Bargaining (議價談判)

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Bargaining (議價談判)

- ▶ Bargaining (就是「討價還價」!)
 - ▶ Process by which economic agents agree on the terms of a deal (個體間討論條件、達成交易的過程)
- Common even in competitive markets
 - ▶ The pit market in NYSE/market experiments
 - ▶ (即使在完全競爭市場也很常見,例如紐約股市的交易坑市場)
 - ▶ Edgeworth Box (Edgeworth, 1881) was created to show range of possible bargaining outcomes (原本是用來研究談判的!)
- ▶ Have you ever bargained with someone? (你有跟別人談判過嗎?)

Bargaining (議價談判)

- Nash (1950, 1951):
 - 1. (Cooperative) Nash Bargaining Solution (奈許談判解)
 - 2. (Non-Cooperative) Nash Equilibrium (奈許均衡)
- Nash could have won two Nobel Prizes...
- ▶ Nash Program: Is NBS the NE/SPE of a particular game? (奈許大哉問: NBS是否為某賽局的NE/SPE?)
 - Yes: Binmore, Rubinstein and Wolinsky (1986)
- ▶ References: BGT, Ch.4, HEE, Ch.4, MGSB, 2nd ed., Ch.14 (參考章節)

2 Bargaining Experiments (兩種談判實驗)

- ▶ Cooperative NBS vs. Non-Cooperative NE
 - ▶ 對應合作賽局NBS和非合作賽局NE,也有兩種談判實驗:
- 1. Unstructured Bargaining Experiments (自由談判實驗)
 - ▶ Free form procedure determined by players (雙方自行決定談判形式過程)
 - ▶ Closer to naturally occurring bargaining (較接近實務上談判)
- 2. Structured Bargaining Experiments (制式談判實驗)
 - ▶ Procedure specified by experimenter (形式過程由實驗者決定)
 - ▶ Game theory makes specific predictions (賽局論能做出明確預測)

Negotiation Research in Applied Psychology

- 3. Negotiation Research: Bazerman et al. (2000)
 - ▶ Bazerman, Magliozzi and Neale (1985) (應用心理學研究)
 - Negotiate over several issues (ex: price/quantity)
 - ▶ Free form communication with fixed deadline (時限内自由溝通討論)
 - Private point schedule (depends on each issue)
 - ▶ 雙方各自知道自己的報酬計分方式,最後須在價格數量等多層面(連續或類別)上達成協議
- ▶ Results: Deals not Pareto-efficient (結果:達成的協議不都有效率)
 - ▶ Affected by systematic heuristics and other cognitive variables (unrelated to game) (受到無關的經驗法則與認知因素影響)

Negotiation Research (協商談判研究)

- ▶ Why not much overlap? (為何沒有交集?)
 - ▶ Game theory assumes too much rationality? (賽局論假設完全理性)
 - ▶ Solvable games are too simplified (解得出來賽局又太簡單)
 - ▶ Hard to apply to Negotiation games? (很難用在協商研究)
- Like the two traditions of experimental economics:
 - ▶ Game experiments are too simplified? (正如賽局論實驗太過簡單)
 - ▶ Hard to apply to market experiments? (很難用賽局論預測市場實驗結果)
- ▶ But research questions are the same! (但兩邊面對一樣的研究問題!!)

- ▶ Test: Nash Bargaining Solution (NBS) (奈許談判解)
 - ▶ The point maximizing the product of utility gains (beyond the disagreement point) (與談判破裂相較讓雙方效用增加量的乘積最大的解)

$$\max_{(x_1,x_2)\in S} (x_1 - d_1)(x_2 - d_2)$$

- Only point satisfying 4 axioms:
 - 1. Pareto Optimality (效率性、不受額外無關選項影響)
 - 2. Symmetry (對稱、不受效用平移伸縮影響)
 - 3. Independence of Irrelevant Alternatives (IIA) (不受額外無關選項影響)
 - 4. Independence from affine utility transformation (不受效用平移伸縮影響)

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Nash Bargaining Solution (NBS奈許談判解)

$$S^* = \arg \max_{(x_1, x_2) \in S} (x_1 - d_1)(x_2 - d_2)$$

$$= \arg \max_{(x_1, x_2) \in S} [u_1(x_1) - u_1(d_1)][u_2(x_2) - u_2(d_2)]$$
is fies:

Satisfies:

- 1. Pareto Optimality: (效率性) $\forall x \in S^*, \exists y \in S, \underline{y} > x \Leftrightarrow y_i \geq x_i \forall i, y_j > x_j$
- 2. Symmetry: (對稱) $d_1 = d_2, (x_1, x_2) \in S^* \Rightarrow (x_2, x_1) \in S^*$
- 3. IA (Independence of Irrelevant Alternatives; 不受額外無關選項影響) S^* solves (T,d) if S^* solves (S,d) and $S^* \subset T \subset S$
- 4. IAT (Independence from affine utility transformation, 不受效用平移伸縮影響)

$$u_1(x) = Ax + B, u_2(x) = Cx + D$$

- ▶ Roth and Malouf (Psychological Review 1979)
- ▶ Player bargain over 100 lottery tickets (雙方談判如何分配100張彩券)
 - Risk neutral if can reduce compound lottery
 - ▶ 用彩券可讓人風險中立地決策(假設人們會把複合機率簡化成單一機率)
 - ▶ 1 ticket = 1% chance winning a prize (每張=1%機率贏得獎金)
 - ▶ Equal (\$1) vs. Unequal Prize (\$1.25/\$3.75)
 - ▶ Full Information vs. Partial Information (know own prize)
 - ▶ 2×2實驗設計: 獎金相同/不同, 資訊透明/不透明
- ▶ NBS: 50-50 split (NBS預測: 不管獎金相同與否、資訊透明與否都是「50-50對分」)

Information	Money	# of Tickets for Player 2 (成員乙所分得的彩券數目)							% of	
(資訊)	Prize (雙方獎金金額)	20	25	30	35	40	45	50	Disagreement (未達成協議的比例)	
Full	1/1	0	0	1	0	1	0	20	0%	
Information (資訊透明)	1.25/3.75	1	6	3	2	2	1	4	14%	
Partial	1/1	0	0	0	0	0	1	14	6%	
Information (資訊不透明)	1.25/3.75	0	0	0	0	0	3	13	0%	
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- Results: Agreements cluster at 50-50
 - ▶ Rare Disagreement (很少未達成協議, 大部分 50-50 對分)
- ▶ 14% Disagree when both know inequality
 - ▶ 雙方清楚知道獎金不平等時,有14%未達成協議(彩券 vs. 金錢平分)
 - Divide tickets or \$\$\$ payoffs equally
 - Sensitive to \$\$\$ payoffs: Violate IAT
 - ▶ 結果受金錢多寡影響,違反「不受效用平移伸縮影響(indep. of affine transformation)」公設
- Rawlsian Bargaining Solution explains this
 - ▶ Follow-up: Roth and Murnighan (ECMA 1982)

Rawlsian Bargaining Solution (羅斯談判解)

$$S^* = \arg\max_{(x_1,x_2)\in S} (x_1-d_1)(x_2-d_2)$$
 Satisfies:
$$= \arg\max_{(x_1,x_2)\in S} [u_1(x_1)-u_1(d_1)][u_2(x_2)-u_2(d_2)]$$

- 1. Pareto Optimality: (效率性) $\forall x \in S^*, \nexists y \in S, \underline{y > x}$
- 2. Symmetry: (對稱) $d_1 = d_2, (x_1, x_2) \in S^* \Rightarrow (x_2, x_1) \in S^*$
- 3. IIA S^* solves (T, d) if S^* solves (S, d), $S^* \subset T \subset S$
- 4. Independence of utility transformation preserving preference order and which player has larger gain

$$x_i \ge y_i \Leftrightarrow u_i(x_i) \ge u_i(y_i)$$
 $x_1 - d_1 \ge x_2 - d_2 \Leftrightarrow u_i(x_1 - d_1) \ge u_i(x_2 - d_2)$

- ▶ Murnighan, Roth and Schoumaker (JRU 1988) Review earlier studies to find: (回顧先前實驗發現)
- ▶ Pairs settle @ final minutes (of 9-12 min) (最後幾分鐘才達成協議)
 - ▶ Convey private info (Stubbornness/Delay Cost)? (表示自己很堅持/可以負擔
- Follow-up: Roth and Schoumaker (AER 1983) 延遲成本?)
 - ▶ First play against computer that gives you a lot (先跟軟弱電腦談判)
- ▶ Expect and get this later from human players (被訓練該多拿)
 - ▶ Strong Reputation (接下來面對真人態度也會較強硬、並且也真的拿比較多)

- ▶ Mehta, Starmer and Sugden (book chapter 1992)
- ▶ Nash Demand Game: (奈許需求實驗) 2 Players
 - ▶ Each state demand (兩人分別列出自己的需求金額,總和 ≤10英鎊就會得到所求)
 - ▶ Get their demand If sum \leq £10, 0 otherwise. (不然都得0)
- Focal point: Players split 4 Aces + 4 deuces (兩人抽4張A/4張2)
 - ▶ Before bargain, players were told:
 - ▶ "4 aces worth £10 together, so to earn \$\$ you have to pool your aces and agree on how to divide the £10."
 - ▶ 焦點:「四張A合起來值十英鎊, 想賺錢就得合作、一起換十英鎊來分」

▶ Results:被告知四張A合起來值十英鎊,因此要賺錢就得把四張A合起來並同意如何平分十英鎊。

實驗結果居然受此敘述(與報酬無關)影響!!

