What Is Market Design?
了解市場設計

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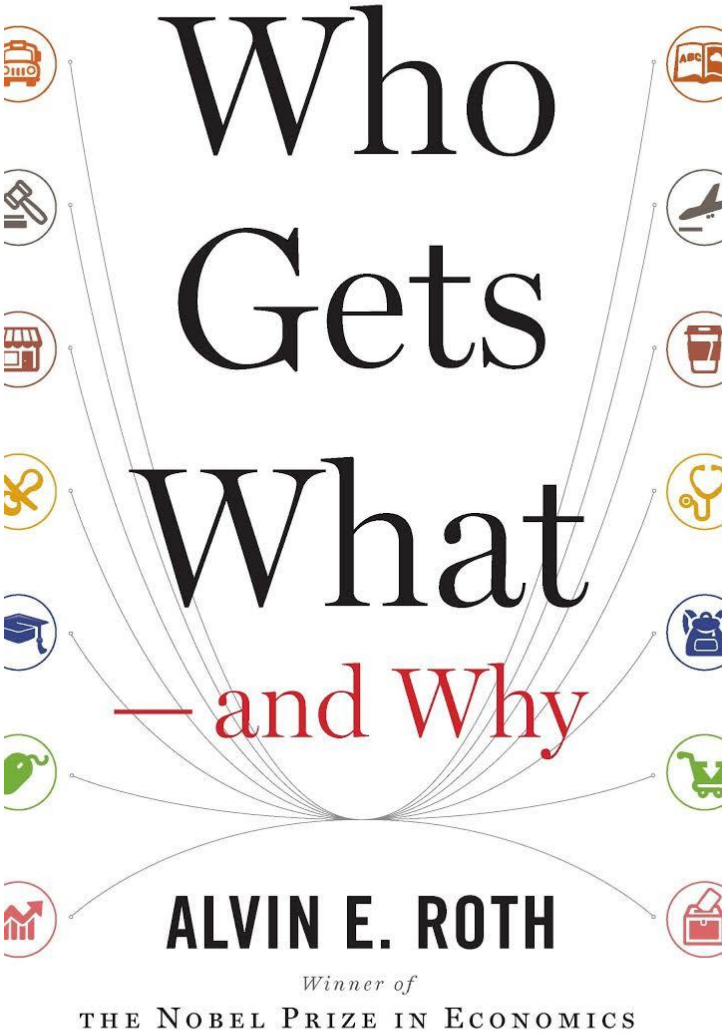
Market Design: Prizing Winning Idea 2012

- Both in the Lab and Field
- Alvin E. Roth (Stanford)

(Keynote of 2013 ESA North American Meeting, Santa Cruz)



Market Design is… (市場設計就是)

- Design an institution to realize gains from trade previously unavailable to solve the problem of (lack of) market failure.
  - 「設計一個制度讓原本無法實現的交易好處得以實現，解決『缺乏市場造成的失靈』(市場失靈)」
- Traditionally, market failure means:
  - Externalities and Public Goods
- But these are lack of market failures
  - Not failures of the market!
  - 傳統上的市場失靈是殃及他人的額外效果(外部性)和可以共享的財貨(公共財)，但這其實是缺乏市場所造成的失靈!

Market Design is Everywhere! (已在你我身邊!)

- Online Trading Platforms
  - 網拍平台讓原本只能讓社區鄰居參與的跳蚤市場擴大參與
- Patents
  - 專利讓知識(可共享的公共財)被發現、發明得到獎勵
- Carbon Market
  - 碳排放市場界定排放權歸屬/減少殃及他人的額外效果
- Social Norms
  - 社會規範為避免竭澤而漁、共同悲劇(Tragedy of Commons)
- Rebuilding Taipei First Fruit and Vegetable Wholesale Market also! (台北第一果菜市場的改建工程也是!)

Examples of Market Design (市場設計的範例)

- Top-Trading Cycle for Agent-Item Matching
  - 人與物的配對市場使用小圈圈優先交換(TTC)演算法
- Delayed Acceptance in Agent-Agent Matching
  - 人與人的配對市場使用延遲接受(DA)演算法
- Auction Design
  - 拍賣設計讓獨佔/獨買者把競爭的力量發揮到極致
- Screening and Signaling
  - 篩選機制與認證標籤克服市場中的資訊落差(asymmetric information)
- Let's see a story regarding Ten Principles of Taiwanese Economics... (來看看鄉民經濟學原理第七條...)

7. 許多產業都很神聖，絕對不能商品化。

- 肝肝相連到天邊(張桂越) (蘋果日報2008/10/24)
- 我有兩個弟弟，一個2004年死了，一個2008年換肝成功。一個在台灣，一個在美國。…
  - 受限法令 有肝無用
- 三弟陷入肝昏迷時… 我們全家大小包括媳婦們的肝，統統願意割一片給三弟，這是「合法的」，卻統統不合比對標準，不是血型不合，就是這個那個的，
- 而三弟幾個當兵的兄弟，肝膽相照，個個身強體壯，血型也對，卻不符合中華民國的法律，見死不能救。
- 我只好鬼鬼祟祟的，聯絡到大陸的換肝掮客，…

