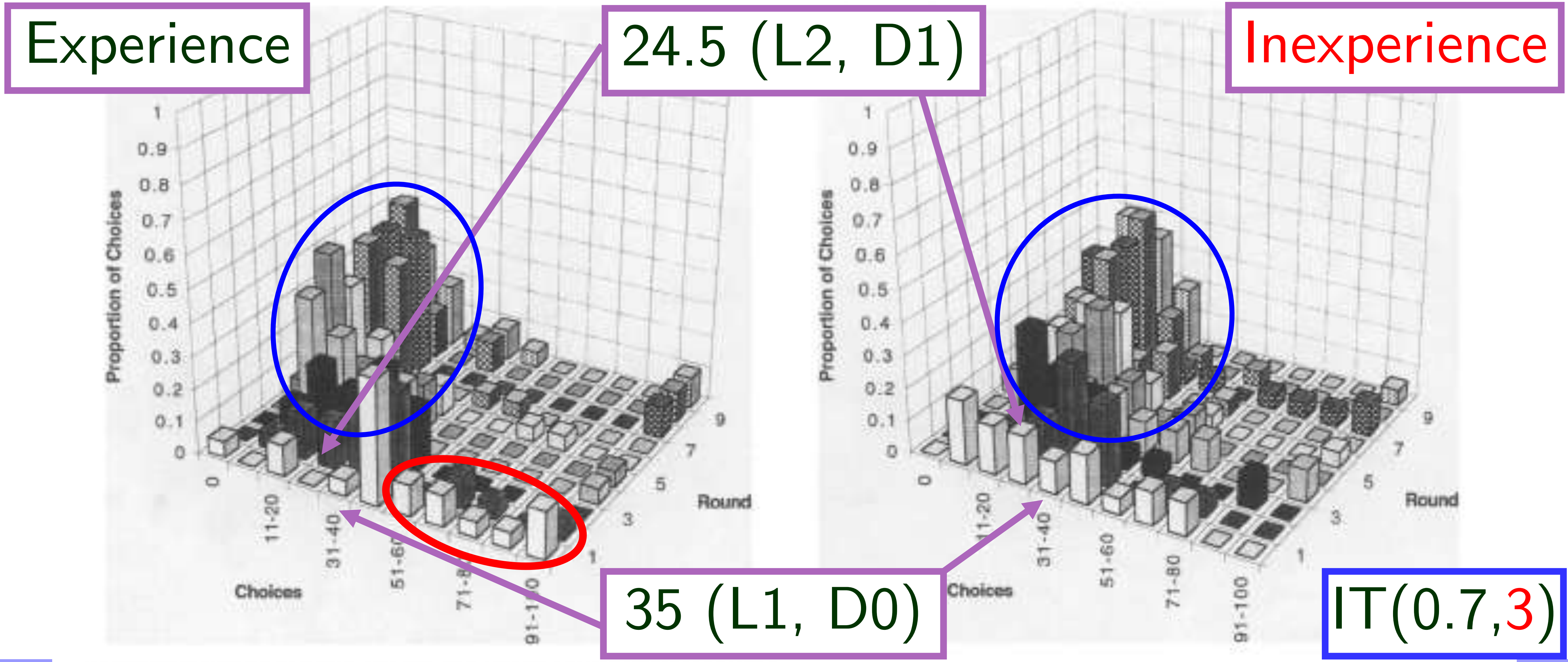


FIGURE 2F. EXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.7, 3)

GURE 2E. INEXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.7, 3)

Camerer, Ho and Weigelt (1998): Experience

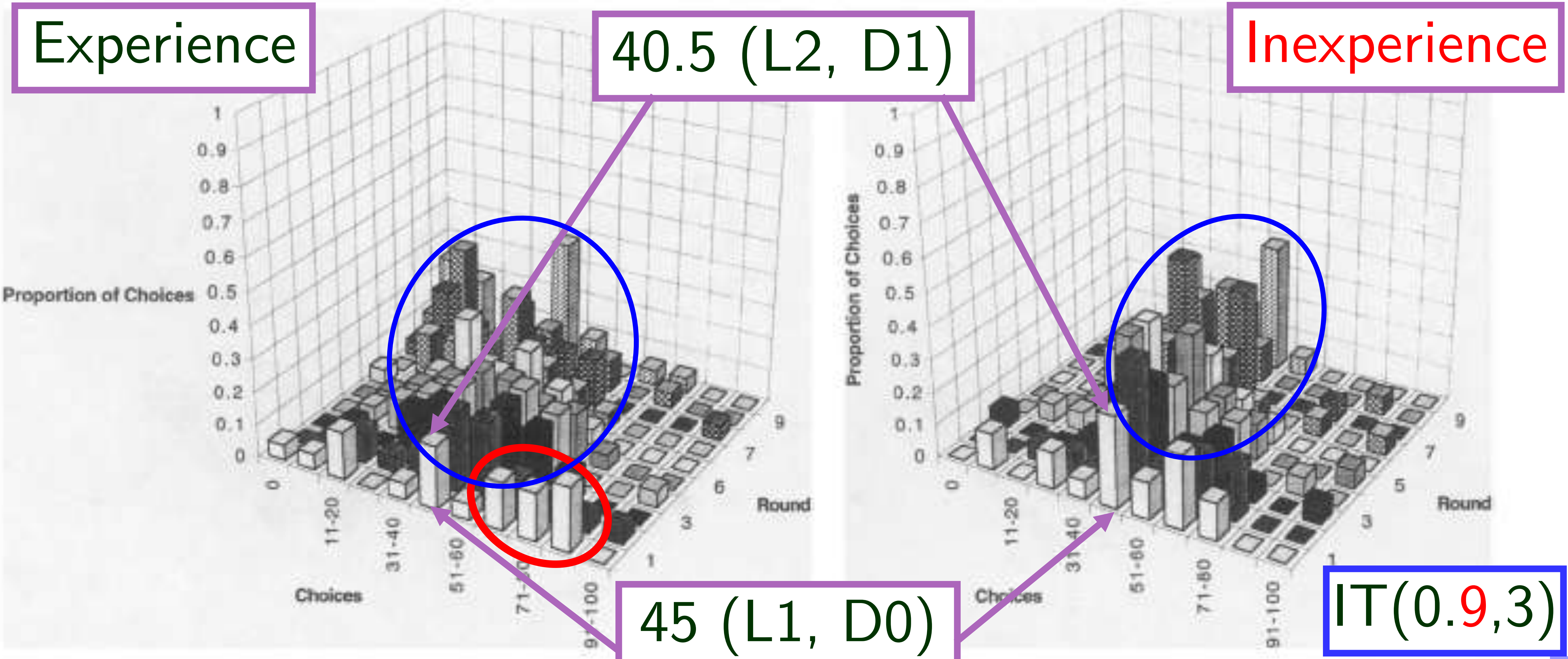


FIGURE 2H. EXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.9, 3)