- Aces split 2-2:
 - ► Agree 50-50 Split (各兩張A就對分)
- ► Aces 1-3: (一張/三張)
 - ▶ Half <u>50-50</u>, (一半對分)
 - ▶ Half 25-75; (另一半要求25-75)
 - ▶ 22% disagree (22%爆掉)

Demand	1A	2A	3A
£2.50	1 1	0	0
£3.00-4.50	5	1	1
£5.00	<u>16</u>	40	<u>17</u>
£5.50-7.00	0	1	11
£7.50	0	0	4
Ν	32	42	33

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Can BGT Explain This? (行為賽局論的解釋)

- ▶ Roth (1985) explains as Coordination Game over allocation focal points 50--50 vs. h--(100--h)
 - ▶ Each favoring one (50 > h whenever 50 < 100-h)
 - ▶ 可用協調賽局解釋: 考慮兩個分配上的協調焦點 50-50 或 h-(100-h)
- ▶ Both simultaneously choose to demand their favorite or acquiesce to the less favorable (兩邊同時選擇「要求有利自己的分配」 或「願接受另一個分配」)
 - ▶ If both demand favorite: Both earn 0 (若都「要求」,兩邊報酬皆為0)
 - ▶ If only one demands favorite: Play focal point (只有一方「要求」, 則按「要求」分)

Can BGT Explain This? (行為賽局論的解釋)

▶ If both acquiesce: Earn average of the two focal points

$$x_1 = (50+h)/2, x_2 = (150-h)/2$$

- ullet 若都「接受另一個」則獲得兩分配平均 $x_{\!\scriptscriptstyle I} = (50+h)/2, \, x_{\!\scriptscriptstyle Z} = (150-h)/2$
- MSE: Players demand with probability

$$p_1 = rac{h-50}{150-h} ext{ and } p_2 = rac{h-50}{h+50}$$

Disagreement rate $= \frac{(h-50)^2}{(150-h)(50+h)}$

Can BGT Explain This? (行為賽局論的解釋)

- PRoth (book chapter 1985) $(h-50)^2$ Disagreement rate $= \frac{(h-50)^2}{(150-h)(50+h)}$
- ▶ Predicted to be $0\% \rightarrow 7\% \rightarrow 10\%$ (過去結果預測隨 $h \uparrow$)
 - for h = 50, 75, 80 by pervious experiments
- ▶ Data: $7\% \rightarrow 18\% \rightarrow 25\%$ (Right direction!)(符合比較靜態預測)
 - ▶ Murnighan et al. (JRU 1988) (理論預測: 未達成協議的比例應該隨 h↑)
 - h = 60, 70, 80, 90 predict 1%, 4%, 10%, 19%
- ▶ Actual data not as good: Constant across h (但實驗結果持平)

Cause of Disagreement: Self-Serving Bias

- ▶ "What is better for me" = "Fair" (對我有利才叫公平)
 - ▶ Add to coordination game explains more disagreement in data (上述協調賽局加入自利偏誤可解釋結果)
- ▶ Same in Kagel, Kim and Moser (GEB 1996):
 - ▶ Ultimatum over 100 tickets (P/R value differently)
 - ▶ 用最後通牒談判分配100張(對雙方價值不同)的彩券
 - ▶ If R unaware of H/L, P_H/P_L propose to give 45%/30%
 - ▶ 回應者不知對方價值高低時,提議者在價值高時會給45%,但價值低時只給30%
 - If aware of H, R will want >50% (Rejection rate = 40%)
 - ▶ 回應者知道對方價值較高會拒絕40%、要求比平分更好, 使得40%提議被拒絕

Babcock et al. (AER 1995, Law & Social Inquiry 1997)

- ▶ Self-serving Bias Experiment: Loewenstein et al. (JLS 1993)
- ▶ Read 27-page actual legal case (讀27頁卷宗:機車騎士告車主)
 - ▶ Motorcyclist sues driver: \$100,000 injury damage
- ▶ Bargain for 30 min. to settle it for ?? dollars
 - ▶ \$5000 legal fees for every 5-min delay
 - ▶ Retired judge imposes award if no agreement
 - ▶ 30分鐘談判和解(訴訟金額\$100k), 每延遲5分鐘須付\$5k律師費(和解不成則由退休法官裁定)
- ▶ First Guess what judge would award (事先預測和解不成法官會如何判)
 - ▶ US\$1 (or 1 Grade Point) for every \$10,000 (實驗中1萬元=1美元/加1分)

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(和解不成來自預期判決結果的落差)

Gap of E(judgment) Predicts Disagreement

- ▶ Baseline: 70% cases settled at period 3-4 (out of 6)
- ▶ E(judgment) differ by \$20,000 (20% of \$100,000)
 - ▶ 70%的控制組在第3-4回合達成和解(總共6回合); 雙方預期判決結果落差在2萬元左右(訴訟金額的20%)

Settlement Stati	E(judgment) Gap (預期判決落差)					
Group (實驗組別)	N	%	periods	(s. e.)	mean (平均)	(s. e.)
Control: Babcock 95'	47	72	3.75	(0.28)	\$18,555	(3,787)
(控制組)			(回合)			
Control: Babcock 97'	26	65	4.08	(0.46)	\$21,783	(3,956)
(控制組)			(回合)			

(提高和解率、更快和解的辦法)

More Pairs Settled (and More Rapidly) if...

- ▶ Don't Know Role when Reading: 94% (in 2.51 pds)
 - But you know own role in reality!
 - ▶ 閱讀前不知自己是哪一方: 94%在2.51回合和解(但現實雙方立場已知!)

Settlement Stati	E(judgment) Gap (預期判決落差)					
Group (實驗組別)	N	%	periods	(s. e.)	mean (平均)	(s. e.)
Control: Babcock 95'	47	- 72	- 3.75	(0.28)	┌\$18,555	(3,787)
Control: Babcock 95' Don't Know Roles	47	94	2.51	(0.21)	\$6,275 ⁼	(4,179)
Control: Babcock 97'	26	65	4.08	(0.46)	\$21,783	(3,956)

(提高和解率、更快和解的辦法)

More Pairs Settled (and More Rapidly) if...

- Or, tell subjects the bias before bargaining and ask them to
- ▶ First List Weakness of Own Case: 96% (in 2.39 periods)
 - ▶ 談判前告知有此偏誤,並請其列出己方立場弱點: 96%在2.39回合和解

Settlement Statistics (和解結果)					E(judgment) Gap (預期判決落差)		
Group (實驗組別)	N	%	periods	(s. e.)	mean (平均)	(s. e.)	
Control: Babcock 95'	47	- 72	- 3.75	(0.28)	┌\$18,555	(3,787)	
Control: Babcock 95' $p < 0.01$ Don't Know Roles	47	•94	2.51	(0.21)	- \$6,275 ⁼	0 (4,179)	
Control: Babcock 97'	26	-65	- 4.08	(0.46)	-\$21.783 $p=0.02$	(3,956)	
Control: Babcock 97' 1st List Weakness	23	9 6	→2.39	(0.34)	\$4,676=0.02	(6,091)	

Summary for Unstructured Bargaining (小結: 自由談判實驗)

- ▶ Focal Points affect bargaining outcome (「焦點」會影響談判結果)
- ▶ Chip Value affect bargaining outcome (籌碼/彩券的價值影響談判結果)
 - ▶ Violate IAT Axiom of NBS (違反奈許談判解的IAT公設, 受效用平移伸縮影響)
- ▶ BGT Explanation: Bargainers may fail to coordinate under multiple focal points (行為賽局論: 在多重「焦點」下協商不易達成協議)
- ▶ Self-Serving Bias predict costly delay/settle (自利偏誤解釋不和解、延遲)
 - ▶ "Outcome favoring me more likely/fair" (對我有利更公平/更可能發生)
 - ▶ Caused by knowing my role when reading case (來自閱讀卷宗時已經 知道自己是哪一方)

Structured Bargaining: Outline (制式談判: 大綱)

- ▶ Finite Alternating-Offer Game (有限回合交互提案,延遲有機會成本)
 - ▶ Pie shrinks or fixed cost to delay (lost wages in strike)
- Infinite Alternating-Offer vs. Random Termination
 - ▶ How can we generate infinitely repeated games?
- Outside Options
 - Will option value affect bargaining outcome?
- Incomplete Information
 - Buyer Value vs. Seller Cost

Finite Alternating-Offer Game (有限回合交互提案)

- Stahl (book 1972) and Rubinstein (ECMA 1982)
 - ▶ Player with higher discount factor δ has advantage
 - ▶ Examples: Wealthy Tourists, Children at Toy Store
 - ▶ 折現因子愈高的一方在談判中有優勢,例如: 觀光客被獅子大開口、小孩在玩具店耍賴
- ▶ 2-Period: Binmore, Shaked and Sutton (AER 1985)
 - ▶ Player 1 offers a division of 100p to player 2
 - If player 2 rejects, makes counteroffer dividing 25p
 - ▶ 成員甲提議如何分配100p,成員乙回應。若拒絕則由他提議分配25p
- ▶ SPE: Player 1 offers 25-75; player 2 accepts 25p

2-Period: Binmore, Shaked and Sutton (AER 1985)

- Experimental Results: mode at 50-50, some 25-75 and others in between (實驗結果: 衆數50-50, 有些在25-75, 其他在兩者之間)
- ▶ Then, player 2 make hypothetical offer as player 1:
 - ▶ Most offer 25-75 (as they would accept as player 2?) (然後成員乙在假設性問題「如果你是成員甲會怎麼提議」之下大部分提議25-75符合SPE)
- Conclusion: Experience as player 2 help them learn SPE quickly, even if they do not play SPE initially
 - ▶ Is Role-Reversal that powerful in learning?!
 - ▶ Triple Confound: Experience, Role Reversal, Incentives

Neelin, Sonnenschein and Spiegel (AER 1988)

- ▶ Have 80 intermediate micro students play in order:
 - ▶ 2-round: Pie size \$5.00 → \$1.25 (2回合: 分配金額\$5.00→\$1.25)
 - ▶ 3-round: Pie size \$5.00 → \$2.50 → \$1.25 (3回合: \$5.00→\$2.50→\$1.25)
 - ▶ 5-round: Pie size \$5.00 \rightarrow \$1.70 \rightarrow \$0.58 \rightarrow \$0.20 \rightarrow \$0.07
 - ▶ (5回合: 分配金額\$5.00→\$1.70→\$0.58→\$0.20→\$0.07)
- ▶ Same SPE: Player 1 offers \$1.25 and player 2 accepts
 - ▶ (三個實驗均有同樣的子賽局完全均衡: 成員甲第一回合提議給成員乙\$1.25, 成員乙就接受了)
 - ▶ 3-round: Offer \$1.25 in round 2 (倒推法: 在三回合的第二回合會提議\$1.25)
 - ▶ 5-round: Offer \$0.07, \$0.13, \$0.45, in round 4, 3, 2 (倒推法: 在五回合的第四、三、二回合分別會提議 \$0.07, \$0.13, \$0.45)

Neelin, Sonnenschein and Spiegel (AER 1988)

▶ 5-round: $\$5.00 \rightarrow \$1.70 \rightarrow \$0.58 \rightarrow \$0.20 \rightarrow \$0.07$

Reject (%)

- ▶ 3-round: $\$5.00 \rightarrow \$2.50 \rightarrow \$1.25$
- ▶ 2-round: \$5.00→\$1.25
- ▶ 2-Round:
 - ▶ Many \$1.25, few \$2.50 (50-50)
 - ▶ Follow SPE, unlike Binmore, et al.
- ▶ 3-Round/5-Round:
 - ▶ Offer 2nd round pie size
 - ▶ As if playing 2-round game!

