

Experiments on Games with Mixed Strategy Equilibrium (混合策略均衡實驗)

Joseph Tao-yi Wang (王道一)
EE-BGT, Lecture 5

Some Comments Regarding 20-Minute Presentations

Joseph Tao-yi Wang (王道一)
EE-BGT

Comments Regarding 20-Minute Presentations

- ▶ Rule 1: Don't use more than 20 slides
 - ▶ It takes on average 1 minute to go over 1 slide
- ▶ Rule 2: Don't use font sizes below 28
 - ▶ Try looking at your slides from far behind
 - ▶ Font sizes < 28 means: You DON'T want people to see it
- ▶ Rule 3: This is a teaser-trailer of the movie
 - ▶ Show the experiment + a snapshot of the results
- ▶ Write down a script and NOT memorize it

Experiments on Games with Mixed Strategy Equilibrium (混合策略均衡實驗)

Joseph Tao-yi Wang (王道一)
EE-BGT, Lecture 5

Games with MSE 有混合策略均衡的賽局

- ▶ Zero-Sum Games (零和賽局)
 - ▶ Rock-Scissor-Paper (剪刀石頭布)
 - ▶ Sports (PK, tennis serves, basketball drives, etc.) (足球罰踢、網球發球、籃球切入或投籃)
 - ▶ Military attack (軍事行動如登陸諾曼地或加萊)
- ▶ Deter Undesired Behavior (嚇阻投機/不希望發生的行為)
 - ▶ Searches of passengers after 9/11 (機場安檢、海關抓走私)
 - ▶ Randomizing across exam questions (老師隨機出題)
- ▶ But, there are folk theories about these games... (但總有一些有趣的「理論」)

玩家公開猜拳遊戲必勝絕招：先出剪刀 (中央社 2007-12-19)

- ▶ 媒體報導，大多數人都知道，在猜拳遊戲中，石頭贏剪刀，剪刀贏布，布勝拳頭，但很少有人知道，如何贏得這個相當普遍的遊戲。現在死忠玩家透露了必殺秘技：

先出剪刀。

L0

- ▶ 英國「每日郵報」報導，研究顯示在這種快速擺出手部姿勢的猜拳遊戲中，石頭是三種猜拳手勢中玩家最喜歡出的一種。

L1

- ▶ 如果你的對手預期你會出石頭，他們就會選擇出布來贏過你，因此你要出剪刀才能贏，因為剪刀贏布。

L2

玩家公開猜拳遊戲必勝絕招：先出剪刀 (中央社 2007-12-19)

- ▶ 報導說，這套剪刀策略讓拍賣商佳士得前年成功贏得一千萬英鎊的生意。一名有錢的日本藝術品收藏家，無法決定要讓哪家拍賣公司來拍賣自己收藏的印象派畫作，於是 he 要求佳士得與蘇富比兩家公司猜拳決定。
- ▶ 佳士得向員工討教猜拳策略，最後在一名主管十一歲的女兒的建議下決定出剪刀。這名女孩現在還在讀書，經常玩猜拳，她推論「所有人都以為你會出石頭」。這代表蘇富比會出布，想要打敗石頭，因此佳士得應該選擇出剪刀。
- ▶ 一如預期，蘇富比最後出布，輸給了佳士得的剪刀，拱手將生意讓給對方。

Mixed-Strategy Equilibrium in Rock-Paper-Scissors

▶ How do you play Rock-paper-scissors (RPS)?

□ 如果你來玩剪刀石頭布，你會出什麼？

▶ What is the MSE here? (剪刀石頭布賽局的均衡為何?)

▶ Mix with probabilities $(1/3, 1/3, 1/3)$ (三者隨機)

▶ Would you really play **this MSE** in RPS?

▶ News article suggests a level-k model... (你真的會按均衡策略來玩嗎?)