FIGURE 2G. INEXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.9, 3)

Camerer, Ho and Weigelt (AER 1998)

- Classification of Types
  - Follow Stahl and Wilson (GEB 1995)
- Level-0: pick randomly from $\mathrm{N}(\mu, \sigma)$
- Level-1: BR to level-0 with noise
- Level-2: BR to level-1 with noise
- Level-3: BR to level-2 with noise

- Estimate type, error using MLE

Camerer, Ho and Weigelt (AER 1998)

Type Distribution...

TABLE 3—MAXIMUM-LIKELIHOOD ESTIMATES AND LOG-LIKELIHOODS FOR LEVELS OF ITERATED DOMINANCE (FIRST-ROUND DATA ONLY)

Parameter estimates	Out data (groups of 3 or 7)		Nagel's data (groups of 16–18)	
	$IT(p, n)$	$FT(p, n)$	$IT(0.5, n)$	$IT(2/3, n)$
ω_0	15.93	21.72	45.83 (23.94)	28.36 (13.11)
ω_1	20.74	31.46	37.50 (29.58)	34.33 (44.26)
ω_2	13.53	12.73	16.67 (40.84)	37.31 (39.34)
ω_3	49.50	34.08	0.00 (5.63)	0.00 (3.28)
μ	70.13	100.50	35.53 (50.00)	52.23 (50.00)
σ	28.28	26.89	22.70	14.72
ρ	1.00	1.00	0.24	1.00
$-LL$	1128.29	1057.28	168.48	243.95

Robustness Checks

- High stakes (Fig.1.3 - small effect lowering numbers)
- Median vs. Mean (Nagel 1999 - same): BGT Fig. 5.1
- $p \times (\text{median of } x_i + 18)$: Equilibrium is inside the range

- Subject Pool Variation:
  - Portfolio managers, Econ PhD, Caltech undergrads
  - CEOs: Caltech Board of Trustees
  - Readers of Financial Times and Expansion
- Experience vs. Inexperience (for the same game)
  - Slonim (EE 2005) – Experience good only for 1st round

Level-k Reasoning

- Theory for Initial Response (BGT, Ch. 5)
  vs. Theory for Equilibration (BGT, Ch. 6)
- First: Stahl and Wilson (GEB 1995)
  - Better: Costa-Gomes, Crawford and Broseta (ECMA 2001)
- Best 1: Camerer, Ho and Chong (QJE 2004)
  - Poisson Cognitive Hierarchy
- Best 2: Costa-Gomes and Crawford (AER 2006)
  - CGC: Level-k Model

Level-k Theory: Stahl and Wilson (GEB 1995)

- Stahl and Wilson (GEB 1995) propose:
- Level-0: Random play
- Level-1: BR to Random play
- Level-2: BR to Level-1
- Nash: Play Nash Equilibrium
- Worldly: BR to distribution of Level-0, Level-1 and Nash types

TABLE IV

PARAMETER ESTIMATES AND CONFIDENCE INTERVALS FOR MIXTURE MODEL WITHOUT RE TYPES

	Estimate	Std. Dev.	95 percent conf. int.	
γ_1	0.2177	0.0425	0.1621	0.3055
μ_2	0.4611	0.0616	0.2014	0.8567
			[0.2360	0.8567]
γ_2	3.0785	0.5743	1.9029	4.9672
			[2.5631	5.0000]
γ_3	4.9933	0.9357	1.9964	5.0000
μ_4	0.0624	0.0063	0.0527	0.0774
ϵ_4	0.4411	0.0773	0.2983	0.5882
γ_4	0.3326	0.0549	0.2433	0.4591
α_0	0.1749	0.0587	0.0675	0.3047
α_1	0.2072	0.0575	0.1041	0.3298
α_2	0.0207	0.0202	0.0000	0.0625
α_3	0.1666	0.0602	0.0600	0.2957
α_4	0.4306	0.0782	0.2810	0.5723
$\mathcal{L}$	-442.727			

Type Distribution...

Level-k Theory: CGCB (ECMA 2001)

- Costa-Gomes, Crawford & Broseta (2001)
- Design 18 2-player NF games to separate:
  - Altruistic (max sum)
  - Optimistic (maximax), Pessimistic (maximin)
  - L1 (=Naïve; BR to L0)
  - L2 (BR to L1)
  - D1/D2 (1/2 round of DS deletion)
  - Sophisticated (BR to empirical)
  - Equilibrium (play Nash)

Level-k Theory: CGCB (ECMA 2001)

- Three treatments (all no feedback):

1. Baseline (B)
   - Mouse click to open payoff boxes
2. Open Box (OB)
   - Payoff boxes always open
3. Training (TS)
   - Rewarded to choose equilibrium strategies

Result 1: Strategies Consistent with Iterated Dominance

- B, OB: 90%, 65%, 15% Equilibrium Play
  - For Equilibria Requiring 1, 2, 3 levels of Iterative Dominance
- TS: 90-100% Equilibrium Play
  - For all levels
- "Game-theoretic reasoning is not computationally difficult, but unnatural."