	Offer (\$)	2-Round	3-Round	5-Round	
	>2.50	-	10%	-	
	2.50	5%	70%	5%	
	2.01-2.49	-	5%	-	
	1.71-2.00	3%	8%	38% 6.7	7º/
	1.70	-	-	35% 7.3	1%
	1.51-1.70	-	-	10%	
	1.26-1.50	45% 119	% 8% 67%	5%	
	1.25	38% 209	<mark>%</mark> –	-	
i	<1.25	10% 100	0% _	8% 100	%

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Neelin, Sonnenschein and Spiegel (AER 1988)

- ▶ Neelin et al. (1988) find stronger support for SPE
 - ▶ Economics undergrads yield different results (經濟系大學生結果不同)
 - ▶ Are they taught backward induction? Also, (因為學過倒推法?)
- ▶ Binmore: (還是實驗說明? Binmore請學生極大化自己的報酬)
 - "YOU WOULD BE DOING US A FAVOR IF YOU SIMPLY SET OUT TO MAXIMIZE YOUR WINNINGS."
- ▶ Janet Neelin (=Janet Currie): (Neelin說上課會討論相關理論)
 - You would be discussing the theory this experiment is designed to test in class."

Ochs and Roth (AER 1989): Opening Offers

- ▶ 2-Round vs. 3-Round Offers to Split \$30
- ▶ Discount Factors δ : 0.4 vs. 0.6 (differ across players)
 - ▶ Bargain over 100 chips (which worth drops differently)
- ▶ 2-Round SPE:
 - ▶ 1st offer = $30\delta_2$
 - Independent of δ_1
- ▶ 3-Round SPE:
 - ▶ 1st offer $< 30\delta_2$

But Experiments show little difference in Period 1!!

Discount	t Factor	2-Roun	d 1 st Offer	3-Round 1st Offer		
δ_1	δ_2	SPE	Period 1	SPE	Period 1	
0.4	0.4	\$12.00	\$13.19	\$7.20	\$13.02	
0.6	0.4	\$12.00	\$14.73	\$4.80	\$14.04	
0.6	0.6	\$18.00	\$13.88	\$7.20	\$13.93	
0.4	0.6	\$18.00	\$14.67	\$10.50	\$13.90	

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Ochs and Roth (AER 1989): Opening Offers

- ▶ Significant learning in 3 cases (1 wrong direction)
- ▶ 2-Round 1st offer = $30\delta_2$: But data dependent of δ_1 !
- ▶ 3-Round 1^{st} offer $< 30\delta_2$: Data drops only in 2 cases! ▶ SPE predicts $25 = \frac{C(8,2) 3}{5}$ (dotted) 3^{rd} counteroffer adds little
- differences:
 - ▶ 17 directions correct in data
 - ▶ binomial p<0.05 Mild Support?

Discount Factor			2-Roun	d 1 st Offer	3-Round 1st Offer		
	δ_1	δ_2	SPE	Period 10	SPE	Period 10	
	0.4	0.4	► \$12.00	\$12.03	\$7.20	\$12.81	
	- 0.6	0.4	···\$12.00	-\$14.34	\$4.80	\$13.17	
	- 0.6	0.6	··\$18.00	\$14.70	···\$7.20	\$13.70	
	→ 0.4	0.6	\$ 18.00	→\$13.57	\$10.50	\$14.23	

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Ochs and Roth (AER 1989): Rejections/Counteroffers

- ▶ 16% opening offers rejected
 - ▶ 15% in Binmore et al. (1985); 14% in Neelin et al. (1988)
- ▶ Counteroffers rejected more (40% of 2nd/54% of 3rd)
 - ▶ Could be Selection Bias: Tough player 2s reject and make

stingy offers

Por, Negative Reciprocity of player 1s (whose offer got rejected)

Discount Factor		2-Round 3	1 st Offer	3-Round 1st Offer		
	δ_1	δ_2	Period 10	Reject	Period 10	Reject
	0.4	0.4	\$12.03	10%	\$12.81	12%
	0.6	0.4	\$14.34	15%	\$13.17	14%
	0.6	0.6	\$14.70	13%	\$13.70	15%
	0.4	0.6	\$13.57	20%	\$14.23	29%

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Ochs and Roth (AER 1989): Rejections/Counteroffers

- ▶ 81% counteroffers are disadvantageous!
 - Less than what was offered to (and rejected by them)
 - ▶ 75% in Binmore et al. (1985); 65% in Neelin et al. (1988)
- Why? Could be subjects...
 - 1. Social Utility: Do not maximize own monetary payoff
 - Example: Also care about relative shares (Bolton, AER 1991)
 - 2. Limited Computation: Do not think ahead sufficiently
 - See MouseLab study below

Bolton (AER 1991): Tournament Incentives

- ▶ Tournament payoffs determined by
 - bargaining performance relative to others in same role
- Social Utility of Bolton (AER 1991)
 - ▶ Besides \$\$ amount, also care about relative shares
- Subjects should never reject equilibrium offers
 - ▶ Since rejecting equilibrium offers in a tournament reduces both relative shares and \$\$ amount

Bolton (AER 1991): Tournament Incentives

- Mixed Experimental Results of Bolton (AER 1991)
 - Inexperience subjects: Tournament similar to control
 - ▶ Higher variance, some difference in rejection rates
 - ▶ 2nd session: Tournament (25%) closer to SPE
 - ▶ 3rd session: Tournament moves back toward equal split
- Carter and McAloon (JEBO 1996):
 - ▶ Tournament does not move Ultimatum offers and acceptance toward self-interest as Bolton (1991) predicts

Harrison and McCabe (bkch 1992): Learning?

- ▶ Binmore et al. (1985): Learn in 1-round of Role-Reversal
- Ochs and Roth (1989) and Bolton (1991): Little Learning
 - ▶ Since subjects suffer only 1-2 rejections over 10 rounds
- ▶ Harrison and McCabe (1992): Repeat 7 times between:
 - ▶ 3-round bargaining: Pie size $100 \rightarrow 50 \rightarrow 25$
 - ▶ 2-round bargaining: Pie size $50 \rightarrow 25$ (subgame of above!)
- ▶ 3-round opening offer converges towards SPE (=25)
 - \blacktriangleright 47 \to 40 \to 41 \to 35 \to 34 \to 30 \to 29
 - ▶ 2-Round opening offer = 24-25 (across all 7 times)

Carpenter (J Conflict Resolution 2003): Fair or SPE?

- ▶ But in Harrison and McCabe, 2-round SPE = 25 = equal split
- What if subgame opening offer is not the fair offer?
- ▶ Carpenter (JCR2003): Alternate between:
 - ▶ 2-round bargaining: Pie size $100 \rightarrow 25$ or 75
 - ▶ 1-round ultimatum: Pie size 25 or 75 (subgame of above!)
- Experimental Results:
 - ▶ Opening offers = 40% of pie size in all games
 - ▶ 2-round bargaining: Slight, insignificant drift towards SPE
 - ▶ So, Harrison and McCabe (1992) worked due to fair offer?

Johnson, Camerer, Sen and Rymon (JET 2002)

- ▶ Failure of backward induction?
- ▶ Play self-interest robots: Think you are self-interest
 - ▶ Subjects should make SPE offers (if they can calculate it)
- ▶ 3-Round pie size hidden in boxes: Click to open
 - MouseLab: Observe information acquisition
- ▶ 3-round: Pie $\$(5.00+\epsilon) \rightarrow \$(2.50+\epsilon) \rightarrow \$(1.25+\epsilon)$
 - ▶ SPE opening offer = \$1.25
 - Average opening offers = \$2.11 (most b/w \$2.00 and \$2.50)
 - ▶ Overall rejection rate = 12% (50% rejection if offer < \$1.80)

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Johnson et al. (JET 2002): Where Are They Looking?

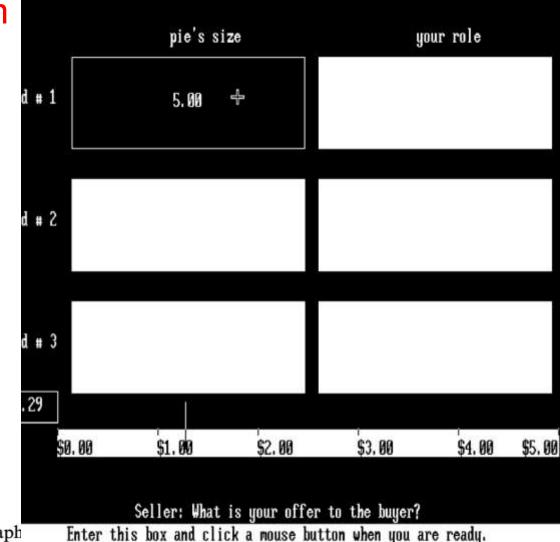
- ▶ $1^{st} \rightarrow 2^{nd} \rightarrow 3^{rd}$ -round: $12.91 \rightarrow 6.67 \rightarrow 1.24$ secs
- Slightly more Forward transitions than Backward
 - ▶ Subjects in 19%/10% of the trials cannot do backward induction since they never opened $2^{nd}/3^{rd}$ -round box

Dound	Dia sira	# of	Trials with No Lookups	Gaze	Transition	n from Row	to Column
Round	Pie Size	Lookups	No Lookups	Time	$\rightarrow 1$	$\rightarrow 2_{\text{Form}}$	$ward \rightarrow 3$
1	\$5.00	4.38	0%	12.91	-	2.55	0.65
2	\$2.50	3.80	19%	6.67	2.10	-	1.24
3	\$1.25	2.12	10%	1.24	0.50	0.88	-

Backward

Johnson, Camerer, Sen & Rymon (2002): Icon Graph

- 1. % Box Shaded = Icon Graph Relative gaze time
- 2. Box Width = Lookups
- 3. Arrows Thickness = # of transitions (>1)
- ▶ 1st box opened most and gazed at longer
- Transitions mostly b/w 1st and 2nd box



Johnson et al. (JET 2002): Icon Graph for Player 1

- % Box Shaded = _
 Relative gaze time
- 2. Box Width = Lookups
- 3. Arrows Thickness = # of transitions (>1)
- ▶ Level-1 (Offer <\$2.00)
 - Look at 2nd box more often and longer than
- ▶ Level-0 (Offer ≥\$2.00)