新聞故事所反映的多層次思考模型預測為何？想知道更多請看課本第五章)

▶ Janken/RPS Robot with 100% winning rate:

▶ v1: <http://www.youtube.com/watch?v=3nxjjztQKtY>

▶ v2: <https://www.youtube.com/watch?v=ZVNnoOcohaU>

Advantages of Games with MSE (此種賽局的優點)

- ▶ Typically have unique equilibrium (有唯一均衡)
 - ▶ All games discussed have unique equilibrium
- ▶ Constant sum: No room for social preference
 - ▶ Not possible to help others without hurting self
(總報酬為常數下通常無社會偏好，因為幫助別人一定傷到自己)
- ▶ Maximin leads to Nash in zero sum (避兇就是均衡)
 - ▶ Maximin is a simple rule: (對方就是要害我如何趨吉避兇)
 - ▶ "I want to maximize the worse case scenario..."
- ▶ A good place to test theory! (這是驗證理論的好地方)

Maximin in Matching Pennies (黑白猜下避兇)

	H	T
H	1	-1
T	-1	1

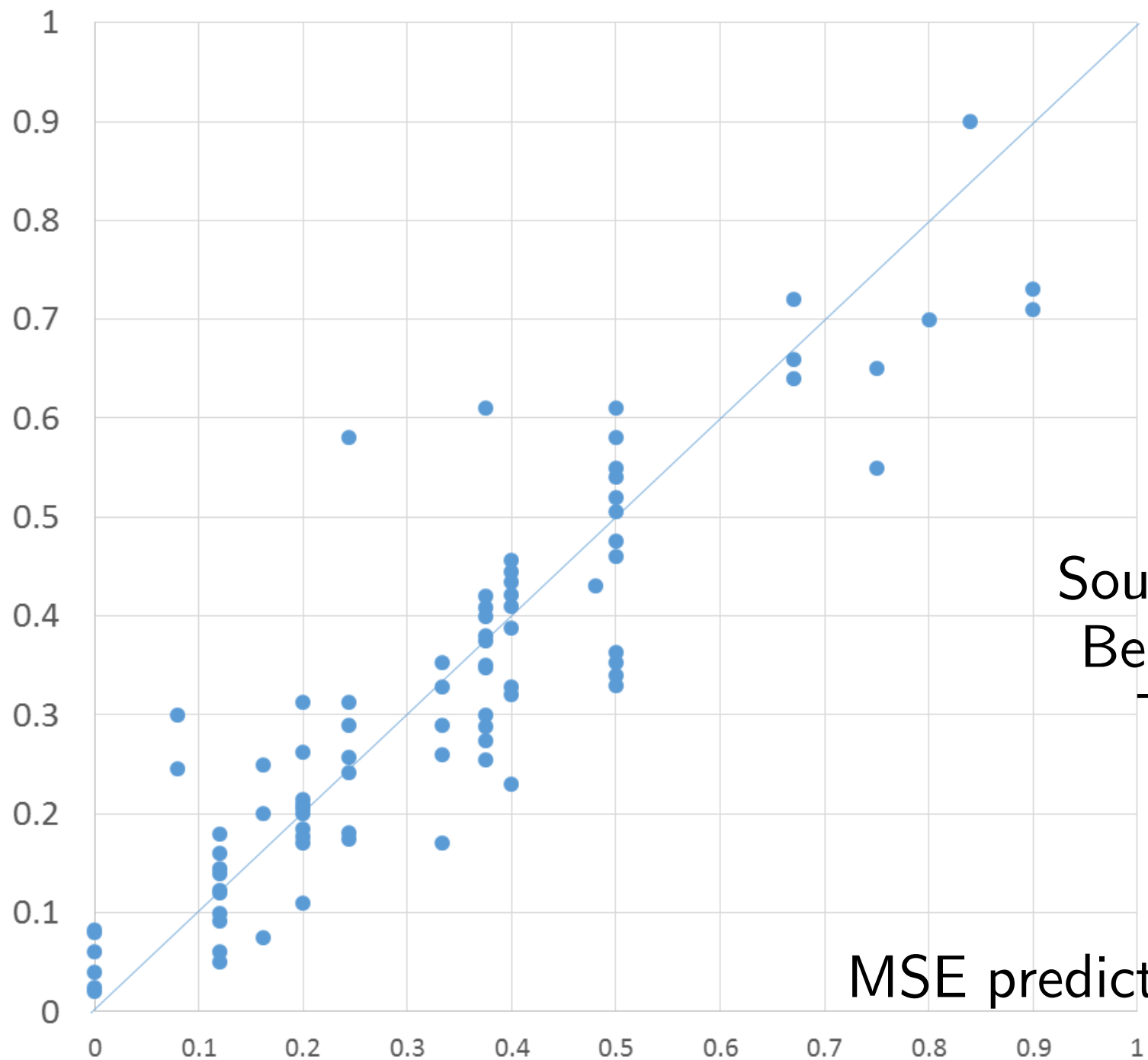
- ▶ **Rowena thinks:** (列子認為)
- ▶ Play H: Worse case = -1 (出正面最慘是對手)
 - ▶ If Colin wants to destroy me (選反面)
- ▶ Play T: Worse case = -1... (出反面最慘...)
- ▶ $(1/2, 1/2)$: Worse case 0^* (一半一半至少不賺不賠)
- ▶ **Same for Colin** (行家所見略同)
- ▶ **This is the MSE!** (這正好是此賽局的混合策略均衡!!)