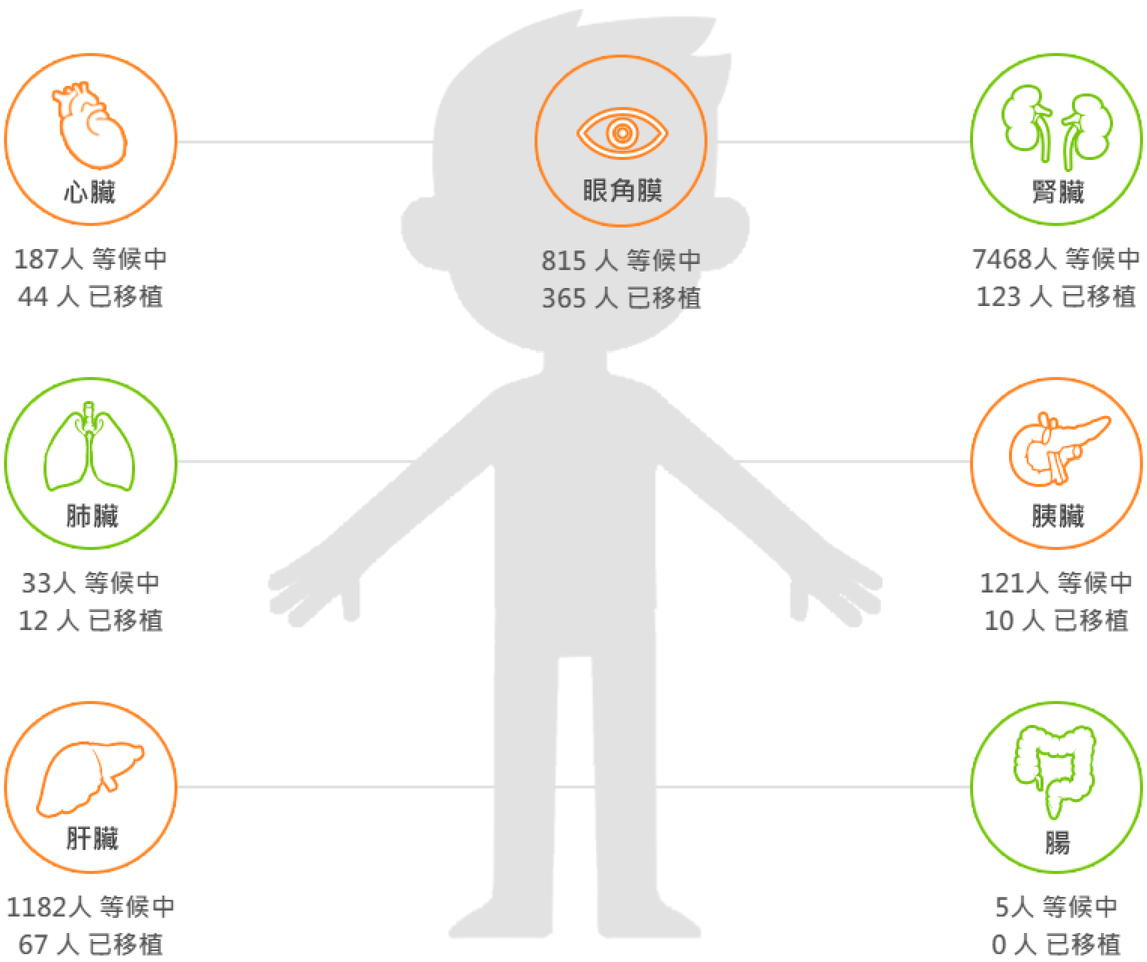
7. 許多產業都很神聖，絕對不能商品化。

- 故事還沒說完。上個月，接到西雅圖的電話，說大弟已進入開刀房，六小時後換肝。今天，大弟換肝手術成功，...
- 對兩個弟弟，一個在台灣，一個在美國，一種肝病兩種命運，我不解神的奧秘，
- 但我知道我們美國家人沒有送一毛錢紅包，沒有求朋友的特權，沒有找什麼參議員，沒有像熱鍋上螞蟻般東奔西跑，沒有用個人的智慧與財力為大弟求得一塊肝，
- 卻順順利利地，在短時間內，可以說是悄悄地換肝成功，不可思議的背後，大有學問：

7. 許多產業都很神聖，絕對不能商品化。

1. 美國社會對器官捐贈的教育普及
   - 供需失衡 自然要搶：台灣的肝病患者排不到、等不到，因為供需失調，幾千個人等一個肝，當然要搶，十八般武藝勢必出籠，送紅包沒用的話，跳進大陸買賣肝臟的漩渦又是何其自然的事。如果國家有健康的機制，誰願意到大陸冒險？(JW: 其實全世界只有一個地方的器官市場沒有供需失衡，你知道是哪裡嗎？不是中國喔！)
2. 盡速成立臨時小組，解決危險個案。有些病人命在旦夕，立法審案冗長費時，有些病人是不能等的
3. 建議立法委員或相關衛生單位，能夠盡速學習與參考國外換肝機制，借他山之石，改善國人換肝機制

7. 許多產業都很神聖，絕對不能商品化。



(財團法人器官捐贈移植登錄中心 107年等候/捐贈移植統計)

7. 許多產業都很神聖，絕對不能商品化。

- 2009年至2017年國內肝、腎臟活體捐贈移植例數
  - 財團法人器官捐贈移植登錄中心 (2009/1/1～2017/12/31)

年度	2009	2010	2011	2012	2013	2014	2015	2016	2017	總計
肝臟	266	344	401	431	447	485	505	428	406	3713
腎臟	90	97	84	73	128	129	104	112	112	929

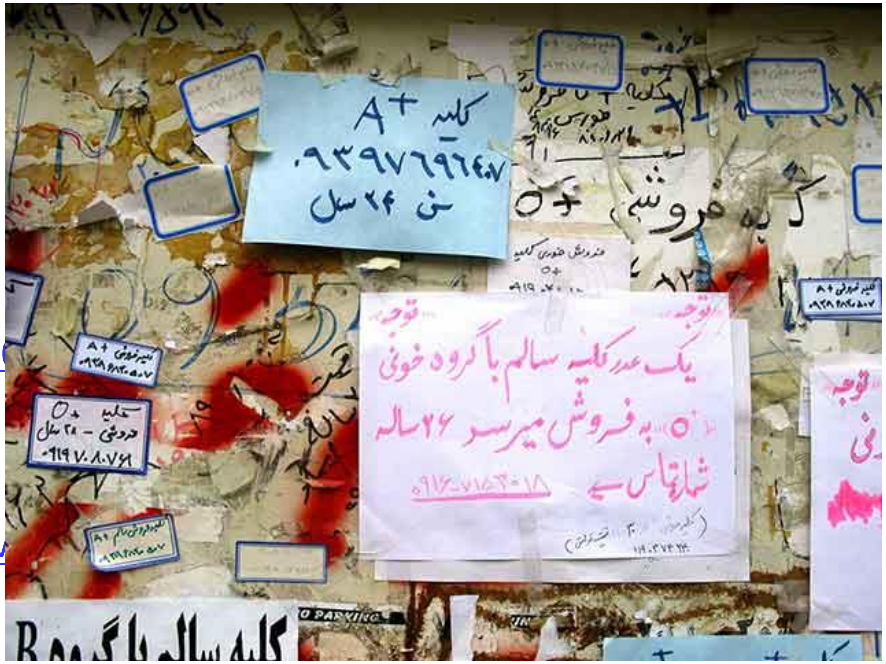
- 公共電視—「獨立特派員」心肝那裡找
  - https://youtu.be/mkRXHcQMAJo?t=1258

Is Selling Organs Acceptable? (器官可以買賣嗎?)

- There is a place you can buy/sell organs legally!
  - 全世界有一個地方可以合法買賣器官…
  - The Guardian posted a touching album of postings on streets around hospitals offering to…
- At Iran!! (伊朗)
  - Kidneys for sale:
    - Iran's trade in organs