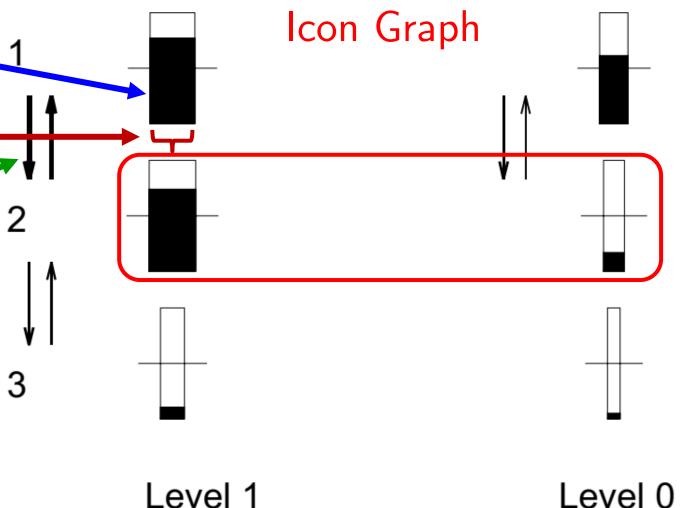
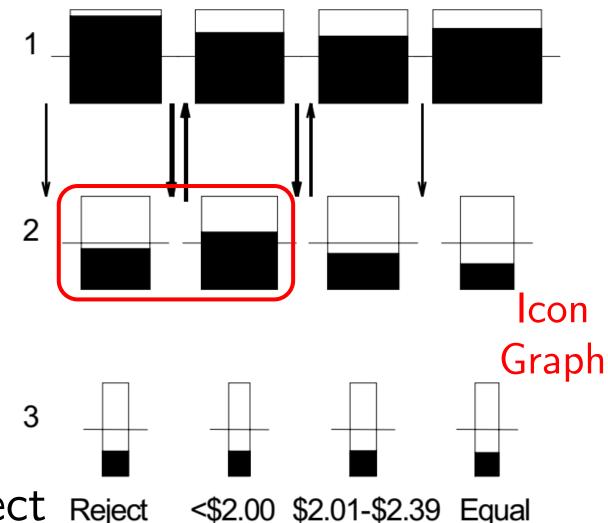


FIG. 4. Icon graphs for player 1 type inferred from first-round offers.

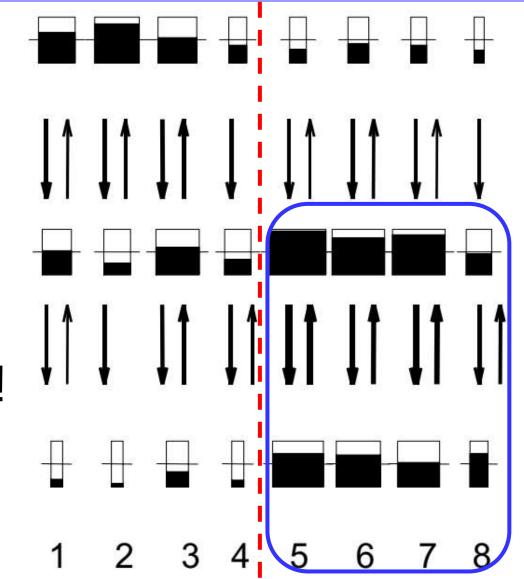
Player 1 Against Robots and Icon Graph for Player 2

- Subjects offer \$1.84 against self-interest robots
 - \$2.11 (against humans)
 - ▶ Still well above SPE (\$1.25)
 - Erasing doubts that opponent are rational insufficient
- ▶ Player 2 who accept <\$2.00
- Look at 2nd box more and longer than Player 2 who reject Reject



Johnson et al. (2002): Teach Backward Induction

- 1. % Box Shaded = Gaze time
- 2. Box Width = Lookups
- 3. Arrows Thickness = Transitions
- ▶ Against self-interest robots:
 - ▶ After Period 4, receive instructions about backward induction
- ▶ Period 5-8: Offer \$1.22 near SPE!
 - Look mostly at 2nd/3rd box and 3 transit between them



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Johnson et al. (2002): Experienced vs. Inexperienced

- Untrained vs. Trained Subjects:
 - ▶ (Trained: Experience playing against self-interest robots)
- Untrained reject low offers from trained (but offer less)
- ▶ Trained subjects learn to raise offers after rejection
 - ▶ Average offer = \$1.60 (still above SPE!)
- ▶ Given such experimental regularities, BGT suggests:
 - Assume subject play truncated games, or
 - Posit decision rules to predict both choices and reasoning processes (and use both choice and lookups to infer rule)

Summary for Alternating-offer Bargaining (小結:輪流出價談判

- Initial offers between equal split and SPE
- ▶ Rejections occur + disadvantageous counteroffers
 - Little learning (except Binmore et al.)
 - May learn when exposed to subgame play even if initial offers accepted (Harrison and McCabe), but only if fair?!
- ▶ Limited Computation/How Bargaining Power Emerge:
 - 1. Players do not look ahead
 - 2. Do not make equilibrium offers to self-interest robots,
 - 3. but do make them after training in backward induction

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(無限重複: 隨機結束 vs. 分配金額縮水)

Random Termination vs. Discounting

- ▶ Discounting factor δ = randomly terminate w/ prob (1δ) (if risk neutral)
- ▶ Zwick, Rapoport and Howard (T&D 1992)
 - ▶ Divide \$30 with random termination(兩人分配\$30, 不限回合但可能隨機結束)
 - ▶ Continuation probabilities 0.90, 0.67, 0.17 (下一回合的機率)
 - ▶ SPE prediction: (均衡預測) 14.21, 12, 4.29
- ▶ Experimental Results: (結果介於50-50平分和均衡之間,最後接受分配與金額縮水的結果類似
 - ▶ Offer <\$15 (between SPE and 50-50); little learning
 - ▶ Accepted final offers: 14.97, 14.76, 13.92
 - ▶ Close to Pie-shrink discounting results: 14.90, 14.64, 13.57

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- \blacktriangleright Instead of pie-shrinking by same discount factor (δ), could pay the same amount of lost wages, profits, etc. (利潤,薪資損失)
- SPE: Strong side (lower delay cost) gets all
 - ▶ 均衡預測: 強勢者(延遲成本低)會全拿
- ▶ Rapoport, Weg and Felsenthal (T&D 1990)
- ▶ Divide 30 shekels (談判分配30舍克勒)
 - ▶ Pseudo-infinite horizon: "Experiment will terminated if it lasted too long." (「假裝」無窮期)
- ▶ Fixed Cost: \$0.10 vs. \$2.50 or \$0.20 vs. \$3.00 (固定延遲成本)

Rapoport, Weg and Felsenthal (1990): \$0.10 vs. \$2.5

- ▶ Strong P offer \$4.4 out of \$30, 83% weak R accepts
- ▶ Weak P offer \$25.4 out of \$30, 33% Strong R accepts Table 4.7

Evnoriment 1	Decision	Equilibrium			
Experiment 1	Decision	Prediction	on 1-6 7-12		13-18
Strong P	Mean Final Offer	\$0.00	\$9.2	\$7.4	\$4.4
$c_1 = \$0.10 < c_2 = \2.5	$\%$ accepted on $1^{\rm st}$	100%	50%	67%	83%
Weak P	Mean Final Offer	\$29.9	\$20.0	\$23.2	\$25.4
$c_1 = \$2.5 < c_2 = \0.10	$\%$ accepted on $1^{\rm st}$	100%	39%	28%	33%
Equal	Mean Final Offer	[\$0, \$27.5]	\$14.8	\$16.1	\$15.6
$c_1\!=c_2\!=\$2.5$	% accepted on 1st	100%	78%	83%	83%

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Bargaining

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Rapoport, Weg and Felsenthal (1990): \$0.20 vs. \$3

- ▶ Strong P offer \$7.9 out of \$30, 61% weak R accepts
- ▶ Weak P offer \$21.6 out of \$30, 28% Strong R accepts Table 4.7

Evnoriment 2	Decision	Equilibrium Periods					
Experiment 2	Decision	Prediction	1-6				
Strong	Mean Final Offer	\$0.00	\$12.8	\$8.6	\$7.9		
$c_1 = \$0.20 < c_2 = \3	$\%$ accepted on 1^{st}	100%	44%	39%	61%		
Weak	Mean Final Offer	\$29.8	\$17.9	\$18.5	\$21.6		
$c_1 = \$3 < c_2 = \0.20	$\%$ accepted on 1^{st}	100%	28%	22%	28%		
Equal	Mean Final Offer	[\$0, \$27.0]	\$14.8	\$14.6	\$14.7		
$c_1\!=c_2^{}\!=\$3$	$\%$ accepted on 1^{st}	100%	94%	94%	94%		

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- Strong support for SPE: In 1st round of bargaining,
 - ▶ 實驗結果支持均衡預測: 在談判開始的第一回合,
 - ▶ Strong P offer \$4.4/\$7.9, so 61-80% weak R accepts
 - ▶ 強勢提議者會提議給對方\$4.4/\$7.9, 且60-80%弱勢回應者會接受
 - ▶ Weak P offer \$21.6/\$25.4 (≪\$30); 30% strong R accepts
 - ▶ Weak P later accepts reality and quickly settles in 2nd (35%) or 3rd-4th (22%) rounds
 - ▶ 弱勢提議者會提議給對方\$21.6/\$25.4,由於遠低於\$30會被大部分強勢回應者拒絕。但他們很快就會面對現實,第二回合有35%、第三、四回合又有22%的組別達成協議

▶ Binmore, Shaked and Sutton (QJE 1989)

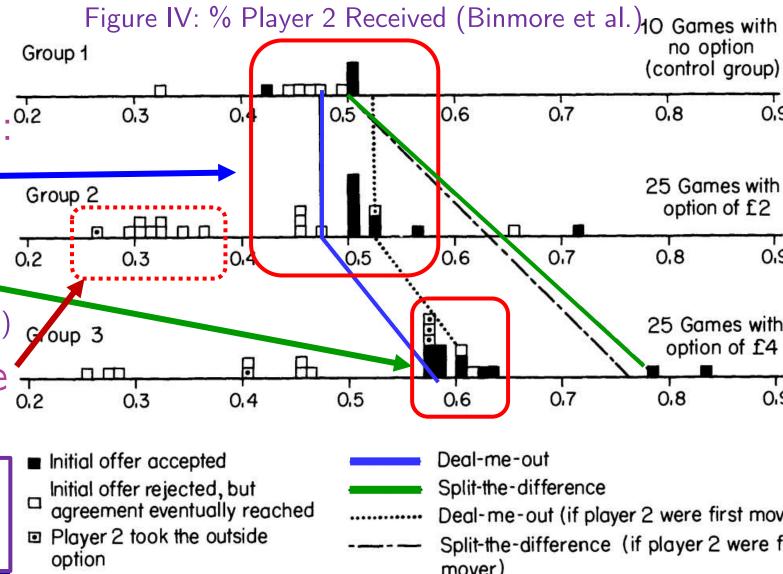
- (雙方談判如何分配
- ▶ 2 players bargain over £7, discount factor $\delta = 0.9$ £7, $frac{1}{2}$ $frac{1}{2}$ $frac{1}{2}$ $frac{1}{2}$
- ▶ Player 2 has outside option of £0, £2, or £4 (若不達成協議成員乙 仍可獲得£0, £2,或£4)
- ▶ Split-the-difference (NBS;平分差額的預測): 50%, 64%, 79%
 - ▶ Divide surplus beyond the threat points (雙方平方超出威嚇點以外的部分)
 - ▶ Offer player 2: £3.5(=0+7/2), £4.5(=2+5/2), £5.5(=4+3/2)
- ▶ Deal-me-out (SPE; 來真的才算數的預測): 47%, 47%, 57%=4/7
 - ▶ Rubinstein-Stahl solution is $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$ (談判解)
 - ightharpoonup Unless credible outside option above cutoff $\left(\frac{\delta}{1-\delta}\right)$ 的威嚇點通通不算數

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Experimental Results

- ► Deal-me-out wins (結果符合來真的才算數)
 - ▶ Outside Option £0/£2: Old Spike around 50% (£0, £2: 50%附近特別多)
 - ▶ Outside Option £4: _____ cluster@57% (£4:集中在57%)
 - Wait 2-3 rounds before accept remaining half

Outside option does affect equilibration!