*We assume preferences satisfy axioms for EU... (假設偏好滿足期望效用公理)

Challenges of Games with MSE (對理論的挑戰)

- ▶ **Epistemic Foundation** (認知基礎：須清楚知道對手的策略)
 - ▶ Requires precise knowledge of opponent strategy
- ▶ **Learning Dynamics may not work** (動態學習不見得好)
 - ▶ Gradient processes spiral away (梯度逼近會螺旋脫離均衡)
 - ▶ No incentive to mix properly at MSE (均衡時亂做沒差)
- ▶ **Randomization can be unnatural**
 - ▶ Especially in repeated play (重複做的話，隨機亂選不太自然)
 - ▶ **Purification** (純化：個體可做不同單純策略，整體看起來「混合」即可)
 - ▶ MSE can occur at population level, not individually

Actual Data
(實驗資料)



Source (資料來源):
Behavioral Game
Theory Ch 3

MSE predictions(均衡預測)

Joker Game: O'Neill (1987) (出鬼牌賽局)

- ▶ Earlier studies: Play between MSE and random
 - ▶ But had computerized opponents and/or low incentives, so hard to interpret the results... (早期實驗結果介於MSE和亂選之間，但通常對手是電腦且不見得有誘因)
- ▶ First "Modern" Studies: O'Neill (PNAS 1987)
- ▶ **Good Design Trick:** (很棒的實驗設計技巧!)
 - ▶ Risk aversion plays no role when there are only two possible outcomes (當實驗結果只有兩種可能時，風險偏好不會影響受試者的決定)

Joker Game: O'Neill (PNAS 1987) (出鬼牌賽局)

	1	2	3	J	MSE	Actual	QRE
1	-5	5	5	-5	0.2	0.221	0.213
2	5	-5	5	-5	0.2	0.215	0.213
3	5	5	-5	-5	0.2	0.203	0.213
J	-5	-5	-5	5	0.4	0.362	0.360
MSE	0.2	0.2	0.2	0.4	- 實際的出牌頻率跟MSE預測很接近 - QRE的預測更接近，但無法解釋「不平均」		
Actual	0.226	0.179	0.169	0.426			
QRE	0.191	0.191	0.191	0.427			

- ▶ Actual frequency quite close to MSE
- ▶ QRE better, but cannot get "imbalances"
- ▶ With $\lambda = 1.313$

Quantal Response Equilibrium (QRE) (手滑反應均衡)

- ▶ QRE - McKelvey and Palfrey (1995)
- ▶ **Better Response**, not best response (更適/非最適)
- ▶ Logit payoff response function: (常用logit報酬反應函數)

$$P(s_i) = \frac{e^{\lambda \cdot \left[\sum_{s_{-i}} P(s_{-i}) u_i(s_i, s_{-i}) \right]}}{\sum_{s_k} e^{\lambda \cdot \left[\sum_{s_{-i}} P(s_{-i}) u_i(s_k, s_{-i}) \right]}}$$

Quantal Response Equilibrium (QRE)

- ▶ $\lambda = 0$: Noise (do not respond to payoffs) (對報酬無反應)
- ▶ $\lambda = \infty$: Nash (perfectly respond to payoffs) (完全反應)

$$P(s_i) = \frac{e^{\lambda \cdot \left[\sum_{s_{-i}} P(s_{-i}) u_i(s_i, s_{-i}) \right]}}{\sum_{s_k} e^{\lambda \cdot \left[\sum_{s_{-i}} P(s_{-i}) u_i(s_k, s_{-i}) \right]}}$$

Response to O'Neill (PNAS 1987) (後續討論)

- ▶ Brown and Rosenthal (ECMA 1990) criticize O'Neill:

- ▶ Overly support MSE (太過支持混合策略均衡)
- ▶ Aggregate tests not good enough (只有總體檢定不夠)

- ▶ So, they run (temporal dependence): (應該檢定跨期相關性)

$$J_{t+1} = \underline{a_0} + \underline{a_1} J_t + \underline{a_2} J_{t-1} \quad (\text{MSE} + \text{Previous Own Choices})$$