https://www.theguardian.com/society/2015/may/1

  - Kidney trade in Iran
    - Wikipedia: en.wikipedia.org/w

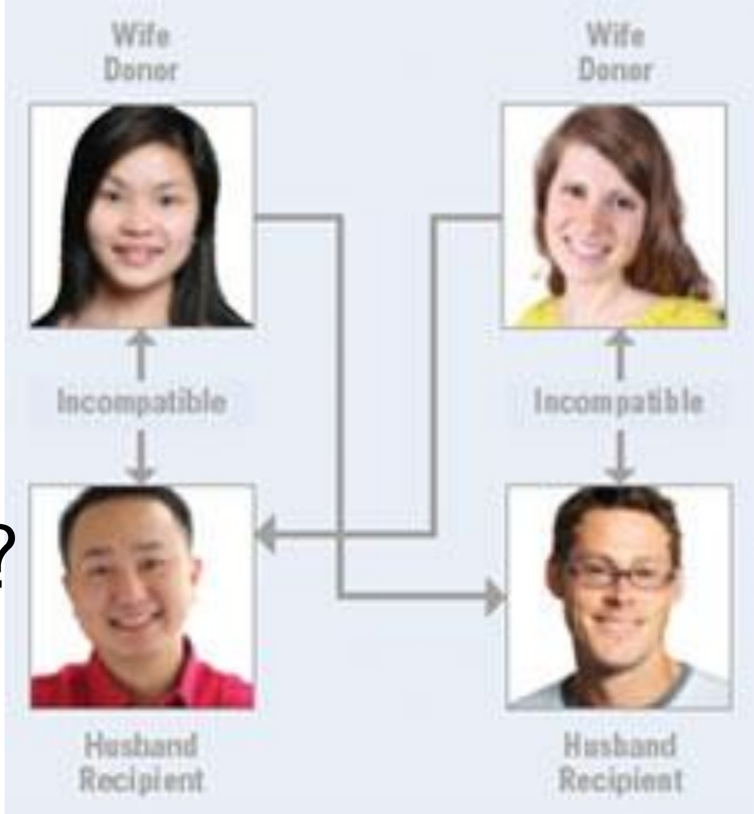


Even If Selling Organs is Not Acceptable

- Should we ban all organ exchanges
  - including those without monetary transfers?
  - 即使不能接受器官買賣，難道沒有金錢交易的器官交換也必須禁止嗎?
- If I want to donate to my wife, but can't
- And you want to donate to your family, too!
  - 假如我想捐腎給家人、但血型不合，你也一樣想捐，但…
- Can I donate to your wife in exchange for you to donate to my wife?
  - 那可以我捐給你家人、「交換」你捐給我家人嗎？

Even If Selling Organs is Not Allowed...

- Kidney SWAP (配對交換捐贈)
  Paired Donor Exchange Transplantation (UCLA Kidney Exchange Program)
- When a donor and a recipient cannot match (blood type,...),
- they can exchange with another pair with similar problems
  - 捐贈者和受贈者血型不合時可跟有類似問題但正好能配合的另一對交換
- What about 3-way-exchange?
  - 那三方捐贈可以嗎?



SWAPs Allowed? Why Not Chain Reaction?

- Chain Transplantation, Kidney Chain: (連鎖捐贈)
  - Altruistic donor gives to a recipient, whose relative donates to a 2nd recipient, whose relative donates...
  - 如果配對交換捐贈可行，那「連鎖反應」呢? 某無私捐贈者捐腎，(無法直接捐贈的)受贈者親屬捐腎給第二位病患，第二位受贈者親屬再繼續捐...

真正的「肝肝相連到天邊」在加州!

- 60 Lives, 30 Kidneys, All Linked (2012/2/18 紐約時報)

From Start to Finish A donation by a Good Samaritan, Rick Ruzzamenti, upper left, set in motion a 60-person chain of transplants that ended with a kidney for Donald C. Terry Jr., bottom right.

Design Agent-to-Item Matching Markets

設計「人/物配對市場」

(坂井豐貴《如何設計市場機制》Ch.1)

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Agent-Item Matching (設計「人/物配對市場」的例子)

- 4 dorm rooms assigned to 4 students:
  - 某棟宿舍有四個房間、住著四位學生，其偏好為:
  - Room 1 to student 1: 4 > 3 > 2 > 1 (住房間1的學生1)
  - Room 2 to student 2: 3 > 4 > 2 > 1 (住房間2的學生2)
  - Room 3 to student 3: 2 > 4 > 1 > 3 (住房間3的學生3)
  - Room 4 to student 4: 3 > 2 > 1 > 4 (住房間4的學生4)
- Example of Agent-to-Item Matching Markets
  - Shapley and Scarf (1974), "On Cores and Indivisibility," Journal Mathematical Economics, 1, 23-37.

Agent-Item Matching Market (人/物配對市場)

- Everyone needs only one item; cannot buy/sell
  - 每人只需要一個、但不允許金錢交易的物品
- Examples: Dorm Rooms, Office Space, Kidney (or other Organs), etc.
  - 宿舍房間、辦公室(使用空間)，腎臟(器官)等等
- What Properties do we wish to see?
  - 我們希望結果符合哪些條件?
  - Shapley and Scarf (1974), "On Cores and Indivisibility," Journal Mathematical Economics, 1, 23-37.

Desirable Properties of Market Design

- 我們希望市場設計的結果符合哪些條件?
- Dormitory Exchange usually requires:
  - 通常生自會設計的換宿制度，需要滿足:

1. Non-Repugnance (不起反感、不涉及金錢交易)
2. Individual Rationality (沒有人換到比目前更糟的房間)
3. Pareto Efficiency = No Pareto Improvement
   - Someone strictly better-off and nobody worse-off
   - Pareto效率:沒有另一個分配可以讓此結果得到Pareto改善，也就是「在不傷害別人的情況下，讓某些人更好」

- Anything else? (還有嗎?)

Individual Rationality (個體自願參與)

- 4 dorm rooms assigned to 4 students:
  - Room 1 to student 1: 4 > 3 > 2 > 1 (住房間1的學生1)
  - Room 2 to student 2: 3 > 4 > 2 > 1 (住房間2的學生2)
  - Room 3 to student 3: 2 > 4 > 1 > 3 (住房間3的學生3)
  - Room 4 to student 4: 3 > 2 > 1 > 4 (住房間4的學生4)
- How can you get everyone on board?
  - Nobody worse-off! (如何設計才能讓個體自願參與呢?)
  - 只要沒有人換到比目前更不喜歡的房間就行了!!
- Just don't give Room 1 to Student 2!
  - 不要強迫分配房間1給學生2就行了! 其他學生不會更糟了!

Pareto Efficiency (如何設計能讓結果更有效率呢?)

- 4 dorm rooms assigned to 4 students:
  - Room 1 to student 1: 4 > 3 > 2 > 1 (住房間1的學生1)
  - Room 2 to student 2: 3 > 4 > 2 > 1 (住房間2的學生2)
  - Room 3 to student 3: 2 > 4 > 1 > 3 (住房間3的學生3)
  - Room 4 to student 4: 3 > 2 > 1 > 4 (住房間4的學生4)

1. Assign Room 3412 to Student 1234 (Better!)
   - Is it Pareto Efficient? Any Pareto Improvement?
   - What if assign Student 12 to Room 43?
   - 分配房間3412給學生1234比原來分配好!但是有達成Pareto效率嗎?有其他分配是更好的Pareto改善嗎?如果給學生12房間43(而非房間34)呢?

Pareto Efficiency (如何設計能讓結果更有效率呢?)

- 4 dorm rooms assigned to 4 students:
  - Room 1 to student 1: 4 > 3 > 2 > 1 (住房間1的學生1)
  - Room 2 to student 2: 3 > 4 > 2 > 1 (住房間2的學生2)
  - Room 3 to student 3: 2 > 4 > 1 > 3 (住房間3的學生3)
  - Room 4 to student 4: 3 > 2 > 1 > 4 (住房間4的學生4)

1. Assign Room 3412 to Student 1234 (Better!)
2. Assign Room 4312 to Student 1234 (Red)
   - Better than Allocation 1! (房間4312給學生1234比分配1好!)
   - Any Pareto Improvement? No, so Pareto Efficient!
     - 有更好的Pareto改善嗎? 沒有! 所以就達到Pareto效率囉!

Pareto Efficiency (如何設計能讓結果更有效率呢?)