Result 2: Estimate Subject Decision Rule

Rules	E(u)	Choice (%)	Choice + Lookup (%)
Altruistic	17.11	8.9	2.2
Pessimistic	20.93	0	4.5
L1 (Naïve)	21.38	22.7	44.8
Optimistic	21.38	0	2.2
L2	24.87	44.2	44.1
D1	24.13	19.5	0
D2	23.95	0	0
Equilibrium	24.19	5.2	0
Sophisticated	24.93	0	2.2

Result 3: Information Search Patterns

Subject / Rule	↕ own payoff		↔ other payoff	
	Predicted	Actual	Predicted	Actual
TS (Equilibrium)	>31	63.3	>31	69.3
Equilibrium	>31 →	21.5	>31	79.0
L1/Optimistic	<31	21.1	-	48.3
Altruistic	<31	21.1	-	60.0
L2	>31	39.4	=31	30.3
D1	>31 →	28.3	>31	61.7

Result 3: Information Search Patterns

- Occurrence (weak requirement)
  - All necessary lookups exist somewhere
- Adjacency (strong requirement)
  - Payoffs compared by rule occur next to each other
- H-M-L-0: % of subjects w/ 67-100%, 34-66% or 0-33% compliance with Adjacency
- H-M-L-0: % of subjects not having 100% compliance with Occurrence

Result 3: Information Search Patterns

- Whole table is large...

TABLE V

AGGREGATE RATES OF COMPLIANCE WITH TYPES' OCCURRENCE AND ADJACENCY FOR TS AND BASELINE SUBJECTS, AND FOR BASELINE SUBJECTS BY MOST LIKELY TYPE ESTIMATED FROM DECISIONS ALONE, IN PERCENTAGES (— VACUOUS)

Treatment (# subjects)	*Altruistic* $J = H,M,L,0$	*Pessimistic* $j = H,M,L,0$	*Naïve* $j = H,M,L,0$	*Optimistic* $j = A,0$	*L2* $j = H,M,L,0$	*D1* $j = H,M,L,0$	*D2* $j = H,M,L,0$	*Equilibrium* $j = H,M,L,0$	*Sophisticated* $j = H,M,L,0$
TS (12)	3,10,50,27	44,7,36,13	83,2,0,15	86,14	76,2,0,22	92,3,1,5	92,3,1,5	96,1,1,3	75,1,1,24
Baseline (45)	14,11,51,24	74,2,11,14	78,4,4,14	85,15	67,14,5,14	52,19,15,14	50,19,15,14	42,23,19,16	39,21,20,21
Altruistic (2)	78,6,11,6	56,8,33,3	53,3,42,3	97,3	47,8,39,6	36,6,56,3	33,8,56,3	31,11,56,3	28,14,56,3
Pessimistic (0)	—,—,—,—	—,—,—,—	—,—,—,—	—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—
Naïve/Optim. (11)	9,5,53,33	85,1,9,5	89,5,3,4	96,4	42,24,3,31	45,22,20,13	43,18,23,16	26,24,28,23	23,23,27,27
L2 (23)	8,12,58,22	72,2,9,17	78,3,0,18	80,20	85,6,3,6	57,20,9,15	54,21,10,15	49,24,12,15	46,22,12,20
D1 (7)	23,21,26,29	59,3,16,23	63,7,6,23	77,23	53,21,6,21	48,17,14,20	45,19,15,21	42,20,17,21	38,14,21,27
D2 (0)	—,—,—,—	—,—,—,—	—,—,—,—	—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—
Equilibrium (2)	6,8,86,0	100,0,0,0	97,3,0,0	100,0	64,36,0,0	69,17,14,0	67,19,14,0	56,25,19,0	53,19,28,0
Sophisticated (0)	—,—,—,—	—,—,—,—	—,—,—,—	—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—

Result 3: Information Search Patterns

- 96% of TS are High Adjacency
  - Only 3% violate Occurrence
- 56% of Equilibrium are High Adjacency
  - 25/19% are M/L
  - 100% Occurrence
  - "Game-theoretic reasoning is not difficult, but unnatural."

Treatment (# subjects)	*Equilibrium* $j = H,M,L,0$	*Sophisticated* $j = H,M,L,0$
TS (12)	96,1,1,3	75,1,1,24
Baseline (45)	42,23,19,16	39,21,20,21
Altruistic (2)	31,11,56,3	28,14,56,3
Pessimistic (0)	—,—,—,—	—,—,—,—
Naïve/Optim. (11)	26,24,28,23	23,23,27,27
L2 (23)	49,24,12,15	46,22,12,20
D1 (7)	42,20,17,21	38,14,21,27
D2 (0)	—,—,—,—	—,—,—,—
Equilibrium (2)	56,25,19,0	53,19,28,0
Sophisticated (0)	—,—,—,—	—,—,—,—

Result 3: Information Search Patterns

- Optimistic has vacuous Adjacency
  - Adjacency = Occurrence

Treatment (# subjects)	*Altruistic* $J = H,M,L,0$	*Pessimistic* $j = H,M,L,0$	*Naïve* $j = H,M,L,0$	*Optimistic* $j = A,0$	*L2* $j = H,M,L,0$	*D1* $j = H,M,L,0$	*D2* $j = H,M,L,0$
TS (12)	3,10,50,27	44,7,36,13	83,2,0,15	86,14	76,2,0,22	92,3,1,5	92,3,1,5
Baseline (45)	14,11,51,24	74,2,11,14	78,4,4,14	85,15	67,14,5,14	52,19,15,14	50,19,15,14
Altruistic (2)	78,6,11,6	56,8,33,3	53,3,42,3	97,3	47,8,39,6	36,6,56,3	33,8,56,3
Pessimistic (0)	—,—,—,—	—,—,—,—	—,—,—,—	—,—	—,—,—,—	—,—,—,—	—,—,—,—
Naïve/Optim. (11)	9,5,53,33	85,1,9,5	89,5,3,4	96,4	42,24,3,31	45,22,20,13	43,18,23,16
L2 (23)	8,12,58,22	72,2,9,17	78,3,0,18	80,20	85,6,3,6	57,20,9,15	54,21,10,15
D1 (7)	23,21,26,29	59,3,16,23	63,7,6,23	77,23	53,21,6,21	48,17,14,20	45,19,15,21
D2 (0)	—,—,—,—	—,—,—,—	—,—,—,—	—,—	—,—,—,—	—,—,—,—	—,—,—,—
Equilibrium (2)	6,8,86,0	100,0,0,0	97,3,0,0	100,0	64,36,0,0	69,17,14,0	67,19,14,0
Sophisticated (0)	—,—,—,—	—,—,—,—	—,—,—,—	—,—	—,—,—,—	—,—,—,—	—,—,—,—