(Guessing $\rightarrow \underline{b_0} J_{t+1}^* + \underline{b_1} J_t^* + \underline{b_2} J_{t-1}^*$ Previous Opponent Choices)

(Previous Outcomes) $\rightarrow \underline{c_1} J_t J_t^* + \underline{c_2} J_{t-1} J_{t-1}^* + \epsilon$

$$J_t = \text{Own Choice}; J_t^* = \text{Other's Choice};$$

- ▶ MSE implies only a_0 is not zero (均衡: 只有 a_0 不是0)

Brown and Rosenthal (ECMA 1990) Results

Effect	Coefficient	% Players w/ $p < 0.05$
Guessing	b_0	8%
Previous Opponent Choices	b_1, b_2	30%
Previous Outcomes	c_1, c_2	38%
Previous Choices + Outcomes	b_1, b_2, c_1, c_2	44%
Previous Own Choices	a_1, a_2	48%
All Effects		62%

Source: Table 3.4, BGT.

Response to O'Neill (PNAS 1987) (後續討論)

- ▶ **Run:** 2 JJJJ 1 2 33 (連發太短)
- ▶ **Too Short runs:** play J twice too rarely (鮮有連續J)
- ▶ **Subjects react to what they see and do** (對歷史有反應)
 - ▶ But most cannot use temporal dependence to guess opponent current action (無法用跨期相關性猜中對方這次行動)
- ▶ **Equilibrium-in-beliefs** somewhat supported (支持信念上的均衡)
 - ▶ Each player may deviate from MSE (每人各自可能偏離)
 - ▶ But these deviations cannot be detected (卻未被破解)

Response to O'Neill (PNAS 1987) (後續討論)

- ▶ **Purification interpretation of MSE** (純化的MSE)
 - ▶ Equilibrium in beliefs, not in mixtures (信念非策略)
- ▶ **Other similar studies** (相關延伸研究)
 - ▶ Rapoport and Boebel (1992): [BGT, Table 3.5]
 - ▶ Mookerjee and Sopher (1997): [BGT, Table 3.6-3.7]
 - ▶ Tang (1996abc, 2001): [BGT, Table 3.8]
 - ▶ Binmore, Swierzbinski and Proulx (2001): [BGT, Table 3.9]

Rapoport and Boebel (GEB 1992): Experiment 1

	C	L	F	I	O	MSE	Actual	QRE
C	10	-6	-6	-6	-6	0.375	0.293	0.286
L	-6	-6	10	10	10	0.250	0.305	0.302
F	-6	10	-6	-6	10	0.125	0.123	0.138
I	-6	10	-6	10	-6	0.125	0.119	0.138
O	-6	10	10	-6	-6	0.125	0.160	0.138
MSE	0.375	0.250	0.125	0.125	0.125	- 實際出牌頻率接近MSE預測(但85%個人拒絕) - QRE的預測更接近, 但無法解釋「不平均」		
Actual	0.352	0.180	0.218	0.099	0.151			
QRE	0.412	0.169	0.140	0.140	0.140			

- ▶ Actual frequency close to MSE
 - ▶ 85% subjects reject χ^2 test
- ▶ QRE better,
 - ▶ $\lambda_1 = 0.248$
 - ▶ (Cannot get imbalances)

Rapoport and Boebel (GEB 1992): Experiment 2

	C	L	F	I	O	MSE	Actual	QRE
C	15	-1	-1	-1	-1	0.375	0.306	0.309
L	-1	-1	15	15	15	0.250	0.324	0.296
F	-1	15	-1	-1	15	0.125	0.100	0.132
I	-1	15	-1	15	-1	0.125	0.115	0.132
O	-1	15	15	-1	-1	0.125	0.155	0.132
MSE	0.375	0.250	0.125	0.125	0.125	- 實驗二報酬和實驗一不同但MSE預測相同 - QRE的預測更接近 - 實驗結果和實驗一類似		
Actual	0.346	0.193	0.202	0.116	0.143			
QRE	0.410	0.184	0.135	0.135	0.135			

▶ Same MSE

▶ Different payoffs

▶ QRE better,

▶ $\lambda_2 = 0.327$

▶ Actual result is similar to Experiment 1

Other Similar Studies (相關延伸研究)

► Stylized Facts: (整體實驗結果)

1. Actual frequencies not far from MSE

► (出牌頻衡很接近MSE)

1. Deviations small but significant

► (跟MSE差距小但統計上顯著)

1. Temporal dependence at individual level

► (個人有跨期相關性)

► Can a theory explain these?