- 4 dorm rooms assigned to 4 students:
  - Room 1 to student 1: 4 > 3 > 2 > 1 (住房間1的學生1)
  - Room 2 to student 2: 3 > 4 > 2 > 1 (住房間2的學生2)
  - Room 3 to student 3: 2 > 4 > 1 > 3 (住房間3的學生3)
  - Room 4 to student 4: 3 > 2 > 1 > 4 (住房間4的學生4)

2. Assign Room 4312 to Student 1234 (Red)
   - What if another allocation is also Pareto efficient?
   - 如果有兩種以上分配都符合Pareto效率怎麼辦?
3. Assign Room 4321 to Student 1234 (also PE)
   - 分配房間4321給學生1234也符合Pareto效率呀! 那要選哪一個?

Will it be Blocked? (看這個設計會不會被小圈圈阻?!)

- 4 dorm rooms assigned to 4 students:
  - Room 1 to student 1: 4 > 3 > 2 > 1 (住房間1的學生1)
  - Room 2 to student 2: 3 > 4 > 2 > 1 (住房間2的學生2)
  - Room 3 to student 3: 2 > 4 > 1 > 3 (住房間3的學生3)
  - Room 4 to student 4: 3 > 2 > 1 > 4 (住房間4的學生4)

2. Assign Room 4312 to Student 1234 (Red)

- Student 2 and 3 will block Allocation 2, since
  - 分配2會被學生2和3私下交易所阻擋!!
- Switching is a Pareto Improvement! (32>31)
  - 因為可以互換讓兩人有Pareto改善(32 > 31)

Cannot Block Strong Core (強力核可就不會被阻擋!)

- 4 dorm rooms assigned to 4 students:
  - Room 1 to student 1: 4 > 3 > 2 > 1 (住房間1的學生1)
  - Room 2 to student 2: 3 > 4 > 2 > 1 (住房間2的學生2)
  - Room 3 to student 3: 2 > 4 > 1 > 3 (住房間3的學生3)
  - Room 4 to student 4: 3 > 2 > 1 > 4 (住房間4的學生4)

2. Assign Room 4312 to Student 1234 (Red)

- Coalition 23 will block Assignment 2 (32>31!)
  - 分配2會被小圈圈(學生2和3)私下交易所阻擋，因為32 > 31!
- Assignment 3 = Strong Core = Cannot block!
  - 分配#3則是強力核可不會被擋! 強力核可(強力核/殼!)又稱「強核心」

Desirable Properties of Market Design

- Non-Repugnance (我們希望市場設計的結果符合哪些條件?至少要不起反感、不涉及金錢交易)

1. Individual Rationality (IR) (個體自願參與)
   - Yourself cannot block (自己一組不會更好、無法阻擋該分配)
2. Pareto Efficiency (PE) (效率)
   - Whole cannot block (所有人一組不會更好、無法阻擋該分配)

- Strong Core automatically satisfies IR and PE
  - No coalition can block (任何小圈圈都不會更好、無法阻擋)
  - 有強力核可,其實就會自動滿足前兩個條件!
  - IR=自己當小圈圈、PE=所有人圍一大圈

No Coalition Can Block Strong Core!!

No coalition can block Strong Core! It's IR and:

- 任何小圈圈都無法阻擋強力核可! 小圈圈都還是會自願參與，且能證明

- Strong Core Exists (強力核可的分配存在)
  - Shapley and Scarf (1974), "On Cores and Indivisibility," Journal Mathematical Economics, 1, 23-37.
- Strong Core is Unique (強力核可的分配唯一)
  - Roth and Postlewaite (1977), "Weak Versus Strong Domination in a Market With Indivisible Goods," Journal Mathematical Economics, 4, 131-137.
- How can we find it? (這麼好的分配要怎麼找出來?)

7 Dorm Rooms Assigned To 7 Students

- Rm 1 to Stud. 1: 5 > 6 > 7 > 1 > 2 > 3 > 4
- Rm 2 to Stud. 2: 3 > 4 > 5 > 6 > 7 > 1 > 2
- Rm 3 to Stud. 3: 4 > 5 > 2 > 7 > 1 > 3 > 6
- Rm 4 to Stud. 4: 1 > 2 > 3 > 4 > 5 > 6 > 7
- Rm 5 to Stud. 5: 4 > 5 > 2 > 3 > 6 > 7 > 1
- Rm 6 to Stud. 6: 7 > 1 > 2 > 3 > 4 > 5 > 6
- Rm 7 to Stud. 7: 1 > 7 > 4 > 5 > 6 > 3 > 2
- Find Strong Core! (請找出7個學生換宿舍強力核可的分配!)
  - Need to check 7!=5040 allocations and 2^7=128 coalitions! (要檢查7!=5040種分配、2^7=128種小圈圈!)

Is Assigning Top-2 Choices Strong Core?

- Rm 1 to Stud. 1: 5 > 6 > 7 > 1 > 2 > 3 > 4
- Rm 2 to Stud. 2: 3 > 4 > 5 > 6 > 7 > 1 > 2
- Rm 3 to Stud. 3: 4 > 5 > 2 > 7 > 1 > 3 > 6
- Rm 4 to Stud. 4: 1 > 2 > 3 > 4 > 5 > 6 > 7
- Rm 5 to Stud. 5: 4 > 5 > 2 > 3 > 6 > 7 > 1
- Rm 6 to Stud. 6: 7 > 1 > 2 > 3 > 4 > 5 > 6
- Rm 7 to Stud. 7: 1 > 7 > 4 > 5 > 6 > 3 > 2

- Top-2: Assign Room 6352471 to Student 1-7
  - 滿足前二志願序:分配房間6352471給學生1-7是強力核可的分配嗎?
  - Or switch 54 to 45! SC? (把54換成45也可! 是強力核可嗎?)

Assigning Top-2 Choices Not Strong Core

- Rm 1 to Stud. 1: 5 > 6 > 7 > 1 > 2 > 3 > 4
- Rm 2 to Stud. 2: 3 > 4 > 5 > 6 > 7 > 1 > 2
- Rm 3 to Stud. 3: 4 > 5 > 2 > 7 > 1 > 3 > 6
- Rm 4 to Stud. 4: 1 > 2 > 3 > 4 > 5 > 6 > 7
- Rm 5 to Stud. 5: 4 > 5 > 2 > 3 > 6 > 7 > 1
- Rm 6 to Stud. 6: 7 > 1 > 2 > 3 > 4 > 5 > 6
- Rm 7 to Stud. 7: 1 > 7 > 4 > 5 > 6 > 3 > 2

- Top-2: Assign Room 6352471 to student 1-7
  - Coalition 145 can block (all switch to favorite!!)
  - 小圈圈145可以阻擋: 他們可都換到第一志願讓學生14更好、學生5沒差

How to Find Strong Core Allocation?

- Rm 1 to Stud. 1: 5 > 6 > 7 > 1 > 2 > 3 > 4
- Rm 2 to Stud. 2: 3 > 4 > 5 > 6 > 7 > 1 > 2
- Rm 3 to Stud. 3: 4 > 5 > 2 > 7 > 1 > 3 > 6
- Rm 4 to Stud. 4: 1 > 2 > 3 > 4 > 5 > 6 > 7
- Rm 5 to Stud. 5: 4 > 5 > 2 > 3 > 6 > 7 > 1
- Rm 6 to Stud. 6: 7 > 1 > 2 > 3 > 4 > 5 > 6
- Rm 7 to Stud. 7: 1 > 7 > 4 > 5 > 6 > 3 > 2

- Top Trading Cycle (用小圈圈優先交換演算法找強力核可分配)
  - All point to 1st choice (所有人都指向自己第一志願)
  - Find Trading Cycle [$1 \rightarrow 5 \rightarrow 4 \rightarrow 1$] (發現小圈圈)

How to Find Strong Core Allocation?