Result 3: Information Search Patterns

- Most frequent types *Altruistic*, *L1* and *L2* have most subjects being High Adjacency (94% Occurrence)

Treatment (# subjects)	*Altruistic* $J = H,M,L,0$	*Pessimistic* $j = H,M,L,0$	*L1/Naïve* $j = H,M,L,0$	*L2* $j = H,M,L,0$	*D1* $j = H,M,L,0$	*D2* $j = H,M,L,0$
TS (12)	3,10,50,27	44,7,36,13	83,2,0,15	76,2,0,22	92,3,1,5	92,3,1,5
Baseline (45)	14,11,51,24	74,2,11,14	78,4,4,14	67,14,5,14	52,19,15,14	50,19,15,14
Altruistic (2)	78,6,11,6	56,8,33,3	53,3,42,3	47,8,39,6	36,6,56,3	33,8,56,3
Pessimistic (0)	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—
Naïve/Optim. (11)	9,5,53,33	85,1,9,5	89,5,3,4	42,24,3,31	45,22,20,13	43,18,23,16
L2 (23)	8,12,58,22	72,2,9,17	78,3,0,18	85,6,3,6	57,20,9,15	54,21,10,15
D1 (7)	23,21,26,29	59,3,16,23	63,7,6,23	53,21,6,21	48,17,14,20	45,19,15,21
D2 (0)	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—
Equilibrium (2)	6,8,86,0	100,0,0,0	97,3,0,0	64,36,0,0	69,17,14,0	67,19,14,0
Sophisticated (0)	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—

Result 3: Information Search Patterns

- 20% *D1* subjects even violate Occurrence
  - More *Equilibrium* and *D1* have High Adjacency for other types

Treatment (# subjects)	*Altruistic* $J = H,M,L,0$	*Pessimistic* $j = H,M,L,0$	*L1/Naïve* $j = H,M,L,0$	*L2* $j = H,M,L,0$	*D1* $j = H,M,L,0$	*D2* $j = H,M,L,0$
TS (12)	3,10,50,27	44,7,36,13	83,2,0,15	76,2,0,22	92,3,1,5	92,3,1,5
Baseline (45)	14,11,51,24	74,2,11,14	78,4,4,14	67,14,5,14	52,19,15,14	50,19,15,14
Altruistic (2)	78,6,11,6	56,8,33,3	53,3,42,3	47,8,39,6	36,6,56,3	33,8,56,3
Pessimistic (0)	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—
Naïve/Optim. (11)	9,5,53,33	85,1,9,5	89,5,3,4	42,24,3,31	45,22,20,13	43,18,23,16
L2 (23)	8,12,58,22	72,2,9,17	78,3,0,18	85,6,3,6	57,20,9,15	54,21,10,15
D1 (7)	23,21,26,29	59,3,16,23	63,7,6,23	53,21,6,21	48,17,14,20	45,19,15,21
D2 (0)	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—
Equilibrium (2)	6,8,86,0	100,0,0,0	97,3,0,0	64,36,0,0	69,17,14,0	67,19,14,0
Sophisticated (0)	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—

Cognitive Hierarchy

- Camerer, Ho and Chong (QJE 2004)
- Poisson distribution of level-k thinkers $f(k|\tau)$
  - τ = mean number of thinking steps
- Level-0: choose randomly or use heuristics
- Level-*k* thinkers use *k* steps of thinking BR to a mixture of lower-step thinkers
  - Belief about others is Truncated Poisson (easy to compute!)
- Explains many data (Does not require initial response!!)

Level-k Theory: Costa-Gomes and Crawford (2006)

- 2-Person Guessing Games (p-beauty contest)
  - Player 1 guesses 300-500, target = 0.7
  - Player 2 guesses 100-900, target = 1.5
  - $0.7 \times 1.5 = 1.05 > 1$...
- Unique Equilibrium: Choose at upper bound (500, 750)
  - In general:
- Target1 x Target2 $>$ 1: Nash = choose at upper bounds
- Target1 x Target2 $<$ 1: Nash = choose at lower bounds

Level-k Theory: Costa-Gomes and Crawford (2006)

- 16 Different Games
- Limits:
- $\alpha = [100, 500]$, $\beta = [100, 900]$,
- $\gamma = [300, 500]$, $\delta = [300, 900]$
- Target: $1 = 0.5$, $2 = 0.7$, $3 = 1.3$, $4 = 1.5$
- No feedback – Elicit Initial Responses

Level-k Theory: Costa-Gomes and Crawford (2006)

- Define Various Types:
- Equilibrium (EQ): BR to Nash (play Nash)
- Defining L0 as uniformly random
  - Based on evidence from past normal-form games
- Level-k types L1, L2, and L3:
- L1: BR to L0
- L2: BR to L1
- L3: BR to L2

Level-k Theory: Costa-Gomes and Crawford (2006)

- Dominance Types:
  - D1: Does one round of dominance and BR to a uniform prior over partner's remaining decisions
  - D2: Does two rounds and BR to a uniform prior
- Sophisticated (SOPH): BR to empirical distribution of others' decisions
  - Ideal type (if all subjects are SOPH, coincide with Equilibrium)
  - See if anyone has a transcended understanding of others' decisions

Level-k

Game	L1	L2	L3	D1	D2	EQ	SOPH
14. β4γ2	600	525	630	600	611.25	750	630
6. δ3γ4	520	650	650	617.5	650	650	650
7. δ3δ3	780	900	900	838.5	900	900	900
11. δ2β3	350	546	318.5	451.5	423.15	300	420
16. α4α2	450	315	472.5	337.5	341.25	500	375
1. α2β1	350	105	122.5	122.5	122.5	100	122
15. α2α4	210	315	220.5	227.5	227.5	350	262
13. γ2β4	350	420	367.5	420	420	500	420
5. γ4δ3	500	500	500	500	500	500	500
4. γ2β1	350	300	300	300	300	300	300
10. α4β1	500	225	375	262.5	262.5	150	300
8. δ3δ3	780	900	900	838.5	900	900	900
12. β3δ2	780	455	709.8	604.5	604.5	390	695
3. β1γ2	200	175	150	200	150	150	162
2. β1α2	150	175	100	150	100	100	132
9. β1α4	150	250	112.5	162.5	131.25	100	187

Level-k Theory: Costa-Gomes and Crawford (2006)

- 43 (out of 88) subjects in the baseline made exact guesses (+/- 0.5) in 7 or more games: (L1, L2, L3, EQ) = (20, 12, 3, 8)

TABLE 1—SUMMARY OF BASELINE AND OB SUBJECTS' ESTIMATED TYPE DISTRIBUTIONS

Type	Apparent from guesses	Econometric from guesses	Econometric from guesses, excluding random	Econometric from guesses, with specification test	Econometric from guesses and search, with specification test
L1	20	43	37	27	29
L2	12	20	20	17	14
L3	3	3	3	1	1
D1	0	5	3	1	0
D2	0	0	0	0	0
Eq.	8	14	13	11	10
Soph.	0	3	2	1	1
Unclassified	45	0	10	30	33

Note: The far-right-hand column includes 17 OB subjects classified by their econometric-from-guesses type estimates.