► (有何理論可以解釋這些實驗結果?)

Response to O'Neill (PNAS 1987) (後續討論)

- ▶ Ask subjects generate random sequences (產生數列)
- ▶ Sequences resemble the underlying statistical process **more closely than** what short random sequences actually do (產生的比真正隨機還要更「隨機」)
- ▶ Too balanced (太平衡)
- ▶ Too few runs (連發太少)
- ▶ Longest run is too short (最長的連發太短)
- ▶ Children don't learn this misconception until 5th grade
 - ▶ A learned mistake (這是一個後天學會的錯誤，小孩子在五年級之前沒有這個問題)

Game Play (賽局實驗) vs. Production (產生數列)

- ▶ Rapoport and Budescu (1992, 1994, 1997)
 - ▶ Compare sequences from a production task to strategies in a constant-sum game (R&B, 1992) (比較產生的數列和零和賽局實驗中的數列)
- ▶ **Condition D:** Matching pennies 150 times 1-by-1
 - ▶ 150次逐次黑白猜
- ▶ **Condition S:** Give sequence of 150 plays at once
 - ▶ 一次給150回合黑白猜的決定
- ▶ **Condition R:** Produce the outcome of tossing an unbiased coin 150 times (產生數列——丟銅板150次的結果)

Game Play (賽局實驗) vs. Production (產生數列)

- ▶ **iid** rejected for 40% (D), 65% (S), 80% (R) of the subjects in the three conditions, so:
 - ▶ 三種分別有40%, 65% 和80%的受試者拒絕 iid 假設
 - ▶ **Game play** reduces deviations from randomness
 - ▶ 真的去玩會讓受試者比較隨機(降低偏離情形)
- ▶ Are subjects better motivated?
 - ▶ 這是因為受試者有更好的誘因,
- ▶ Or, are their working memory interfered and randomize "memory-lessly"?
 - ▶ 還是因為他們的腦部運作(工作記憶)受到干擾, 以致於「忘記過去, 努力面前」?

3-action Matching Pennies

	1	2	3	MSE
1	2	-1	-1	$1/3$
2	-1	2	-1	$1/3$
3	-1	-1	2	$1/3$

MSE	$1/3$	$1/3$	$1/3$
-----	-------	-------	-------

► Rapoport and Budescu (1994)

Runs in 3-act

▶ Game and Production are close...

▶ Game closer to iid than Production!

Pattern	Game (%)	Production (%)	iid (%)
xx	26.9%	27.2%	33.3%
xyx	19.6%	20.9%	22.2%
xyy	19.6%	21.0%	22.2%
xxx	7.3%	6.3%	11.1%
xxxx	2.0%	1.8%	3.7%
xxxy	5.3%	4.5%	7.4%
yxxx	5.4%	4.5%	7.4%
xyxx	5.6%	3.5%	7.4%
xxyx	5.8%	3.7%	7.4%

Other Play in 3-action Matching Pennies

Pattern	Game (%)	Production (%)	iid (%)
xy	73.1%	72.8%	66.7%
xyx	23.7%	16.0%	22.2%
xyz	29.7%	35.9%	22.2%
yxzx	9.6%	7.8%	7.4%
xyxz	9.9%	7.9%	7.4%
xyzx	12.1%	17.3%	7.4%

Source: Table 3.10, BGT.

A Limited Memory Model (有限記憶模型)

- ▶ Subjects only remember last m elements
- ▶ Chose the $(m+1)$ st to balance the number of H and T choices in the last $(m+1)$ flips (受試者只記得最後 m 回合，第 $(m+1)$ 回合做決定來平衡正反面在 $(m+1)$ 次中出現的次數。如果 m 很小就會正反變換太頻繁)
- ▶ If m is small, alternate choices too frequently
- ▶ Experimental Data: (Should all be 0.5 if iid)
 - ▶ $P(H|H)=0.42$, $P(H|HH)=0.32$, $P(H|HHH)=0.21$
- ▶ Requires $m=7$ to generate this (Magic 7?)
 - ▶ 實驗結果：如果iid的話應該都是0.5，但需要 $m=7$ 才能解釋實驗結果

Explicit Randomization (使用亂數產生器)