- Rm 2 to Stud. 2: 3 > 6 > 7 > 2
- Rm 3 to Stud. 3: 2 > 7 > 3 > 6
- Rm 6 to Stud. 6: 7 > 2 > 3 > 6
- Rm 7 to Stud. 7: 7 > 6 > 3 > 2

- Top Trading Cycle Algorithm (小圈圈優先交換演算法)
  - Cycle [1 → 5 → 4 → 1] trade 1st (小圈圈就優先交換)
  - All point to remaining 1st (其餘人都指向剩下房間的第一志願)
  - Find Trading Cycle [2 → 3 → 2] (發現兩人小圈圈)
  - And Self Cycle [7 → 7] (還有自我小圈圈)
  - [2 → 3 → 2] and [7 → 7] trade 2nd (這些也優先交換)

How to Find Strong Core Allocation?

- Rm 6 to Stud. 6: 6
- Top Trading Cycle Algorithm (小圈圈優先交換演算法)
  - Cycle [1 → 5 → 4 → 1] trade 1st (小圈圈優先交換)
  - [2 → 3 → 2] and [7 → 7] trade 2nd (這些也優先交換)
  - All point to remaining 1st (其餘人繼續指剩下房間的第一志願)
  - Only Self Cycle [6 → 6] left (這時候只剩下6自己跟自己交換)
- Algorithm ends if all allocated (所有人都分配完就終止)
- In general, TTC ends in finite time
- Finds the unique Strong Core allocation

(一般來說，TTC演算法會在有限時間終止、找到唯一滿足強力核可的分配)

TTC Algorithm Finds Strong Core!

- Rm 1 to Stud. 1: [5] > 6 > 7 > 1 > 2 > 3 > 4
- Rm 2 to Stud. 2: [3] > 4 > 5 > 6 > 7 > 1 > 2
- Rm 3 to Stud. 3: 4 > 5 > [2] > 7 > 1 > 3 > 6
- Rm 4 to Stud. 4: [1] > 2 > 3 > 4 > 5 > 6 > 7
- Rm 5 to Stud. 5: [4] > 5 > 2 > 3 > 6 > 7 > 1
- Rm 6 to Stud. 6: 7 > 1 > 2 > 3 > 4 > 5 > [6]
- Rm 7 to Stud. 7: 1 > [7] > 4 > 5 > 6 > 3 > 2

- Strong Core: Assign Rm 5321467 to stud.1-7
  1. [1 → 5 → 4 → 1] trade (強力核可: 房間5321467給學生1-7)
  2. [2 → 3 → 2] and [7 → 7] trade/[6 → 6] left

Find Strong Core in 6-on-6 Market!!

- Rm 1 to Stud. 1: 3 > 6 > 1 > 2 > 4 > 5
- Rm 2 to Stud. 2: 1 > 6 > 2 > 3 > 4 > 5
- Rm 3 to Stud. 3: 2 > 6 > 5 > 1 > 3 > 4
- Rm 4 to Stud. 4: 3 > 1 > 6 > 2 > 5 > 4
- Rm 5 to Stud. 5: 4 > 1 > 2 > 6 > 3 > 5
- Rm 6 to Stud. 6: 4 > 1 > 2 > 3 > 5 > 6

- Top Trading Cycle (小圈圈優先交換演算法)
  - All point to 1st choice (所有人都指向自己第一志願)
  - Find Trading Cycle [$1 \to 3 \to 2 \to 1$] (發現小圈圈)
  - Cycle [$1 \to 3 \to 2 \to 1$] trade 1st (小圈圈就優先交換)

Find Strong Core in 6-on-6 Market!!

- Rm 4 to Stud. 4: 6 > 5 > 4
- Rm 5 to Stud. 5: 4 > 6 > 5
- Rm 6 to Stud. 6: 4 > 5 > 6

- Top Trading Cycle (小圈圈優先交換演算法)
  - Cycle [1 → 3 → 2 → 1] trade 1st (小圈圈優先交換)
  - All point to remaining 1st (其餘人都指向剩下房間的第一志願)
  - Find Trading Cycle [4 → 6 → 4] (發現兩人小圈圈)
  - Cycle [4 → 6 → 4] trade 2nd (這個也優先交換)
  - All point to remaining 1st (其餘人繼續指剩下房間的第一志願)
  - Only Self Cycle [5 → 5] left (這時候只剩下5自己跟自己交換)

Find Strong Core in 6-on-6 Market!!

- Rm 1 to Stud. 1: 3 > 6 > 1 > 2 > 4 > 5
- Rm 2 to Stud. 2: 1 > 6 > 2 > 3 > 4 > 5
- Rm 3 to Stud. 3: 2 > 6 > 5 > 1 > 3 > 4
- Rm 4 to Stud. 4: 3 > 1 > 6 > 2 > 5 > 4
- Rm 5 to Stud. 5: 4 > 1 > 2 > 6 > 3 > 5
- Rm 6 to Stud. 6: 4 > 1 > 2 > 3 > 5 > 6

- TTC assigns Rm 312654 to Student 1-6
  - 小圈圈優先演算法把房間312654給學生1-6

1. [1 → 3 → 2 → 1] trade ([1→3→2→1]優先交換)
2. [4 → 6 → 4] trade/[5 → 5] left ([4→6→4]交換/5跟自己)

Why is Strong Core Rule Better?

- TTC is a Strong Core Rule (TTC演算法是強力核可制度)
  - It cannot be blocked, and it is (除了不會被小圈圈阻擋，還)
- Strategy-Proof (SP) (滿足對策免疫，因為謊報偏好只會更糟!)
  - Honesty is the Best Policy since lying is worse!
- Can you see TTC is strategy-proof? (看得出來嗎?)
- If not, we may need to remind subjects (需要提醒)
- This rule is strategy-proof, so it is best for you to report truthfully! (本規則對策免疫，所以誠實為上策!!)
- Other rules strategy-proof? (還有哪些規則也滿足對策免疫?)

Only Strong Core Rule Satisfies All Three:

1. Strategy-Proof (SP) (對策免疫，因為謊報偏好只會更糟!)
   - Honesty is the Best Policy since lying is worse!
2. Individual Rationality (IR) (個體自願參與)
   - Yourself cannot block (自己一組不會更好、無法阻擋該分配)
3. Pareto Efficiency (PE) (效率)
   - Whole cannot block (所有人一組不會更好、無法阻擋該分配)

- Non-TTC Rules Can Only Satisfy 2 out of 3!
  - Jinpeng Ma (馬金朋) (1994), "Strategy-proofness and the strict core in a market with indivisibilities," International Journal of Game Theory, 23(1), 75-83.
    (只有強力核可制度TTC能同時符合三個條件，不接受就只能三選二!)

Room 567 Vacated for New Student 567...