Level-k Theory: Costa-Gomes and Crawford (2006)

- No Dk types
- No SOPH types
- No L0 (only in the minds of L1...)
- Deviation from Equilibrium is cognitive
- Cannot distinguish/falsify Cognitive Hierarchy
  - BR against lower types, not just L(k-1)
- But distribution is not Poisson (against CH)
  - Is the Poisson assumption crucial?

Level-k Theory: Costa-Gomes and Crawford (2006)

- Pseudotypes: Constructed with subjects' guesses in 16 games (pseudo-1 to pseudo-88)
- Specification Test: Compare the likelihood of subject's type with likelihoods of pseudotypes
  - Should beat at least $87/8 = 11$ pseudotypes since:
    - $\Pr($random type beats other 7 pre-set types$) = 1/8$
  - Unclassified if failed

Level-k Theory: Costa-Gomes and Crawford (2006)

- Omitted Type Test: Find clusters that
  - (a) Look like each other, (b) not like pre-set types
  - High pseudotype likelihood within, but low outside
- CGC find 5 small clusters; total = 11 of 88 subjects
- Other clusters? Maybe, but size smaller than 2/88 (2.3%)
- Smaller clusters could be treated as errors
  - Models for 2.3% of population is not general enough to make it worth the trouble (No point to build one model per subject!)

Level-k Theory: Costa-Gomes and Crawford (2006)

- Large fraction of subjects' deviations from equilibrium explained by Level-k model
  - (that can be explained by a model)
- Although the model explains only half+ of subjects' deviations from equilibrium,
- it may still be optimal for a modeler to treat the rest of the deviations as errors
  - Since the rest is not worth modeling...

Does Level-k Explain Hide-and-Seek Games?

- Aggregate RTH Hide-and-Seek Game Results:
- Both Hiders and Seekers over-choose central A
- Seekers choose central A even more than hiders

	A	B	A	A
Hiders (624)	0.2163	0.2115	0.3654	0.2067
Seekers (560)	0.1821	0.2054	0.4589	0.1536

Hide-and-Seek Game: Crawford and Iriberri (2007)

- Can a strategic theory explain this?
- Level-k: Each role is filled by Lk types: $L0$, $L1$, $L2$, $L3$, or $L4$ (probabilities to be estimated)
  - Note: In Hide and Seek the types cycle after $L4$...
- High types anchor beliefs in a naive $L0$ type and adjusts with iterated best responses:
  - $L1$ best responds to $L0$ (with uniform errors),
  - $L2$ best responds to $L1$ (with uniform errors), ...
  - Lk best responds to $L(k\text{-}1)$ (with uniform errors), etc.

Hide-and-Seek Game: Anchoring Type Level-0

- *L0* Hiders and Seekers are symmetric
  - Favor salient locations equally

1. Favor B: choose with probability $q > 1/4$ (More B)
2. Favor end A: choose with prob. $p/2 > 1/4$ (Less B)

- Choice probabilities are ($p/2$, q , 1 - p - q, $p/2$) for locations:

A	B	A	A

Hide-and-Seek Game: Anchoring Type Level-0

1. Favor B: choose with probability $q > 1/4$ (More B)
2. Favor end A: choose with prob. $p/2 > 1/4$ (Less B)

- Note: Specification of Anchoring Type $L0$ is the key to model's explanatory power
  - See Crawford and Iriberri (AER 2007) for other $L0$
  - But cannot use uniform $L0$ (coincide with equilibrium)

A	B	A	A

Hide-and-Seek: Crawford and Iriberri (2007)

- More (or less) attracted to B: $p/2<q$ ($p/2>q$)
- L1 Hiders choose central A

TABLE 2—TYPES' EXPECTED PAYOFFS AND CHOICE PROBABILITIES IN RTH'S GAMES WHEN $p > 1/2$ AND $q > 1/4$

Hider	Expected payoff (More B)	Choice probability (More B)	Expected payoff (Less B)	Choice probability (Less B)	Seeker	Expected payoff (More B)	Choice probability (More B)	Expected payoff (Less B)	Choice probability (Less B)
L0 (Pr. r)					*L0* (Pr. r)				
A	–	$p/2$	–	$p/2$	A	–	$p/2$	–	$p/2$
B	–	q	–	q	B	–	q	–	q
A	–	$1-p-q$	–	$1-p-q$	A	–	$1-p-q$	–	$1-p-q$
A	–	$p/2$	–	$p/2$	A	–	$p/2$	–	$p/2$
L1 (Pr. s)					*L1* (Pr. s)				
A	$1-p/2 < 3/4$	0	$1-p/2 < 3/4$	0	A	$p/2 > 1/4$	0	$p/2 > 1/4$	1/2
B	$1-q < 3/4$	0	$1-q < 3/4$	0	B	$q > 1/4$	1	$q > 1/4$	0
A	$p+q > 3/4$	1	$p+q > 3/4$	1	A	$1-p-q < 1/4$	0	$1-p-q < 1/4$	0
A	$1-p/2 < 3/4$	0	$1-p/2 < 3/4$	0	A	$p/2 > 1/4$	0	$p/2 > 1/4$	1/2
L2 (Pr. t)					*L2* (Pr. t)				

Hide-and-Seek: Crawford and Iriberri (2007)

- More (or less) attracted to B: $p/2 < q$ ($p/2 > q$)
- L1 Seekers avoid central A (pick B or end A)