- ▶ Observe the randomization subjects want to play (觀察人們為亂數產生器設定何種機率)
 - ▶ Bloomfield (1994), Ochs (1995b), Shachat (2002)
- ▶ Explicit Randomization: (使用亂數產生器)
 1. Allocate 100 choices to either strategies (決定100張牌兩邊各放幾張)
 2. Choices are shuffled and computer selects one (讓電腦隨機打一張)
 - ▶ Deviations cannot be due to cognitive limit! (還偏離均衡就不是因為不能產生亂數)
- ▶ **Result:** Deviations from MSE small but significant
- ▶ About 10% purists (偏離MSE很小但顯著。10%「單純的人」)

Explicit Randomization (使用亂數產生器)

- ▶ Ex: Ochs (1995b) - Matching Pennies (黑白猜)
 - ▶ Row player payoff of (H, H): $1 \rightarrow 9 \rightarrow 4$ (改列子報酬)
- ▶ MSE: Column strategy changes, but row is same!
 - ▶ 行家的MSE會改變；列子的反而不會變
- ▶ Allocate 10 plays of H or T (分配十個選擇給正或反)
 - ▶ Becomes a 10-play sequence (變成「做十次的數列」)
- ▶ Note: Random draw without replacement
 - ▶ This is not exactly randomization of MSE...
 - ▶ 註：這是隨機抽取不放回，不是真的MSE...

Matching Pennies (Baseline)

	H	T
H	1, 0	0, 1
T	0, 1	1, 0

- ▶ MSE:

- ▶ R: (0.500, 0.500)

- ▶ C: (0.500, 0.500)

- ▶ Actual Frequency: (實際頻率)

- ▶ R: (0.500, 0.500)

- ▶ C: (0.480, 0.520)

- ▶ QRE:

- ▶ R: (0.500, 0.500)

- ▶ C: (0.500, 0.500)

Matching Pennies (Game 2)

	H	T
H	9, 0	0, 1
T	0, 1	1, 0

▶ MSE:

▶ R: (0.500, 0.500)

▶ C: (0.100, 0.900)

▶ Actual Frequency: (實際頻率)

▶ R: (0.600, 0.400)

▶ C: (0.300, 0.700)

▶ QRE:

▶ R: (0.649, 0.351)

▶ C: (0.254, 0.746)

Matching Pennies (Game 3)

	H	T
H	4, 0	0, 1
T	0, 1	1, 0

▶ MSE:

▶ R: (0.500, 0.500)

▶ C: (0.200, 0.800)

▶ Actual Frequency: (實際頻率)

▶ R: (0.540, 0.460)

▶ C: (0.340, 0.660)

▶ QRE:

▶ R: (0.619, 0.381)

▶ C: (0.331, 0.669)

MSE in Field Context (實際現場的MSE)

- ▶ Rapoport and Almadoss (2000)
- ▶ Patent Races Game (競相專利賽局)
 - ▶ Two firms with endowment $e = 5$ (兩家廠商, 各有財產)
 - ▶ Invest 1, 2, ..., $e = 5$ (integer)
 - ▶ Win $r = 8$ or 20 if invest the most
- ▶ Unique MSE:
 - ▶ Invest e with prob. $1 - e/r$, invest others with prob. $1/r$
 - ▶ Not obvious! (37.5%/75%) (12.5%/5%)

Patent Race Results (競相專利賽局實驗結果)

(Table 3.14)	Game L ($e = 5, r = 8$)		Game H ($e = 5, r = 20$)	
Investment	MSE	Actual	MSE	Actual
0	12.5%	16.9%	5.0%	14.1%
1	12.5%	11.6%	5.0%	5.5%
2	12.5%	8.8%	5.0%	5.3%
3	12.5%	11.8%	5.0%	5.3%
4	12.5%	9.0%	5.0%	6.9%
5	37.5%	41.8%	75.0%	62.8%