- Rm 1 to Stud. 1: 5 > 6 > 7 > 1 > 2 > 3 > 4
- Rm 2 to Stud. 2: 3 > 4 > 5 > 6 > 7 > 1 > 2
- Rm 3 to Stud. 3: 4 > 5 > 2 > 7 > 1 > 3 > 6
- Rm 4 to Stud. 4: 1 > 2 > 3 > 4 > 5 > 6 > 7
- Rm 5 empty/N5: 4 > 5 > 2 > 3 > 6 > 7 > 1
- Rm 6 empty/N6: 7 > 1 > 2 > 3 > 4 > 5 > 6
- Rm 7 empty/N7: 1 > 7 > 4 > 5 > 6 > 3 > 2

- Can randomly assign New Student 567 to Room 567 and then use TTC
  - 畢業空出房間? 可隨機分配空房間567給新生567再跟舊生一起做TTC

If Only Assign Priority to New Student 567...

- Rm 1 to Stud. 1: 5 > 6 > 7 > 1 > 2 > 3 > 4
- Rm 2 to Stud. 2: 3 > 4 > 5 > 6 > 7 > 1 > 2
- Rm 3 to Stud. 3: 4 > 5 > 2 > 7 > 1 > 3 > 6
- Rm 4 to Stud. 4: 1 > 2 > 3 > 4 > 5 > 6 > 7
- Rm 5 empty/N5: 4 > 5 > 2 > 3 > 6 > 7 > 1
- Rm 6 empty/N6: 7 > 1 > 2 > 3 > 4 > 5 > 6
- Rm 7 empty/N7: 1 > 7 > 4 > 5 > 6 > 3 > 2

- Assign 1st to tenant, then by priority (優先給現住戶再按照排序)
  - Priority 123 to New Student 567 (新生567排序為123)
  - Priority 4-7 for Student 1-4 (舊生1234排序為4567)

Revised Top Trading Cycle Algorithm (reTTC)

1 ← Rm 1 to Stud. 1: 5 > 6 > 7 > 1 > 2 > 3 > 4
2 ← Rm 2 to Stud. 2: 3 > 4 > 5 > 6 > 7 > 1 > 2
3 ← Rm 3 to Stud. 3: 4 > 5 > 2 > 7 > 1 > 3 > 6
4 ← Rm 4 to Stud. 4: 1 > 2 > 3 > 4 > 5 > 6 > 7
5 ← Rm 5 empty/N5: 4 > 5 > 2 > 3 > 6 > 7 > 1
5 ← Rm 6 empty/N6: 7 > 1 > 2 > 3 > 4 > 5 > 6
5 ← Rm 7 empty/N7: 1 > 7 > 4 > 5 > 6 > 3 > 2

- Student point to 1^{st} choice; Room to priority (學生指向第一志願，房間指向優先)
  - Tenant or Top Priority (stud 5) (現住戶/排序第一學生5)
  - Find Cycle [1 → 5 → 5 → 4 → 4 → 1 → 1]

Revised Top Trading Cycle Algorithm (reTTC)

$2 \leftarrow$ Rm 2 to Stud. 2: 3 > 6 > 7 > 2

$3 \leftarrow$ Rm 3 to Stud. 3: 2 > 7 > 3 > 6

$6 \leftarrow$ Rm 6 empty/N6: 7 > 2 > 3 > 6

$6 \leftarrow$ Rm 7 empty/N7: 7 > 6 > 3 > 2

- Top Trading Cycle Algorithm (小圈圈優先交換演算法)
  - $[1 \rightarrow 5 \rightarrow 5 \rightarrow 4 \rightarrow 4 \rightarrow 1 \rightarrow 1]$ trade 1st (優先交換)
  - Students point to remaining 1st (剩下學生指向剩下房間中的第一志願，剩下房間指向剩下學生中的第一優先)
  - Rooms point to remaining priority (發現兩個小圈圈)
  - Find Cycle $[2 \rightarrow 3 \rightarrow 3 \rightarrow 2 \rightarrow 2]$ & $[6 \rightarrow 7 \rightarrow 6]$
  - Only N7/Rm 6 left $[7 \rightarrow 6 \rightarrow 7]$ (只剩學生7和房間6一組)

Revised Top Trading Cycle Algorithm (reTTC)

- Rm 1 to Stud. 1: [5] > 6 > 7 > 1 > 2 > 3 > 4
- Rm 2 to Stud. 2: [3] > 4 > 5 > 6 > 7 > 1 > 2
- Rm 3 to Stud. 3: 4 > 5 > [2] > 7 > 1 > 3 > 6
- Rm 4 to Stud. 4: [1] > 2 > 3 > 4 > 5 > 6 > 7
- Rm 5 empty/N5: [4] > 5 > 2 > 3 > 6 > 7 > 1
- Rm 6 empty/N6: [7] > 1 > 2 > 3 > 4 > 5 > 6
- Rm 7 empty/N7: 1 > 7 > 4 > 5 > [6] > 3 > 2

- reTTC assigns Rm 5321476 to Student 1-7
  - [1 → 5 → 5 → 4 → 4 → 1 → 1] trade 1^{st} (優先交換)
  - [2 → 3 → 3 → 2 → 2], [6 → 7 → 6], [7 → 6 → 7]

Roth Use reTTC to Design Kidney Exchange

- Stud. 1 in Rm 1 → Patient 1 with Donor 1,
- ... (住房間i 的學生$i \rightarrow$ 有親友願意捐腎i 的病患i)
- Stud. n in Rm n → Patient n with Donor n,
- Empty Room $(n+1)$ → Deceased Donor
- N$(n+1)$ → Patient $(n+1)$ on waitlist (w/od Donor)
- 空房間$(n+1)$→屍腎 / 新生$(n+1)$→等候名單上(無捐腎親友)病患$(n+1)$

- Since Deceased Donors are rare, empty rooms are actually waitlist (由於屍腎太少，空房間其實是等候名單)
  - Deceased donors appear 1-by-1; algorithm adjusts real-time (實務上屍腎是一個個臨時出現的，演算法必須即時調整)

What If Rm 0 Pops Up (Only Stud. 4 Likes It)

- Rm 1 to Stud. 1: 5 > 6 > 7 > 1 > 2 > 3 > 4 > 0
- Rm 2 to Stud. 2: 3 > 4 > 5 > 6 > 7 > 1 > 2 > 0
- Rm 3 to Stud. 3: 4 > 5 > 2 > 7 > 1 > 3 > 6 > 0
- Rm 4 to Stud. 4: 0 > 1 > 2 > 3 > 4 > 5 > 6 > 7
- Rm 5 to Stud. 5: 4 > 5 > 2 > 3 > 6 > 7 > 1 > 0
- Rm 6 to Stud. 6: 7 > 1 > 2 > 3 > 4 > 5 > 6 > 0
- Rm 7 to Stud. 7: 1 > 7 > 4 > 5 > 6 > 3 > 2 > 0

- Strong Core: Assign Rm 5321467 to stud.1-7
  - Pareto Improvement [$4 \to 0$, $1 \to 7$, $7 \to 1$] (出現Pareto改善: 學生4換到空房間0/讓出房間1給學生7/讓出房間7給學生6)