TABLE 2—TYPES' EXPECTED PAYOFFS AND CHOICE PROBABILITIES IN RTH'S GAMES WHEN $p > 1/2$ AND $q > 1/4$

Hider	Expected payoff (More B)	Choice probability (More B)	Expected payoff (Less B)	Choice probability (Less B)
L0 (Pr. r)				
A	–	$p/2$	–	$p/2$
B	–	q	–	q
A	–	$1-p-q$	–	$1-p-q$
A	–	$p/2$	–	$p/2$
L1 (Pr. s)				
A	$1-p/2 < 3/4$	0	$1-p/2 < 3/4$	0
B	$1-q < 3/4$	0	$1-q < 3/4$	0
A	$p+q > 3/4$	1	$p+q > 3/4$	1
A	$1-p/2 < 3/4$	0	$1-p/2 < 3/4$	0
L2 (Pr. t)				

Seeker	Expected payoff (More B)	Choice probability (More B)	Expected payoff (Less B)	Choice probability (Less B)
L0 (Pr. r)				
A	–	$p/2$	–	$p/2$
B	–	q	–	q
A	–	$1-p-q$	–	$1-p-q$
A	–	$p/2$	–	$p/2$
L1 (Pr. s)				
A	$p/2 > 1/4$	0	$p/2 > 1/4$	1/2
B	$q > 1/4$	1	$q > 1/4$	0
A	$1-p-q < 1/4$	0	$1-p-q < 1/4$	0
A	$p/2 > 1/4$	0	$p/2 > 1/4$	1/2
L2 (Pr. t)				

Hide-and-Seek: Crawford and Iriberri (2007)

- More (or less) attracted to B: $p/2 < q$ ($p/2 > q$)
- L2 Hiders choose central A with prob. in [0,1]

TABLE 2—TYPES' EXPECTED PAYOFFS AND CHOICE PROBABILITIES IN RTH'S GAMES WHEN $p > 1/2$ AND $q > 1/4$

Hider	Expected payoff	Choice probability	Expected payoff	Choice probability	Seeker	Expected payoff	Choice probability	Expected payoff	Choice probability
	More B		Less B			More B		Less B	
L1 (Pr. *s*)					*L1* (Pr. *s*)				
A	$1-p/2 < 3/4$	0	$1-p/2 < 3/4$	0	A	$p/2 > 1/4$	0	$p/2 > 1/4$	1/2
B	$1-q < 3/4$	0	$1-q < 3/4$	0	B	$q > 1/4$	1	$q > 1/4$	0
A	$p+q > 3/4$	1	$p+q > 3/4$	1	A	$1-p-q < 1/4$	0	$1-p-q < 1/4$	0
A	$1-p/2 < 3/4$	0	$1-p/2 < 3/4$	0	A	$p/2 > 1/4$	0	$p/2 > 1/4$	1/2
L2 (Pr. *t*)					*L2* (Pr. *t*)				
A	1	1/3	1/2	0	A	0	0	0	0
B	0	0	1	1/2	B	0	0	0	0
A	1	1/3	1	1/2	A	1	1	1	1
A	1	1/3	1/2	0	A	0	0	0	0

Hide-and-Seek: Crawford and Iriberri (2007)

- More (or less) attracted to B: $p/2 < q$ ($p/2 > q$)
- L2 <u>Seekers</u> choose central A for sure

TABLE 2—TYPES' EXPECTED PAYOFFS AND CHOICE PROBABILITIES IN RTH'S GAMES WHEN $p > 1/2$ AND $q > 1/4$

Hider	Expected payoff (More B)	Choice probability (More B)	Expected payoff (Less B)	Choice probability (Less B)	Seeker	Expected payoff (More B)	Choice probability (More B)	Expected payoff (Less B)	Choice probability (Less B)
L1 (Pr. *s*)					*L1* (Pr. *s*)				
A	$1 - p/2 < 3/4$	0	$1 - p/2 < 3/4$	0	A	$p/2 > 1/4$	0	$p/2 > 1/4$	1/2
B	$1 - q < 3/4$	0	$1 - q < 3/4$	0	B	$q > 1/4$	1	$q > 1/4$	0
A	$p + q > 3/4$	1	$p + q > 3/4$	1	A	$1-p-q < 1/4$	0	$1-p-q < 1/4$	0
A	$1 - p/2 < 3/4$	0	$1 - p/2 < 3/4$	0	A	$p/2 > 1/4$	0	$p/2 > 1/4$	1/2
L2 (Pr. *t*)					*L2* (Pr. *t*)				
A	1	1/3	1/2	0	A	0	0	0	0
B	0	0	1	1/2	B	0	0	0	0
A	1	1/3	1	1/2	A	1	1	1	1
A	1	1/3	1/2	0	A	0	0	0	0

Hide-and-Seek: Crawford and Iriberri (2007)

- More (or less) attracted to B: $p/2<q$ ($p/2>q$)
- L3 Hiders avoid central A

TABLE 2—TYPES' EXPECTED PAYOFFS AND CHOICE PROBABILITIES IN RTH'S GAMES WHEN $p > 1/2$ AND $q > 1/4$

Hider	Expected payoff (More B)	Choice probability (More B)	Expected payoff (Less B)	Choice probability (Less B)	Seeker	Expected payoff (More B)	Choice probability (More B)	Expected payoff (Less B)	Choice probability (Less B)
L2 (Pr. *t*)					*L2* (Pr. *t*)				
A	1	1/3	1/2	0	A	0	0	0	0
B	0	0	1	1/2	B	0	0	0	0
A	1	1/3	1	1/2	A	1	1	1	1
A	1	1/3	1/2	0	A	0	0	0	0
L3 (Pr. *u*)					*L3* (Pr. *u*)				
A	1	1/3	1	1/3	A	1/3	1/3	0	0
B	1	1/3	1	1/3	B	0	0	1/2	1/2
A	0	0	0	0	A	1/3	1/3	1/2	1/2
A	1	1/3	1	1/3	A	1/3	1/3	0	0

Hide-and-Seek: Crawford and Iriberri (2007)

- More (or less) attracted to B: $p/2<q$ ($p/2>q$)
- L3 Seekers choose central A with probability in [0,1]