Binmore, Proulx, Samuelson & Swierzbinski (EJ 1998)

- Nash Demand Game: Player 1 and player 2 divide \$10
 - ▶ State demands x_1 , x_2 : Split accordingly if $x_1 + x_2 \le 10$
 - ▶ Otherwise, both earn zero
- ▶ Outside Option: Player 2 can opt out to earn α for sure
 - $\alpha = \$0.90, \$2.50, \$4.90, \6.40 or \$8.10
- ▶ Since most demand \$5, opt out if $\alpha = \$6.40$ or \$8.10?
 - ▶ Forward Induction: Player 2 expects to earn $> \alpha$ if opt in
 - ▶ Player 1 should anticipate this and demand $\$(10 \alpha)$

Binmore, Proulx, Samuelson & Swierzbinski (EJ 1998)

- ▶ Results support Deal-me-out: $x_2 = \max\{\$5, \$\alpha\}$
 - ▶ 11-17% player 1 demand > $\$(10-\alpha)$; many player 2s opt out!

BGT, Table 4.8

Value of Option (α)	% of Player 2s Opt Out	Median x_2 if Opt In	$egin{array}{c} Median \ x_1 \ if \ Opt \ In \end{array}$	% of Player 1s Demand >\$(10–α)
\$0.90	0.0%	\$4.97	\$4.90	0.0%
\$2.50	1.0%	\$4.95	\$4.90	0.0%
\$4.90	33.4%	\$5.00	\$4.65	0.9%
\$6.40	59.8%	\$6.40	\$3.20	11.1%
\$8.10	80.9%	\$8.10	\$1.65	17.0%

Bargai 95% CI = [\$0.95, \$4.50] ph Tao-yi Wang

Forsythe, Kennan and Sopher (book chapter 1991)

- ▶ Split R = \$2.40/\$1.20 per period for 4/8 periods
 - ▶ Strong player threat point \$1.40/\$0.70: 58% = 7/12 of R
 - ▶ Could gain \$1.00/\$0.50 in each; Free-form message online

pie=(Surplus beyond threat point) x T	Game I (pie=\$4)	Game II (pie=\$4)	Game III (pie=\$8)	Game IV (pie=\$8)
Per-period Revenue (R)	\$2.40	\$1.20	\$2.40	\$1.20
Number of Periods (T)	4	8	8	16
Threat Points (s, w)	(\$1.40, \$0)	(\$0.70, \$0)	(\$1.40, \$0)	(\$0.70, \$0)
Split-the-difference	(\$7.60, \$2)	(\$7.60, \$2)	(\$15.20, \$4)	(\$15.20, \$4)
Deal-me-out	(\$5.60, \$4)	(\$5.60, \$4)	(\$11.20, \$8)	(\$11.20, \$8)

Forsythe, Kennan and Sopher (1991): Predictions

- ▶ Split-the-difference: Split the surplus (\$1.00/\$0.50)
 - ▶ Game I : $(\$1.40 + \$0.50) \times 4 = \$7.60 \text{ vs. } \$2.00 = \$0.50 \times 4$
 - Figure II: $(\$0.70 + \$0.25) \times 8 = \$0.25 \times 8$

pie=(Surplus beyond threat point) x T	Game I (pie=\$4)	Game II (pie=\$4)	Game III (pie=\$8)	Game IV (pie=\$8)
Per-period Revenue (R)	\$2.40	\$1.20	\$2.40	\$1.20
Number of Periods (T)	4	8	8	16
Threat Points (s, w)	(\$1.40 , \$0)	(\$0 . 70 , \$0)	(\$1.40, \$0)	(\$0.70, \$0)
Split-the-difference	(\$7.60, \$2)	(\$7.60, \$2)	(\$15.20, \$4)	(\$15.20, \$4)
Deal-me-out	(\$5.60, \$4)	(\$5.60, \$4)	(\$11.20, \$8)	(\$11.20, \$8)

Forsythe, Kennan & Sopher (bk ch 1991): Predictions

- ▶ Deal-me-out: Strong gets threat point (\$1.40/\$0.70)
 - ▶ Game I: $$1.40 \times 4 = $5.60 \text{ vs. } $4.00 = ($2.40 $1.40) \times 4$
 - Figure II: $\$0.70 \times 8 = (\$1.20 \$0.70) \times 8$

pie=(Surplus beyond threat point) x T	Game I (pie=\$4)	Game II (pie=\$4)	Game III (pie=\$8)	Game IV (pie=\$8)
Per-period Revenue (R)	\$2.40	\$1.20	\$2.40	\$1.20
Number of Periods (T)	4	8	8	16
Threat Points (s, w)	(\$1.40 , \$0)	(\$0 . 70 , \$0)	(\$1.40, \$0)	(\$0.70, \$0)
Split-the-difference	(\$7.60, \$2)	(\$7.60, \$2)	(\$15.20, \$4)	(\$15.20, \$4)
Deal-me-out	(\$5.60, \$4)	(\$5.60, \$4)	(\$11.20, \$8)	(\$11.20, \$8)

Forsythe, Kennan & Sopher (bk ch 1991): Predictions

- ▶ Split-the-difference: Split the surplus (\$1.00 or \$0.50)
 - From Fig. 6.50 and Fig. 6.50 are III: $(\$1.40 + \$0.50) \times 8 = \$15.60$ vs. $\$4.00 = \0.50×8
 - Game IV: $(\$0.70 + \$0.25) \times 16 = \$0.25 \times 16$

	<u> </u>				
	pie=(Surplus beyond threat point) x T	Game I (pie=\$4)	Game II (pie=\$4)	Game III (pie=\$8)	Game IV (pie=\$8)
Р	er-period Revenue (R)	\$2.40	\$1.20	\$2.40	\$1.20
١	lumber of Periods (T)	4	8	8	16
	Threat Points (s, w)	(\$1.40, \$0)	(\$0.70, \$0)	(\$1.40 , \$0)	(\$0.70, \$0)
	Split-the-difference	(\$7.60, \$2)	(\$7.60, \$2)	(\$15.20, \$4)	(\$15.20, \$4)

(\$5.60, \$4)

Deal-me-out

(\$5.60, \$4) (\$11.20, \$8) (\$11.20, \$8)

Forsythe, Kennan & Sopher (bk ch 1991): Predictions

- ▶ Deal-me-out: Strong gets threat point (\$1.40/\$0.70)
 - ▶ Game I: $$1.40 \times 8 = $11.20 \text{ vs. } $8.00 = (2.40 $1.40) \times 8$
 - Figure II: $\$0.70 \times 16 = (1.20 \$0.70) \times 16$

F	oie=(Surplus beyond threat point) x T	Game I (pie=\$4)	Game II (pie=\$4)	Game III (pie=\$8)	Game IV (pie=\$8)
Pe	er-period Revenue (R)	\$2.40	\$1.20	\$2.40	\$1.20
N	umber of Periods (T)	4	8	8	16
-	Threat Points (s, w)	(\$1.40, \$0)	(\$0.70, \$0)	(\$1.40 , \$0)	(\$0.70, \$0)
	Split-the-difference	(\$7.60, \$2)	(\$7.60, \$2)	(\$15.20, \$4)	(\$15.20, \$4)
	Deal-me-out	(\$5.60, \$4)	(\$5.60, \$4)	(\$11.20, \$8)	(\$11.20, \$8)

Forsythe, Kennan and Sopher (bk ch 1991): Results

- ▶ Joint-cost theory: Shorter strike length in I/III (not supported!)
- ▶ Strong/weak get 5-10%/70% surplus: Support Deal-me-out!

pie=(Surplus beyond threat point) x T	Game I (pie=\$4)	Game II (pie=\$4)	Game III (pie=\$8)	Game IV (pie=\$8)
Per-period Revenue (R)	\$2.40	\$1.20	\$2.40	\$1.20
Number of Periods (T)	4	8	8	16
Threat Points (s, w)	(\$1.40, \$0)	(\$0.70, \$0)	(\$1.40, \$0)	(\$0.70, \$0)
Split-the-difference	(\$7.60, \$2)	(\$7.60, \$2)	(\$15.20, \$4)	(\$15.20, \$4)
Deal-me-out	(\$5.60, \$4)	(\$5.60, \$4)	(\$11.20, \$8)	(\$11.20, \$8)
Average Strike Length	0.74	1.17	1.47	1.44

Average Payoffs (s, w) (\$5.77,\$3.09) (\$6.49,\$2.53) (\$11.62,\$6.14) (\$12.50,\$5.97)

Forsythe, Kennan and Sopher (book chapter 1991)

- Messages sent mostly verbal tug-of-war between
 - ▶ (Everyday) Fairness vs. Game-theoretic Bargaining Power
 - ▶ Strong: "I will still earn plenty even if you fail to agree."
 - ▶ Weak: Beg for equal divisions/at least split surplus 50-50
- Promise to repay in the next and take large share now
- Other logs:
 - ▶ In response to how "cute" he is,
 - "...Are you in a frat[ernity]? If so maybe I'll deal and maybe I won't. Depends on whatever."