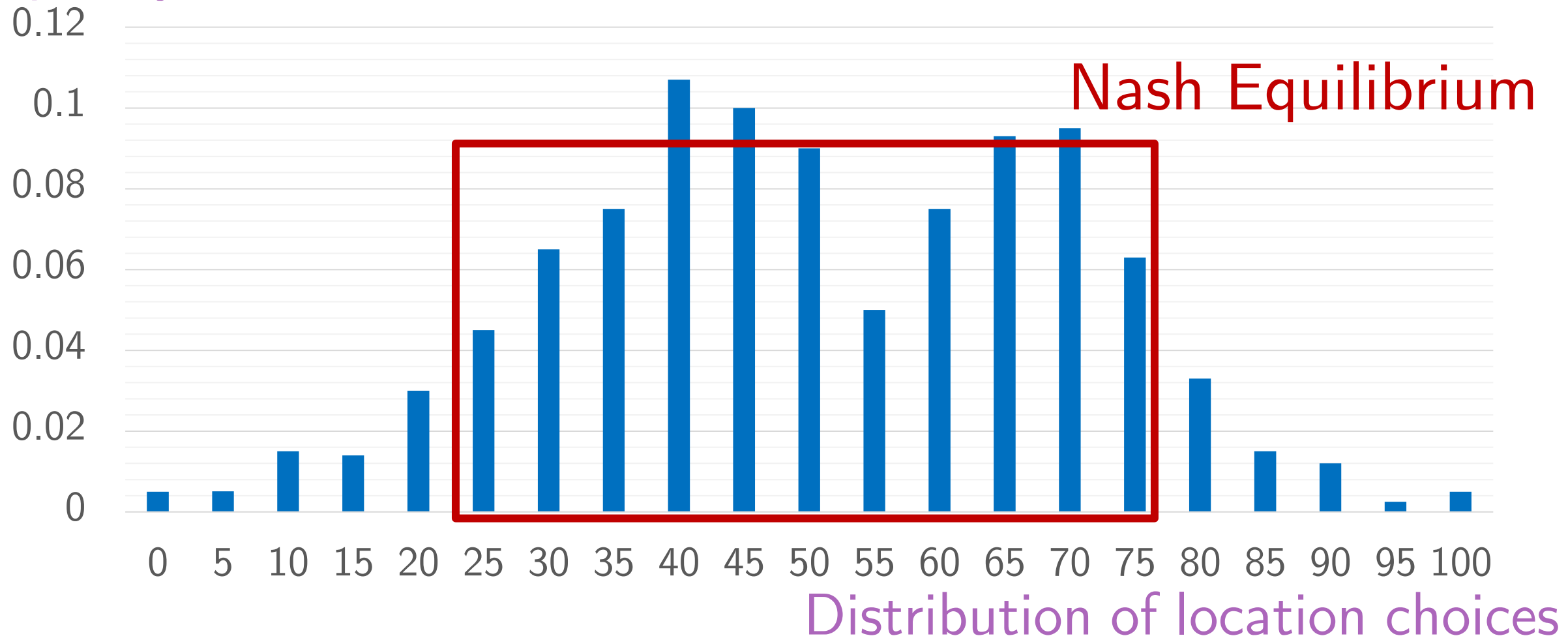
MSE in Field Context (實際現場的MSE)

- ▶ **3 Firm Hotelling:** Collins and Sherstyuk (2000)
 - ▶ 2-Firm: Brown-Kruse, Cronshaw and Schenk (1993)
 - ▶ 4-Firm: Huck, Muller and Vreind (2002)
- ▶ **Location Games (3 Firm Hotelling Model)**
 - ▶ Three firms simultaneously choose $[0,100]$
 - ▶ Consumers go to nearest firm
 - ▶ Profits proportional to units sold
- ▶ **Unique MSE:** Randomize uniformly $[25,75]$

MSE in Field Context (實際現場的MSE)

Frequency

Source: Figure 3.2, BGT; Based on Colins and Sherstyuk (2000).



Two Field Studies

- ▶ **Walker and Wooders (2001)**

- ▶ Serve decisions (L or R) of tennis players in 10 Grand Slam matches

- ▶ **Result:**

- ▶ Win rates across two different directions are not statistically different ($p < 0.10$ for only 2/40)
 - ▶ Players still exhibit some over-alteration in serve choices through temporal dependence ($p < 0.10$ for 8/40)
 - ▶ Weaker than lab subjects