TABLE 2—TYPES' EXPECTED PAYOFFS AND CHOICE PROBABILITIES IN RTH'S GAMES WHEN $p > 1/2$ AND $q > 1/4$

Hider	Expected payoff	Choice probability	Expected payoff	Choice probability	Seeker	Expected payoff	Choice probability	Expected payoff	Choice probability
	More B		Less B			More B		Less B	
L2 (Pr. *t*)					*L2* (Pr. *t*)				
A	1	1/3	1/2	0	A	0	0	0	0
B	0	0	1	1/2	B	0	0	0	0
A	1	1/3	1	1/2	A	1	1	1	1
A	1	1/3	1/2	0	A	0	0	0	0
L3 (Pr. *u*)					*L3* (Pr. *u*)				
A	1	1/3	1	1/3	A	1/3	1/3	0	0
B	1	1/3	1	1/3	B	0	0	1/2	1/2
A	0	0	0	0	A	1/3	1/3	1/2	1/2
A	1	1/3	1	1/3	A	1/3	1/3	0	0

Hide-and-Seek: Crawford and Iriberri (2007)

- More (or less) attracted to B: $p/2<q$ ($p/2>q$)
- L4 Hiders avoid central A

TABLE 2—TYPES' EXPECTED PAYOFFS AND CHOICE PROBABILITIES IN RTH'S GAMES WHEN $p > 1/2$ AND $q > 1/4$

Hider	Expected payoff (More B)	Choice probability (More B)	Expected payoff (Less B)	Choice probability (Less B)	Seeker	Expected payoff (More B)	Choice probability (More B)	Expected payoff (Less B)	Choice probability (Less B)
L3 (Pr. u)					*L3* (Pr. u)				
A	1	1/3	1	1/3	A	1/3	1/3	0	0
B	1	1/3	1	1/3	B	0	0	1/2	1/2
A	0	0	0	0	A	1/3	1/3	1/2	1/2
A	1	1/3	1	1/3	A	1/3	1/3	0	0
L4 (Pr. v)					*L4* (Pr. v)				
A	2/3	0	1	1/2	A	1/3	1/3	1/3	1/3
B	1	1	1/2	0	B	1/3	1/3	1/3	1/3
A	2/3	0	1/2	0	A	0	0	0	0
A	2/3	0	1	1/2	A	1/3	1/3	1/3	1/3

Hide-and-Seek: Crawford and Iriberri (2007)

- More (or less) attracted to B: $p/2<q$ ($p/2>q$)
- L3 Seekers avoid central A

TABLE 2—TYPES' EXPECTED PAYOFFS AND CHOICE PROBABILITIES IN RTH'S GAMES WHEN $p > 1/2$ AND $q > 1/4$

Hider	Expected payoff (More B)	Choice probability (More B)	Expected payoff (Less B)	Choice probability (Less B)	Seeker	Expected payoff (More B)	Choice probability (More B)	Expected payoff (Less B)	Choice probability (Less B)
L3 (Pr. *u*)					*L3* (Pr. *u*)				
A	1	1/3	1	1/3	A	1/3	1/3	0	0
B	1	1/3	1	1/3	B	0	0	1/2	1/2
A	0	0	0	0	A	1/3	1/3	1/2	1/2
A	1	1/3	1	1/3	A	1/3	1/3	0	0
L4 (Pr. *v*)					*L4* (Pr. *v*)				
A	2/3	0	1	1/2	A	1/3	1/3	1/3	1/3
B	1	1	1/2	0	B	1/3	1/3	1/3	1/3
A	2/3	0	1/2	0	A	0	0	0	0
A	2/3	0	1	1/2	A	1/3	1/3	1/3	1/3

Hide-and-Seek Game: Explain Stylized Facts

- Given *L0* playing $(p/2, q, 1-p-q, p/2)$,
  - *L1* Hiders choose central A (avoid *L0* Seekers)
  - *L1* Seekers avoid central A (search for *L0* Hiders)
- *L2* Hiders choose central A with prob. in [0,1]
- *L2* Seekers choose central A for sure
- *L3* Hiders avoid central A
- *L3* Seekers choose central A with prob. in [0,1]
- *L4* Hiders and Seekers both avoid central A

Hide-and-Seek Game: Explain Stylized Facts

- To reproduce the stylized facts, need
  - Heterogeneous Population (L0, L1, L2, L3, L4) $= (r, s, t, u, v)$ with $r = 0$, t & u large, s not too large
- Need $s < (2t+u)/3$ (More B), or $s < (t+u)/2$ (Less B)
  - estimated $r = 0$, s=19%, t=32%, u=24%, v=25%

Total	$p < 2q$	$p > 2q$	Total	$p < 2q$	$p > 2q$
A	$rp/2+(1-\varepsilon)[t/3+u/3]$ $+(1-r)\varepsilon/4$	$rp/2+(1-\varepsilon)[u/3+v/2]$ $+(1-r)\varepsilon/4$	A	$rp/2+(1-\varepsilon)[u/3+v/3]$ $+(1-r)\varepsilon/4$	$rp/2+(1-\varepsilon)[s/2+v/3]$ $+(1-r)\varepsilon/4$
B	$rq+(1-\varepsilon)[u/3+v]$ $+(1-r)\varepsilon/4$	$rq+(1-\varepsilon)[t/2+u/3]$ $+(1-r)\varepsilon/4$	B	$rq+(1-\varepsilon)[s+v/3]$ $+(1-r)\varepsilon/4$	$rq+(1-\varepsilon)[u/2+v/3]$ $+(1-r)\varepsilon/4$
A	$r(1-p-q)+(1-\varepsilon)[s+t/3]$ $+(1-r)\varepsilon/4$	$r(1-p-q)+(1-\varepsilon)[s+t/2]$ $+(1-r)\varepsilon/4$	A	$r(1-p-q)+(1-\varepsilon)[t+u/3]$ $+(1-r)\varepsilon/4$	$r(1-p-q)+(1-\varepsilon)[t+u/2]$ $+(1-r)\varepsilon/4$
A	$rp/2+(1-\varepsilon)[t/3+u/3]$ $+(1-r)\varepsilon/4$	$rp/2+(1-\varepsilon)[u/3+v/2]$ $+(1-r)\varepsilon/4$	A	$rp/2+(1-\varepsilon)[u/3+v/3]$ $+(1-r)\varepsilon/4$	$rp/2+(1-\varepsilon)[s/2+v/3]$ $+(1-r)\varepsilon/4$