- ▶ After breakup: "Go back to Burge [dorm] and roll in the barf,"
- "...I am a junior and live in Mayflower and I'd love for you to stop by sometime and visit"
- ▶ Binmore, Morgan, Shaked and Sutton (GEB 1991)
 - Exercise outside option: Exogenous vs. Voluntary
 - ▶ Low outside options matter if sometimes exercised exogenously
 - Results do show difference!
- Malleable Fairness: Subject answer to "What division seemed most fair" depends on their own experience

Summary for Random Termination/Outside Options

- Random Termination vs. Discounting Future Payoff
 - ▶ Similar results (more rejections under random termination)
- ▶ Fixed-Cost Games vs. Fixed-Discounting Games
 - ▶ Lopsided Division vs. Equal Splits
 - Can social preferences explain both results?
- Outside Options matter only if yields more than SPE
 - Unlike Nash Bargaining Solution (Split-the-difference)
 - ▶ High-option subjects need time to learn they get little surplus

Incomplete Information (資訊不透明)

- Add Asymmetric Information to bargaining
 - ▶ 在談判實驗中加入資訊不透明的情形
- ▶ More realistic, but (更符合真實)
 - ▶ Hard to bargain for a bigger share AND convey information at the same time (但是很難同時傳遞訊息又獅子大開口)
- Might need to turn down an offer to signal patience or a better outside option
 - ▶ 可能必須要用拒絕某一方案來展示自己有更好的外部選項

- Rapoport, Erev, and Zwick (MS 1995)
- ▶ Seller: Own item (worthless to herself) (賣方: 擁有1單位商品留著沒用)
- ▶ Buyer: Private reservation price is uniform[0,1]
 - ▶ (買方:保留價格為均匀分配,實際價值只有自己才知道)
- ▶ Seller makes an offer each period (每回合賣方提議價格)
- ▶ Common discount factor δ (相同的折現率 δ)
 - ▶ Unique Sequential Equilibrium (唯一的序列均衡)
 - ightharpoonup Seller Offer: Buyer Accepts if $p_t \leq v \cdot \frac{1-\gamma \cdot \delta}{1-\gamma \cdot \delta}$

 $1-\delta$

- ▶ Unique Sequential Equilibrium: (唯一的序列均衡)
- Seller Offer: (賣方提議)

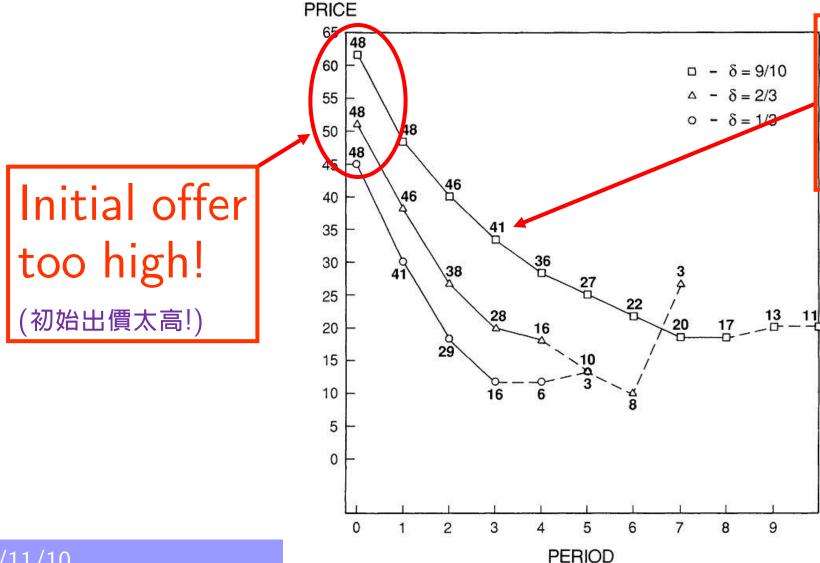
$$p_0 = \gamma \cdot \frac{1 - \delta}{1 - \gamma \cdot \delta},$$

- Subsequently: $p_t = p_0 \cdot \gamma^t$,
 - ▶ (接下來)
- Buyer Accepts if $p_t \leq v \cdot \frac{1-\delta}{1-\gamma \cdot \delta}$ (買方接受底線為)

- lackbox Complicate Strategy: Depend on δ (均衡策略很複雜且跟 δ 有關)
 - Price discriminate high/low-value buyers
 - ▶ Price declines slow enough so high-value buyers will not want to wait (對保留價格不同的買方實施價格歧視, 價格下降速度慢到讓高保留價格者不願意等待。但受試者做得到嗎?)
- Can subjects get these in experiments?
 - ▶ Different *δ* : H (0.90), M (0.67), L (0.33) (不同折現率)
 - ▶ Opening p₀: H (0.24), M (0.36), L (0.45) (初始出價)
 - ▶ Discount γ: H (0.76), M (0.68), L (0.55) (降價幅度)

(賣方對保留價格未知的買方提議)

Seller Make Offer to Informed Buyer



Decline Rate Amazingly Close!

(但降價幅度很接近理論預測)

- ▶ Can subjects get these in experiments?
 - ▶ Different δ: H (0.90), M (0.67), L (0.33) (不同折現率)
 - ▶ Opening p₀: H (0.24), M (0.36), L (0.45) (初始出價)
 - ▶ Discount *y* : H (0.76), M (0.68), L (0.55) (降價幅度)
- lackbox Buyers accept the $1^{
 m st}$ or $2^{
 m nd}$ offer below v (出價<v —兩回合就接受)
 - ▶ Accept offers too soon (買方接受得太早/應該再等一會兒)
- Sellers ask for higher prices (than equilibrium)
 - ▶ 跟均衡相比,賣方初始出價太高,但實際降價幅度非常接近理論預測
 - ▶ But empirical discount γ : H (0.81), M (0.68), L (0.55)

Strikes and 1-Sided Information (資訊不透明與罷工)

- Forsythe, Kennan and Sopher (AER 1991)
 - ▶ Only Informed bargainer I sees pie size π_g or π_b (只有一方 I 知道總金額)
- ▶ Uninformed U can strike to shrink pie by γ (不知情的一方 U 可罷工 使金額縮水 γ)
- ▶ What happens in free-form bargaining? (自由談判的實驗結果會如何?)
- ▶ Myerson (1979): Revelation Principle (顯示真實原則)
- 1. announces true state (I 宣布真實狀況)
- 2. U strikes to shrink pie by γ_g or γ_b (U 罷工會讓金額變成 γ_g 或 γ_b)
- 3. I gives U (based on true state) x_g or x_b (根據真實狀況 I 給 U x_g 或 x_b)

Strikes and 1-Sided Information (資訊不透明與罷工)

▶ IC requires: (誘因符合限制式)

$$(\gamma_g - \gamma_b)\pi_b \le x_g - x_b \le (\gamma_g - \gamma_b)\pi_g$$

▶ Interim Incentive Efficiency requires: (中間誘因效率)

$$\gamma_g = 1, x_g - x_b = (1 - \gamma_b)\pi_g$$

- lacksquare Strike $(\gamma_b < 1)$ if and only if $p\pi_g > \pi_b$
 - ▶ 罷工 $(\gamma_b < 1)$ 的充分必要條件
- ▶ Deriving this is complicated... (解出這些條件很複雜...)
 - ▶ Could ANY subject get close to this? (會有人解出來嗎?)

Strikes and 1-Sided Information (資訊不透明與罷工)

- ▶ Random Dictator (RD) Axiom: (隨機獨裁分配公設)
 - ▶ Agree fair mix between each being dictator to propose mechanism (同意隨機決定由誰獨裁決定分配機制)
- ▶ Then: (則)

$$\gamma_g = 1, x_g = \frac{\pi_g}{2}, \gamma_b = \frac{1}{2}, x_b = 0 \text{ if } p\pi_g > \pi_b$$

$$\gamma_g = 1, x_g = \frac{\pi_b}{2}, \gamma_b = 1, x_b = \frac{\pi_b}{2} \text{ if } p\pi_g < \pi_b$$

Strikes and 1-Sided Information (資訊不透明與罷工)

- ▶ This is a win-win experiment: (這是一個雙贏實驗)
 - ▶ Success if theory predictions are close (如果結果符合理論預測就驗證了理論)
 - ▶ If not, will point to which assumption fails (不符合可看出哪個假設出問題)
- ▶ Forsythe, Kennan and Sopher (AER 1991): (文字溝通10分鐘)
 - ▶ 10 minute sessions; written messages
- ▶ Is Myerson (1979) confirmed? (顯示真實原則是否被驗證?)
 - ▶ Surprisingly yes, though not perfect... (出乎意料地正確, 但是還不夠完美...)

Strike Condition Off (罷工條件不成立) $p\pi_g < r$

Game (賽局)	p	State (狀況)	π	$oldsymbol{\pi}_U$	π_I	Total (加總)	Strike (罷工)
		b	2.80				
111	0 5	g	4.20				
	0.5	aver.	2 50	1.50	1.80	3.29	6.0%
		pred.	3.50	1.40	2.10	3.50	0.0%
	0.25	b	2.40				
1\ /		g	6.80				
ZUZ5/11/1U		aver.	3.50	1.21	2.04	3.24	7.4%
		pred.		1.20	2.30	3.50	0.0%

Strike Condition Off (罷工條件不成立)

$p\pi_g < \pi_b$

p	State (狀況)	π	π_U	π_I	Total (加總)	Strike (罷工)
	b	2.80	1.47	1.18	2.66	5.2%
0.5	g	4.20	1.52	2.41	3.93	6.5%
0.5	aver.	2 50			3.29	6.0%
	pred.	5.50	1.40	2.10	3.50	0.0%
0.25	b	2.40	1.08	1.04	2.12	11.8%
	g	6.80	1.58	5.03	6.61	2.9%
	aver.	2 50	1.21	2.04	3.24	7.4%
	pred.		1.20	2.30	3.50	0.0%
	0.5	0.5 g $aver.$ $pred.$ b g 0.25 g $aver.$	$0.5 = \begin{array}{c} b & 2.80 \\ g & 4.20 \\ aver. \\ pred. \\ \end{array}$ $0.25 = \begin{array}{c} b & 2.40 \\ g & 6.80 \\ aver. \\ pred. \\ \end{array}$	$0.5 = \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$0.5 \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$0.5 \begin{array}{c ccccccccccccccccccccccccccccccccccc$

Strike Condition On (罷工條件成立)

 $p\pi_g > \pi_b$

Game (賽局)	p	State (狀況)	π	π_U	π_I	Total (加總)	Strike (罷工)
		b	1.00				
	0.5	g	6.00				
	0.5	aver.	3.50	1.05	2.00	3.05	13.0%
		pred.		1.50	1.75	3.25	7.1%
ZUZ5/11/1U	0.75	b	2.30				
		g	3.90				
		aver.	3.50	1.41	1.76	3.18	9.3%
		pred.		1.46	1.75	3.21	8.3% rao-yr vvang

Strike Condition On (罷工條件成立)