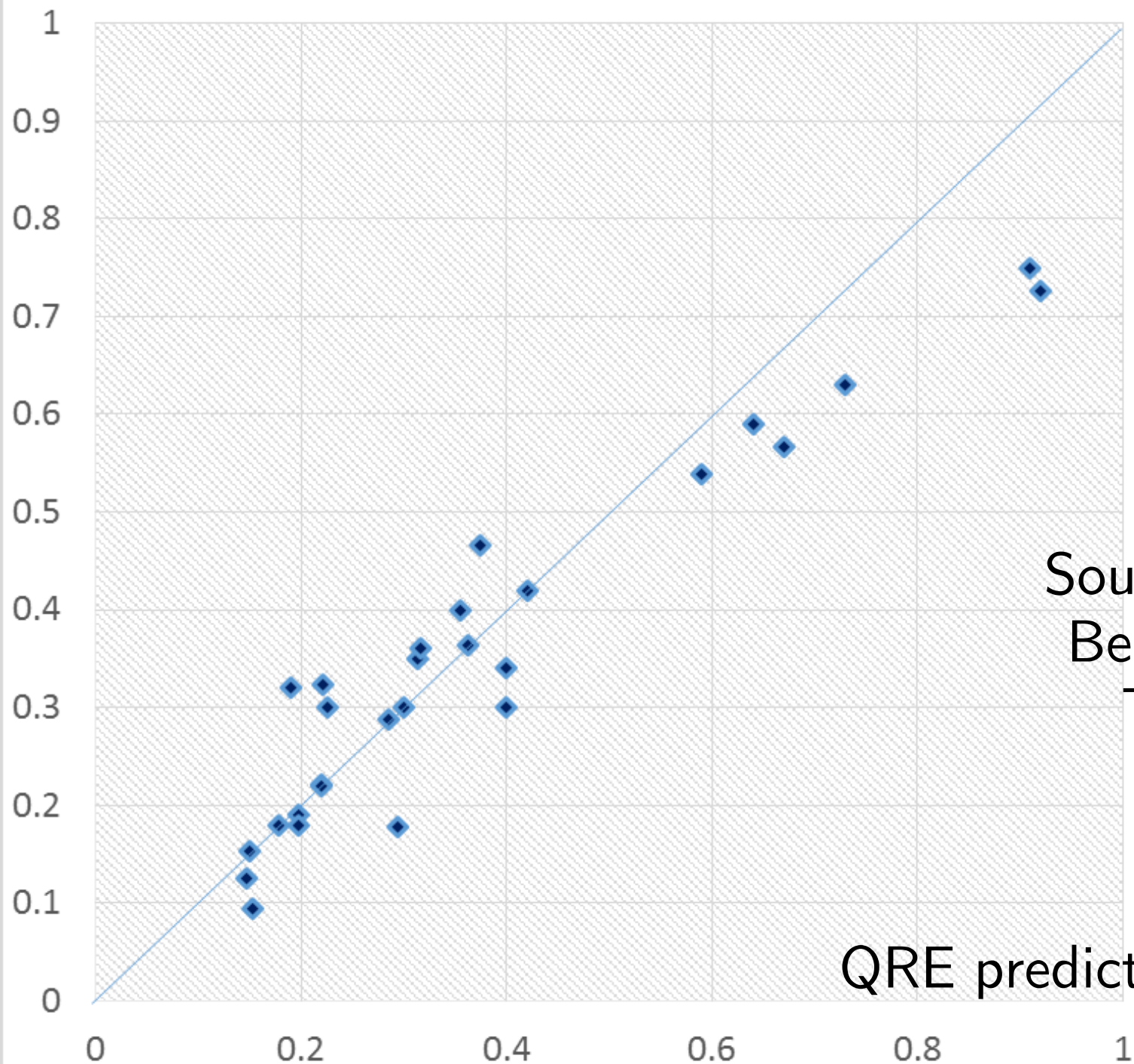
Two Field Studies

- ▶ **Palacios-Huerta (2001): Soccer Penalty Kicks**
 - ▶ Code both kicker and goalie's choices
 - ▶ No selection bias (look at all games)
- ▶ Win rates are equal; no serial dependence
 - ▶ Not surprising since penalty kicks are few and are often done by different players
- ▶ Recent: Huang, Hsu, and Tang (AER 2007)
 - ▶ Chen-Ying Huang (here at NTU)

Conclusion

- ▶ Take-Home Message:
- ▶ Aggregate frequencies of play are close to MSE but the deviations are statistically significant.
- ▶ QRE seems to fit behaviors well
- ▶ Temporal dependence frequently observed

Actual Data
(實驗資料)



Source (資料來源):
Behavioral Game
Theory Ch 3

QRE predictions(均衡預測)

Conclusion

- ▶ With explicit randomization, the existence of purists hint on **equilibrium in beliefs**
 - ▶ Players cannot guess what opponents are doing
 - ▶ Beliefs about opponents are correct on average
 - ▶ But, they may not be randomizing themselves
- ▶ **Field-Lab-Theory:** Ostling, Wang, Chou and Camerer (2011), "Testing Game Theory in the Field: Evidence from Swedish Poisson LUPI Lottery Games," *American Economic Journal: Microeconomics*, 3(3), 1-33.