Hide-and-Seek: Out of Sample Prediction

- Estimate on one treatment and predict other five treatments
  - 30 Comparisons: 6 estimations, each predict 5
- This Level-k Model with symmetric *L0* beats other models (LQRE, Nash + noise)
  - Mean Squared prediction Error (MSE) 18% lower
  - Better predictions in 20 of 30 comparisons

Hide-and-Seek Level-k Model Ported to Joker Game

- Can Level-k thinking from the Hide-and-Seek Game predict results of other games?
  - Try O'Neill (1987)'s Joker Game
- Stylized Facts:
  - Aggregate Frequencies close MSE
  - Ace Effect (A chosen more often than 2 or 3)
    - Not captured by QRE

Joker Game: O'Neill (1987) (出鬼牌賽局)

	A	2	3	J	MSE	Actual	QRE
A	-5	5	5	-5	0.2	0.221	0.213
2	5	-5	5	-5	0.2	0.215	0.213
3	5	5	-5	-5	0.2	0.203	0.213
J	-5	-5	-5	5	0.4	0.362	0.360
MSE	0.2	0.2	0.2	0.4			
Actual	0.226	0.179	0.169	0.426			
QRE	0.191	0.191	0.191	0.427			

- 實際的出牌頻率跟MSE預測很接近
- QRE的預測更接近，但無法解釋「不平均」

- Actual frequency quite close to MSE
- QRE better, but cannot get "imbalances"

Hide-and-Seek Level-k Model Ported to Joker Game

- Level-*k* model w/ symmetric *L0* (favor A&J)
- *L0* : $(a, (1-a-j)/2, (1-a-j)/2, j)$, $a, j > 1/4$
  - A and J, are face cards and end locations, are more salient than 2 and 3...
- Higher *Lk* type BR to *L(k-1)* (Table A3-A4)
- Challenge: To get the Ace Effect (without L0), need a population of almost all L4 or L3
  - This is an empirical question, but very unlikely

Hide-and-Seek Level-k Model Ported to Joker Game

- Could there be no Ace Effect in the initial rounds of O'Neil's data?
  - The Level-k model predicts a Joker Effect instead!
- Crawford and Ireberri asked for O'Neil's data
  - And they found...
- Initial Choice Frequencies
  - (A, 2, 3, J) = (8%, 24%, 12%, 56%) for Player 1
  - (A, 2, 3, J) = (16%, 12%, 8%, 64%) for Player 2

Table 5. Comparison of the Leading Models in O'Neill's Game

Model	Parameter estimates	Player	A	2	3	J	MSE
Observed frequencies (25 Player 1s, 25 Player 2s)		1	0.0800	0.2400	0.1200	0.5600	-
		2	0.1600	0.1200	0.0800	0.6400	-
Equilibrium without perturbations		1	0.2000	0.2000	0.2000	0.4000	0.0120
		2	0.2000	0.2000	0.2000	0.4000	0.0200
Level-*k* with a role-symmetric *L0* that favors salience	$a > 1/4$ and $j > 1/4$	1	0.0824	0.1772	0.1772	0.5631	0.0018
	$3j - a < 1$, $a + 2j < 1$	2	0.1640	0.1640	0.1640	0.5081	0.0066
Level-*k* with a role-symmetric *L0* that favors salience	$a > 1/4$ and $j > 1/4$	1	0.0000	0.2541	0.2541	0.4919	0.0073
	$3j - a < 1$, $a + 2j > 1$	2	0.2720	0.0824	0.0824	0.5631	0.0050
Level-*k* with a role-symmetric *L0* that avoids salience	$a < 1/4$ and $j < 1/4$	1	0.4245	0.1807	0.1807	0.2142	0.0614
		2	0.1670	0.1807	0.1807	0.4717	0.0105
Level-*k* with a role-asymmetric *L0* that favors salience for locations for which	$a_1 < 1/4$, $j_1 > 1/4$; $a_2 > 1/4$, $j_2 < 1/4$	1	0.1804	0.2729	0.2729	0.2739	0.0291
player is a seeker and avoids it for locations for which player is a hider	$3j_1 - a_1 < 1$, $a_1 + 2j_1 < 1$, $3a + j > 1$	2	0.1804	0.1804	0.1804	0.4589	0.0117

Conclusion

- Limit of Strategic Thinking: 2-3 steps
- Theory (for initial responses?!)

- Level-k Types:
  - Stahl-Wilson (GEB 1995), CGCB (ECMA 2001)
  - Costa-Gomes and Crawford (AER 2006)
  - Ho and Su (MS 2013)
  - Chen, Huang and Wang (GEB 2018)
- Cognitive Hierarchy:
  - Camerer, Ho and Chong (QJE 2004)

Applications of Level-k Thinking

- p -Beauty Contest:
  - Costa-Gomes and Crawford (AER 2006)
  - Chen, Huang and Wang (GEB 2018)
- MSE:
  - Hide-and-Seek: Crawford and Iriberri (AER 2007)
  - LUPI: Ostling, Wang, Chou and Camerer (AEJmicro 2011)
- Auctions:
  - Overbidding: Crawford and Iriberri (AER 2007)
  - Repeated eBay Auctions: Wang (2006)

More Applications

- Coordination-Battle of the Sexes (Simple Market Entry Game):
  - Camerer, Ho and Chong (QJE 2004)
  - Crawford (2007)
- Pure Coordination Games:
  - Crawford, Gneezy and Rottenstreich (AER 2008)
- Pre-play Communication:
  - Crawford (AER 2003)
  - Ellingsen and Ostling (AER 2011)

More Applications

- Strategic Information Communication:
  - Crawford (AER 2003)
  - Cai and Wang (GEB 2006)
  - Kawagoe and Takizawa (GEB 2008)
  - Wang, Spezio and Camerer (AER 2010)
  - Brown, Leveno and Camerer (AEJmicro 2012)
  - Lai, Lim and Wang (GEB 2015)
  - Battaglini, Lai, Lim and Wang (APSR 2019)
  - Fong and Wang (FBE 2023)