Roth Use reTTC to Design Kidney Exchange

- Kidney swap is a Pareto Improvement
  - Others not better-off even if we ban kidney swap!
  - (交換捐贈是Pareto改善。即使不允許交換捐贈，其他人還是維持現狀)
- Kidney chains give priority to those who can continue the chain reaction
  - Not Pareto Improvement (Waitlist pushed back)
  - Unless Altruistic Donor donates only if chain reaction (or if chain ends on the waitlist)
  - 連鎖捐贈則把優先機會讓給能起連鎖反應的人。若不允許連鎖捐贈，會給等候名單上的第一個人。並非Pareto改善，除非無償捐贈者「只有激起連鎖反應才願意捐」或是連鎖反應的終點回到等候名單上的第一個人

活體腎移植 配對系統7月上線 (2018/1/29)

  - 聯合報 記者修瑞瑩／台南報導
- …美國知名女藝人席琳娜因為紅斑性狼瘡病症損及腎臟，由閨蜜捐腎移植，重啓演藝事業，
- 財團法人器官捐贈移植登錄中心董事長、健保署長李伯璋表示，國內目前活體腎臟捐贈，為避免有買賣行為，只限於配偶及五親等家屬，沒辦法像美國連閨蜜也能捐贈，
- 但線上配對，等於突破只有親人才能捐贈的限制。
- 李伯璋表示，器官捐贈中心繼推動器官捐贈者家人可優先獲得他人器官捐贈，再推動活體腎臟線上配對，相關計畫報衛福部審查後，7月上路。

活體腎移植 配對系統7月上線 (2018/1/29)

- 線上配對是指需要移植的患者與願意捐贈的親人，能與其他病患及親人一起配對，相互捐贈，
  - 例如A、B、C3名患者都在等待換腎，親人也願意捐贈，與患者配對不合，
  - 經過線上配對後，可能A的親人捐腎給B，B的親人捐贈給C，C的親人再捐贈給A。
- 移植醫師表示，部分醫師認為新制效果有限，但以美國實施多年經驗來看，確實可提高配對成功機率。
- 過去親人間如果配對不成，例如血型不合或交叉試驗陽性，...能與其他患者親人配對成功，是另一條出路。

Market Design @ Taiwan
市場設計：台灣國中會考

Joseph Tao-yi Wang (王道一)
Lecture 11, EE-BGT

志願難填 教團：學生陷賽局困境

(2014/6/9國語日報)國教行動聯盟昨天痛批，升學制度儼然變成賭博式賽局，學生想進理想學校，竟得猜測別人的志願怎麼填，陷入「賽局理論」困境。

- (國教行動聯盟理事長王立昇表示，志願序納入超額比序計分，填錯會被扣分，加上第一次免試分發後，基北區約有六千個學生可能放棄錄取考特招，所以預測別人填哪些志願、會不會放棄一免，成了填寫志願的重要因素
- 王立昇指出，「賽局理論」是研究遊戲中個體預測對方和己方行為，所產生的影響，並分析最佳策略。現在的十二年國教，已經讓學生面臨一樣的困擾。

填志願諜對諜　國教盟驚爆：學生想輕生

國中會考成績上周四公布後，家長學生茫然不知如何選填志願

國教行動聯盟今上午公開呼籲教育部，今年取消志願序計分或採3-7個志願為群組，差一個群組扣1分，以免學生陷入選填志願的博弈賽局中，填志願淪為諜對諜。

(2014/6/7蘋果日報)

填志願諜對諜　國教盟驚爆：學生想輕生

(2014/6/7蘋果日報)

國教行動聯盟理事長王立昇表示，…教育部應公布更多資訊並延長志願表繳交時間，讓學生有更充足資訊能錄取最理想的學校。他進一步表示，學生為了上好學校，同學間已互相猜忌，打探彼此第一志願是什麼做為自己選填志願的參考，陷入博弈賽局中，解決方法只有取消志願序計分，或擴大為群組計分，降低傷害。

制度變數多 教團憂入學如賽局 (2014/6/8)

- (中央社記者許秩維) 國教行動聯盟今天說，國教入學制度變數多，恐陷賽局理論，孩子得預測他人如何填志願，聯盟籲取消志願序計分。
- 國教行動聯盟舉行記者會，憂心國教入學制度陷入賽局理論的困境，讓學生和家長寢食難安。
- 國教行動聯盟理事長王立昇表示，目前國教入學制度面臨幾個問題，如志願序計分，由於不知別人如何填志願，要進入自己理想的學校就可能有很多變數，導致陷入賽局理論的困境，學生家長難以填志願。

Taiwan High School Choice

- History School Choice in Taiwan
  - Old: Sequential Dictator based on Exam Score
  - New System in 2014
- Exam-exempt School Choice based on:
  - # of ABC from Joint Exam (會考)
  - Self-reported School Choice Rankings
  - Other factors (that all get the same score)
  - Chinese composition: Grade 1-6
  - A++, A+, A, A-, etc.

Taiwan School Choice: A Simplified Model

- How can we analyze this?
  - Simplify to obtain a tractable model/example
  - Implement in the lab
- What are key elements of the situation?
- What are the key results to reproduce?
- Next: Run lab experiments to

1. Test the model
2. Try alternative institutions
3. Teach parents/policy makers

Taiwan School Choice: A Simplified Model

- Three schools: A, B, C
- Three students: 1 & 2 are type a, 3 is type c
- Student Payoffs: $u(A) = h, \;\; u(B) = 1, \;\; u(C) = 0$
- School Payoffs: $v(a) = 1, \;\; v(c) = 0$
- Actions: Self-report School Choice Rankings

  $$S = \{ABC, BAC, ACB, CAB, CBA, BCA\}$$
- Assign everyone to their first choice
  - Ties broken by student type/grade, then random
  - Remaining students assigned to remaining schools

Taiwan School Choice: A Simplified Model

- This is manipulable (=not strategy-proof)
  - Truthful Reporting of Ranking is not BR!
- Suppose all students truthfully report ABC
- Outcome: Student 1, 2 go to schools A, B (randomly) and student 3 goes to school C
  - Schools ABC get students of type aac
- But: Student 3 could gain by misreporting!

$$U_3(\underline{\underline{BAC}}) = u(B) = 1 > u(C) = 0 = U_3(ABC)$$

Taiwan School Choice: A Simplified Model

- What is the Nash Equilibrium of the game?

1. Student 3 reports BAC
2. Student 1 & 2 report ABC with prob. p,
   report BAC with prob. $(1-p)$

- Outcome:
- p^2 : School ABC get students of type aca
  - When both Student 1 & 2 report ABC...
- $1-p^2$: School ABC get students of type aac

Taiwan School Choice: A Simplified Model

3 reports BAC; 1,2 report ABC/BAC with $(p, 1-p)$

- For Student 1 (and 2) to mix, need: $\boxed{1+p=h}$

$$U_1(ABC) = p\left(\frac{1}{2}\cdot \underline{\underline{u(A)}} + \frac{1}{2}\cdot \underline{\underline{u(C)}}\right) + (1-p)\cdot \underline{\underline{u(A)}}$$

$$= p\left(\frac{1}{2}\cdot \underline{\underline{h}} + \frac{1}{2}\cdot \underline{\underline{0}}\right) + (1-p)\cdot \underline{\underline{h}} = \left(1-\frac{p}{2}\right)h$$

$$U_1(BAC) = p\cdot \underline{\underline{u(B)}} + (1-p)\left(\frac{1}{2}\cdot \underline{\underline{u(B)}} + \frac{1}{2}\cdot \underline{\underline{u(A)}}\right)$$

$$= p\cdot \underline{\underline{1}} + (1-p)\left(\frac{1}{2}\cdot \underline{\underline{1}} + \frac{1}{2}\cdot \underline{\underline{h}}\right) = \frac{1+p}{2} + \frac{1-p}{2}\cdot h$$