$p\pi_g > \pi_b$

Game (賽局)	p	State (狀況)	π	π_U	π_I	Total (加總)	Strike (罷工)
		b	1.00	0.31	0.30	0.61	39.0%
	0 5	g	6.00	1.78	3.70	5.48	8.7%
·	0.5	aver.	2 50	1.05	2.00	3.05	13.0%
		pred.	3.50	1.50	1.75	3.25	7.1%
	0.75	b	2.30	1.06	0.84	1.90	17.2%
ZUZ5/11/10		g	3.90	1.53	2.07	3.59	7.9%
		aver.	2 50	1.41	1.76	3.18	9.3%
		pred.	3.50	1.46	1.75	3.21	8.3%

Sealed-Bid in Bilateral Bargaining (密封投標的雙邊談判)

- ▶ Both buyers and sellers have private information (買賣雙方 都各自知道自己的成本/保留價格)
- ▶ Sealed-Bid Mechanism (密封投標機制)
 - ▶ Both write down a price (雙方都寫下一個價格)
 - lacktriangle Trade at the average if $p_b>p_s$ (當 $p_b>p_s$ 則以均價成交)
 - ▶ Call Market: Many buyers vs. many sellers (公開喊價: 許多買方 vs. 許多賣方)
- Two-Person Sealed-Bid Mechanism
 - ▶ One form of bilateral bargaining (雙人密封投標機制: 一種特定的雙邊談判)

Two-Person Sealed-Bid Mechanism (雙人密封投標機制)

- ▶ Buyer V: uniform[0,100]; Seller C: uniform[0,100]
 - ▶ 買方價值 *V*

賣方成本 C

- Piecewise-linear Equilibrium: (not unique)
 - ▶ Chatterjee and Samuelson (1983) (一個分段線性均衡)
 - Myerson and Satterthwaite (1883): Maximize ex ante gains

$$p_b = \left\{ egin{array}{ll} V & ext{if } V < 25 \ rac{25}{3} + rac{2}{3}V & ext{if } V \geq 25 \ p_s = \left\{ egin{array}{ll} 25 + rac{2}{3}C & ext{if } C < 75 \ C & ext{if } C \geq 75 \ \end{array}
ight.$$

Two-Person Sealed-Bid Mechanism (雙人密封投標機制)

- Radner and Schotter (JET 1989): 8 sessions
- ▶ 1, 2, 8: Baseline as above (場次1,2,8為對照組/如上所述)
- ▶ 3: Trade at price (v + c + 50) / 3 if v > (c+25)
 - ightharpoonup Should bid their values $v=V,\ c=C$ (場次3改變交易價格決定方式,讓誠實下標買方價值/賣方成本為上策)
- $lacksymbol{+}$ 4: Price =v, (Buyers should bid v=V/2) (場次4價格為買方出價) 買方出價應為價
- ▶ 5,6: Alternative distribution for more learning 値之半/場次5,6 改分配增加學習)
 - ▶ Distribution with more trade (for learning): m=0.438
- ▶ 7: Face-to-face bargaining (場次7為面對面談判)

Estimated	Seller	Bid Fur	nction SI	ope(用資	料估計量方出	出價函數斜率)
		Cutoff (前段)		Above Cu		
Session (場次)	β	\hat{eta}	t-stat (t值)	β	\hat{eta}	t-stat (t值)
1	0.67	0.58	(-1.38)	1	0.97	(-0.32)
2	0.67	0.74	(1.28)	1	1.07	(0.14)
8	0.67	0.75	(1.65)	1	1.07	(0.17)
3	1	1.06	(1.04)	1	0.67	(-0.58)
5	0.438	0.48	(0.87)	1	1.00	(0.60)
6 (-20)	0.438	0.57*	(2.16)	1	0.97	(-0.79)
6 (21-)	0.438	0.52	(1.20)	1	0.95	(-0.69)
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Below Cutoff (前段)

Above Cutoff (後段)

		(133124)			(103117)	
Estimated	Buyer	Bid Fu	nction S	ope (用資	料估計置方出	價函數斜率)
Session (場次)	β	\hat{eta}	t-stat (t值)	β	\hat{eta}	t-stat (t值)
1	1	1.00	(0.01)	0.67	0.85*	(4.14)
2	1	0.91	(-0.52)	0.67	1.06	(1.28)
8	1	0.91	(-0.14)	0.67	0.80*	(2.32)
3	1	0.92	(-0.88)	1	0.73*	(-2.64)
4	0.5	0.55	(0.66)	0.5	0.58*	(2.32)
5	1	0.80*	(-4.17)	0.438	0.50	(1.12)
6 (-20)	1	0.85	(-1.40)	0.438	0.40	(0.56)
6 (21-)	1	1.11	(0.70)	0.438	0.32	(-1.55)

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Face-to-face Yields 110% Efficiency (面對面可達到110%效率)

- ▶ Some truthfully reveal; others do not (因部分人講真話)
- Radner and Schotter (JET 1989, p.210):
 - ▶ The success of the face-to-face mechanism, if replicated, might lead to a halt in the search for better ways to structure bargaining in situations of incomplete information. (如果面對面談判的成功可重複驗證,那也許就不必再費心尋找資訊不全下、更好的制式談判方式了)
 - It would create, however, a need for a theory of such structured bargaining in order to enable us to understand why the mechanism is so successful.

(反而需要更好的制式談判理論來解釋為什麼面對面的方式這麼成功)

- ▶ Schotter, Snyder and Zheng (GEB 2000)
 - ▶ Subjects draw valuations from asymmetric distribution as in Session 5-7 of Radner and Schotter (1989)
 - ▶ Distribution with more trade (for learning): m=0.438
- ▶ Let agents bargains face-to-face for you (派代表幫你談判)
 - Buyers tell agent maximum willing-to-bid
 - Sellers tell agent minimum willing-to-bid
 - ▶ Agents paid percentage of surplus/fixed fee for each trade

- ▶ Results of Schotter, Snyder and Zheng (GEB 2000):
 - ▶ Give maximum reservation price below true valuation
 - ▶ m=0.78 with percentage-fee vs. m=0.70 with fixed-fee
 - ▶ Between predicted m=0.438 and truthful revelation (m=1)
- ▶ Rapoport and Fuller (1995)
 - ▶ Replicate Radner and Schotter (1989) with:
 - Strategy method and asymmetric value distribution
 - ▶ 詢問完整策略、買方價值/賣方成本分配不對稱

- ▶ Daniel, Seale and Rapoport (J Math Psych 1998)
 - ▶ Experiment 1 replicate asymmetric value distribution (R-F)
- ▶ Buyer V: uniform[0, 200]; Seller C: uniform[0, 100]
- Linear bid function for seller: $p_s = 50 + \frac{2}{3}C$
- Piecewise-linear bid function for buyer: $p_b = \begin{cases} 1 & \text{function for buyer: } p_b = \begin{cases} 1 & \text{function for buyen: } p_b = \begin{cases} 1 & \text{function for buyen: } p_b = \begin{cases} 1 & \text{function for buyen: } p_b = \begin{cases} 1 & \text{function for buyen: } p_b = \begin{cases} 1 & \text{function for buyen: } p_b = \begin{cases} 1 & \text{function for buyen: } p_b = \begin{cases} 1 & \text{function for buyen: } p_b = \begin{cases} 1 & \text{function for buyen: } p_b = \begin{cases} 1 & \text{function for buyen: } p_b = \begin{cases} 1 & \text{function for buyen: } p_b = \end{cases} \end{cases}$
 - \hat{m} =0.56 (near 2/3)
 - \hat{m} =0.28 (near 0)

if V < 50

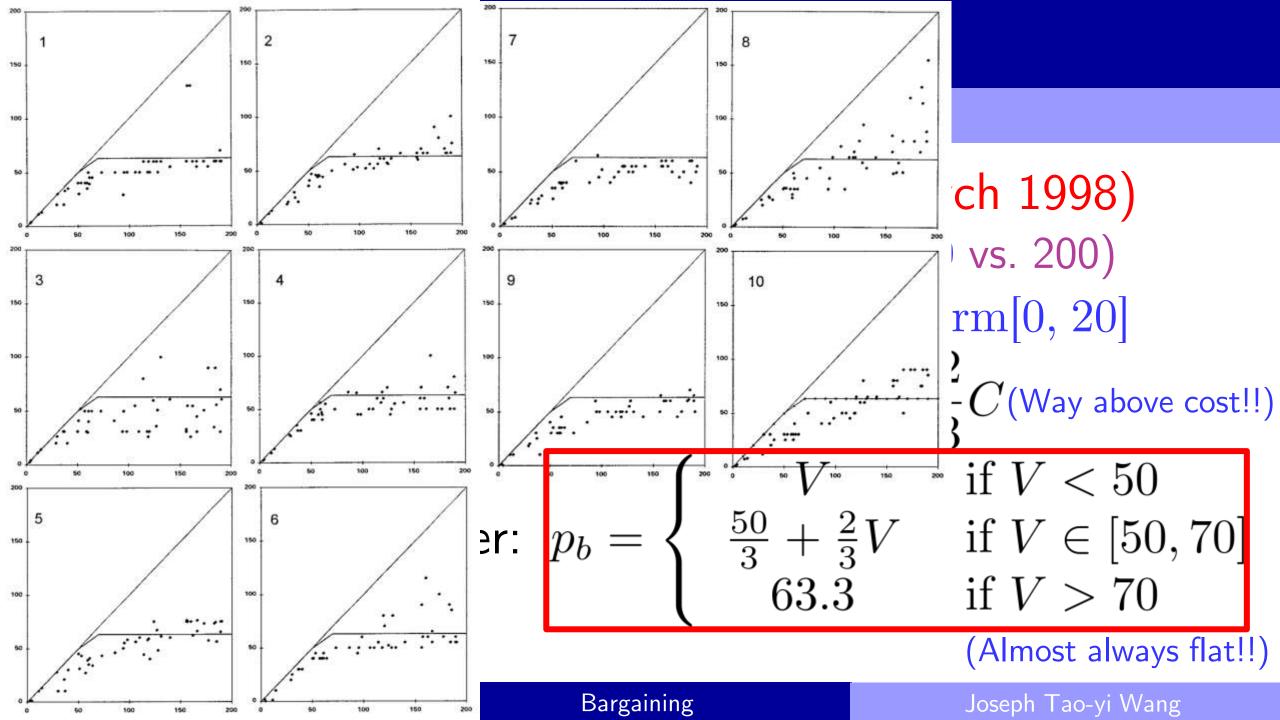
if $V \in [50, 150]$

116.7 if V > 150

- ▶ Daniel, Seale and Rapoport (J Math Psych 1998)
 - ▶ Experiment 2: Extremely asymmetric value distribution
- ightharpoonup Buyer V: uniform[0, 200]; Seller C: uniform[0, 20]
- Linear bid function for seller: $p_s = 50 + \frac{2}{3}C$ (Way above cost!!)

Piecewise-linear bid function for buyer:
$$p_b = \begin{cases} V & \text{if } V < 50 \\ \frac{50}{3} + \frac{2}{3}V & \text{if } V \in [50, 70] \\ 63.3 & \text{if } V > 70 \end{cases}$$

(Almost always flat!!)





Daniel, Seale and Rapoport (1998): Experiment 1

Buyer V: uniform[0, 200]; Seller C: uniform[0, 100]

$$p_b = \begin{cases} V & \text{if } V < 50\\ \frac{50}{3} + \frac{2}{3}V & \text{if } V \in [50, 150]\\ 116.7 & \text{if } V > 150 \end{cases}$$

$$p_s = 50 + \frac{2}{3}C$$

	Buyer (Value Range)			Buyer	Seller		
	0-50	51-150	151-200	R^2	a	β	R^2
Prediction	1.00	0.67	0.00		50.0		-
Mean (DSR-Exp1)	0.88	0.61	0.16	0.87	39.0	0.73	0.67
Median(RDS-Exp1)	0.89	0.64	-0.08	0.88	26.3	0.84	0.83

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Daniel, Seale and Rapoport (1998): Experiment 2

Buyer V: uniform[0, 200]; Seller C: uniform[0, 20]

$$p_b = \begin{cases} V & \text{if } V < 50\\ \frac{50}{3} + \frac{2}{3}V & \text{if } V \in [50, 70]\\ 63.3 & \text{if } V > 70 \end{cases}$$

$$p_s = 50 + \frac{2}{3}C$$

	Buyer (Value Range)			Buyer	Seller		
	0-50	51-70	71-200	R^2	α	β	R^2
Prediction	1.00	0.67	0.00	-	50.0	0.67	-
Median(DSR-Exp2)	0.78	0.46	0.21	0.76	39.0	0.73	0.67

▶ Overbid in first 10 rounds, then learn to bid much lower

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- Rapoport, Daniel and Seale (1998): Experiment 2 Seller C: uniform[0, 200]; Buyer V: uniform[100, 200]
 - ▶ Flip buyer-seller asymmetry; fixed pairing (買方價值/賣方成本的分配

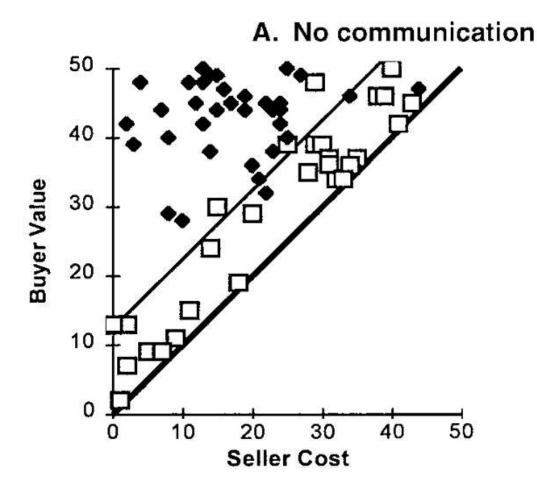
$$p_s = \begin{cases} 83.3 & \text{if } C < 50 \\ 50 + \frac{2}{3}C & \text{if } C \in [50, 150] \\ C & \text{if } C > 150 \end{cases}$$
 不對稱反過來、固定配對)
$$p_b = 16.7 + \frac{2}{3}V$$

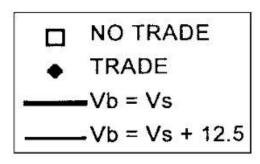
	Seller	Seller	Buyer				
	151-200	51-150	0-50	R^2	α	β	R^2
Prediction	1.00	0.67	0.00	-	16.7	0.67	-
Median(RDS-Exp2)	0.95	0.62	0.05	0.91	15.0	0.71	0.80

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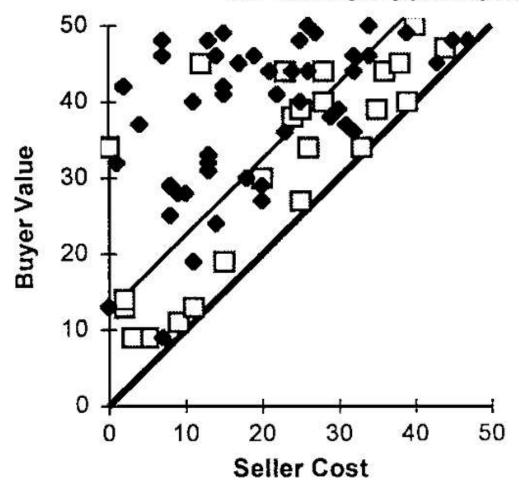
- ▶ Valley et al. (GEB 2002): Communication (溝通)
- Buyer/Seller Values/Costs: uniform[0, \$50]
 - ▶ 買方價值/賣方成本的分配都是 uniform[0, \$50] (透過出價來談判)
 - Bargain by stating bids; 7 periods; no rematch
 - ▶ Half had no feedback (七回合不重複配對/一半沒有看結果)
- No communication: Sealed-bid in 2 minutes (沒有溝通: 2分鐘内 密封投標)
- ▶ Written communication: Exchange messages for 13 minutes before final bid (文字溝通: 13分鐘傳紙條交換意見)
- ▶ Face-to-face: Pre-game communication (當面: 事前溝通)

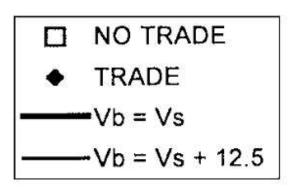
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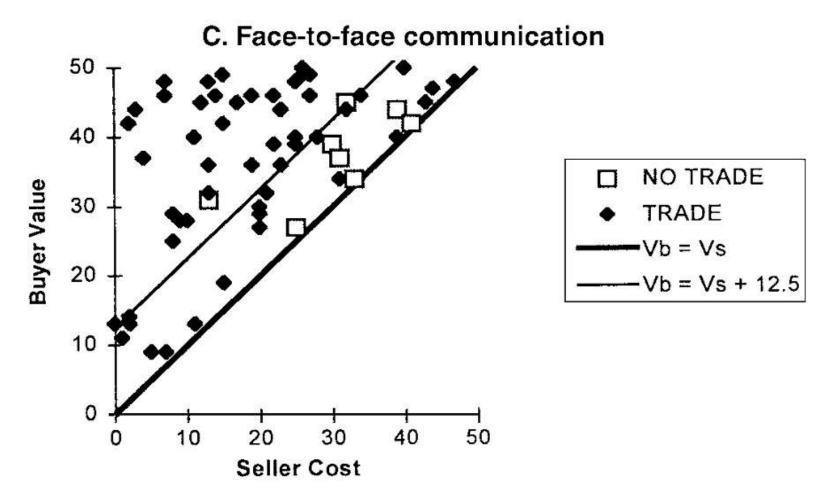












- Empirical bid function slope = 0.7 (near 2/3)
 - ▶ 實驗資料估計出來的出價函數斜率(=0.7)接近三分之二
- ▶ Why are there "gains of communication"?
 - ▶ 「溝通的好處」哪來?
- ▶ Slope of buyer bids against seller bids=0.6
- Buyers bid higher when seller bids higher
 - ▶ 賣家出價約為買家的六成且賣家出價愈高、買家出價愈高
 - Mutual bidding of values (common in students)
 - Mutual revelation of values (common in students)
 - ▶ (學生受試者更傾向)一起用真實價值/成本出價或一起揭露

- ▶ Coordinating on a price (40%文字/70%當面協調相同出價)
 - ▶ Happens 40% in written, 70% in face-to-face
- ▶ Not truth-telling (only 1/3) (講真話只有1/3旦未協調)
 - TT not coordinated (4% written, 8% face)
- ▶ Feel each other out; give enough surplus
 - ▶ Modal equal split of surplus (彼此試探後給足交易好處)
- Variance of surplus doubles (by mismatch)
 - ▶ (大多平分交易好處, 但交易好處的變異數倍增, 因為協調不成)

Summary for Asymmetric Information

- More Realistic!
 - ▶ Bargainers know their V, others only know the asymmetry
- Experimental Results surprisingly support theory!
 - One-sided bargaining: Sellers make sequential offers
 - Initial offer too high, but decline at predicted rates
 - Strike as predicted if bargain over large/small surplus
 - ▶ Sealed-bid mechanism: Confirm piece-wise linear equilibrium
 - ▶ Pre-play Communication: Efficiency higher than predicted
 - Used by players to agree on (self-enforcing) one-price equilibrium

Conclusion (結論)

- ▶ Unstructured Bargaining (自由談判: 焦點/競爭的焦點)
 - ▶ Focal divisions; competing focal points
 - ▶ Self-serving bias (erased by veil of ignorance or stating weakness of own case) (自立偏誤可以無知之幕或找己方弱點抗衡)
- ▶ Structured Bargaining (制式談判)
 - ▶ Deviate toward equal splits (朝平分偏離均衡預測/因社會偏好?)
 - Social preference models could explain this
 - ▶ But Johnson et al. (JET 2002) suggest deviation due to limited look-ahead (但MouseLAB結果顯示也可能來自無法「無限往前看」的有限理性)

Conclusion (結論)

- ▶ Outside options affect bargaining divisions only if threats are credible (威嚇點來真的才算數)
 - ▶ Lower fixed cost player gets everything (延遲成本低全拿)
- ▶ Information Asymmetry: One-Sided (單邊資訊不對等)
 - ▶ Revelation Principle + Random Dictator: Good
 - ▶ 顯示真實原則+隨機獨裁者公設的預測被實驗結果證實
 - ▶ Bazaar mechanism: (檢驗賣家一再降價直到買家接受的機制)
 - ▶ Offers decline as theory predicts, but start too high and respond to δ wrongly since buyers accept too early (降價幅度符合預測,但是初始開價太高,因買家會太快接受)

Conclusion (結論)

- ▶ Bilateral Bargaining: Two-Sided (雙邊資訊不對等談判)
 - ▶ Sealed-bid mechanism (密封投標機制): between truthful revelation & piecewise-linear equilibrium
- ▶ Players over-reveal values in face-to-face
 - ▶ Too honest, but "more efficient" (當面太誠實/超效率)
 - ▶ Communication → agree on a single price (溝通→合意單—價格)
- ▶ Why theory does better in sealed-bid than alternative-offer bargaining? (為何密封投標比反覆提議更合乎理論?)
 - Is sealed-bid cognitively more transparent?