Taiwan School Choice: A Simplified Model

- Why is this a Nash Equilibrium?
  - Student 1 & 2 report ABC with prob. $p = h - 1$
- For Student 3, we need $p > 0.555(0.55496)$

$$f(p) = U_3(BAC) - U_3(ABC) \geq 0$$
$$= p^2 \cdot 1 - (1-p)^2 \cdot h$$
$$= p^2 - (1-p) \cdot (1-p^2)$$

- Since $f'(p) = 2p + (1-p^2) + 2p(1-p) > 0$

$f(p)$ increasing $\Rightarrow 1 + p = h > 1.555(0.55496)$

Conclusion (for the Example) 結論

- Nash Equilibrium of this 3-student game:

1. Student 3 untruthfully reports BAC
2. Student 1 & 2 mix between truthful & untruthful reports ABC/ BCA, $(p, 1-p)$

- Outcome:
- p^2 : School ABC get students of type aca
  - When both Student 1 & 2 report ABC…
- $1-p^2$: School ABC get students of type aac

Possible Extensions:

1. Is Cardinal Utility Required?
   - Ordinal preferences is fine if exists p so that

$$\left(\frac{p}{2}\right) \cdot C + \left(1 - \frac{p}{2}\right) \cdot A \sim \left(\frac{1+p}{2}\right) \cdot B + \left(\frac{1-p}{2}\right) \cdot A$$

2. What if students have different preferences?
   - Different Risk Attitudes?
3. What if there are more students/schools?
4. What if schools can also act strategically?
5. What is a Good Alternative Mechanism?

A Simple Theory of Matching (R-S, Ch.2)

- Gale & Shapley (1962); Roth & Sotomayor (1990)
- Finite Set of Students S and Schools C
- 1-1 Matching, Strict (Ordinal) Preferences:
  - $c \succ_s \tilde{c}$: Student s prefers School c to $\tilde{c}$
  - $s \succ_c \tilde{s}$: School c prefers Student s to $\tilde{s}$
  - $i \succ_j \emptyset$: i is acceptable to j
- A matching is $\mu : S \cup C \rightarrow S \cup C \cup \{\emptyset\}$

$$\mu(s) = c_{\in C \cup \{\emptyset\}} \quad \Leftrightarrow \quad \mu(c) = s_{\in S \cup \{\emptyset\}}$$

A Simple Theory of Matching (R-S, Ch.2)

- Matching μ blocked by individual i if $\emptyset \succ_i \mu(i)$
- Matching μ blocked by pair s, c if
  - $$c \succ_s \mu(s) \text{ and } s \succ_c \mu(c)$$
- Matching is stable if it is blocked by neither
  - Core = Set of all stable matchings
  - A stable matching is Pareto efficient
- Theorem (Gale-Shapley, R-S Theorem 2.8)
  - Exists a stable matching in any 1-1 matching market

Deferred Acceptance Algorithm

- Step 1: Students apply to their first choices
  - Schools tentatively hold most preferred student and reject all others
- Step t (2 and above): Students rejected in Step $t-1$ apply to next highest choice
  - Schools tentatively hold most preferred student (new or held) and reject all others
- Stop when no more new applications
  - Happens in finite time!

DA Algorithm: Taiwan School Choice Model

- 3 schools: A, B, C; 3 students: a, b, c
  - Student Payoffs: $u(A) = h, \;\; u(B) = 1, \;\; u(C) = 0$
  - School Payoffs: $v(a) = 1, \;\; v(b) = 0.999, \;\; v(c) = 0$
- Step 1: All students apply to school A
  - School A holds student a and rejects b, c
- Step 2: Students b, c apply to school B
  - School B holds student b and rejects c
- Step 3: Students c applies to school C
  - School C holds student c and terminates DA!

Deferred Acceptance Algorithm

- Proof of Theorem (Gale-Shapley)
  - DA gives matching where no student/school applies to/holds unacceptable schools/students
- Matching μ not blocked by any individual!
  - If $c \succ_s \mu(s) \neq c$, s was rejected by c before in DA
  - But in DA, c rejects only if it sees better choice!
  - Hence, $\mu(c) \succ_c s$
- Matching μ not blocked by any pair!
- Resulting Matching μ of DA is stable. QED

DA Algorithm: Taiwan School Choice Model

- What does stable mean in the field?!
- Roth (1984):
  - stable ones successfully used
  - continue to be used (unstable ones abandoned)
- Few complaints in Taiwan?!
- A student-proposing DA algorithm yields:
- Student-optimal stable matching
    - (superior to all other stable matching)
  - Proof of Theorem? See R-S Theorem 2.12

DA Algorithm: Marriage Matching

- Male-optimal stable matching
  - (superior to all other stable matching)

= Female-pessimal
  - (inferior to all other stable matching)

- In contrast, A female-proposing DA leads to
  - Female-optimal/male-pessimal stable matching
- Why is proposing power less important school choice?
  - Student/School Preferences More Aligned?

Rural Hospital Theorem (R-S Th'm 2.22)

- The same set of students/schools are left unmatched in all stable matching
- This means:
  - A loser is a loser in any stable matching (魯蛇到哪裡都是魯蛇)
  - Cannot expect any stable-matching mechanism to solve rural hospital problem (偏遠地區醫療)
- Proof?

Proof of Rural Hospital Theorem

- Student-optimal stable matching $\overline{\mu}$
- Alternative stable matching μ
- $\overline{\mu}$ is student-optimal:
  - Students matched in μ also matched in $\overline{\mu}$
- $\overline{\mu}$ is school-pessimal:
  - Schools matched in $\overline{\mu}$ also matched μ
- # of matches are the same in any match
- Same set of students/schools matched in $\overline{\mu}, \mu$

Truthful Reporting and Strategy-Proofness

- Main problem of the new system in Taiwan:
  - People want to misrepresent their preferences!
- Mechanism: Rule that yields a matching from (reported) preferences
- A mechanism is strategy-proof if reporting true preferences is a dominant strategy for everyone
  - The new system in Taiwan is not strategy-proof
  - Is DA strategy-proof?

Truthful Reporting and Strategy-Proofness

- In fact, no stable mechanism is strategy-proof! (R-S Theorem 4.4)
  - But, by Dubins and Freedman 1981, Roth 1982:
- Theorem (R-S Theorem 4.7): The student-proposing DA is strategy-proof for students.
- Why DA (old system in Taiwan) is good:
  1. Stable
  2. Students prefer it to all other stable matching
  3. Strategy-proof for students

Truthful Reporting and Strategy-Proofness

1. Strategy-proof → Manipulable
   - Degree of strategy-proofness (instead of Y/N)
2. 1-1 → Many-to-one
   - Schools can accept up to q_c students (quota)
   - Existence of stable many-to-one matching market
   - X-proposing DA → X-optimal stable matching
   - Rural Hospital Theorem (fill same # of students)
   - Student-proposing DA strategy-proof for students
   - No stable mechanism strategy-proof for schools
3. Problem for Married Couples?!