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STRATEGIC INFORMATION TRANSMISSION

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"Oh, what a tangled web we weave, when first we practice to deceive!"
—Sir Walter Scott

The paper is about…

- Information transmission
- Communication
- Rational behavior
- Deception
- Lying
- Truth-telling
- Interests
- However: very general, very broad concept

Explaining by using math!

LEMMA 2: *If* $V(a_{i-1}, a_i, a_{i+1}, b) = 0$ *for* $0 \le a_{i-1} < a_i < a_{i+1} \le 1$, *then* $U_1^S(\bar{y}(a, a_i), a_i, b) > 0$ *and* $V_1(a, a_i, a_{i+1}, b) < 0$ *for all* $a \in [0, a_{i-1}]$, *and* $U_1^S(\bar{y}(a_i, a), a_i, b) < 0$ *and* $V_3(a_{i-1}, a_i, a, b) < 0$ *for all* $a \in [a_{i+1}, 1]$.

PROOF: Since $U^S(\bar{y}(a_{i-1}, a_i), a_i, b) = U^S(\bar{y}(a_i, a_{i+1}), a_i, b)$ by hypothesis, $\bar{y}(a_i, a_{i+1}) > \bar{y}(a_{i-1}, a_i)$, and $U_{11}^S(\cdot) < 0$, $U_1^S(y, a_i, b) > 0$ for $y \le \bar{y}(a_{i-1}, a_i)$ and $U_1^S(y, a_i, b) < 0$ for $y \ge \bar{y}(a_i, a_{i+1})$. The Lemma follows from the definition of V because $\bar{y}(\cdot)$ is strictly increasing in both of its arguments. *Q.E.D.*

(13) $$U^S(\bar{y}(a_i, a_{i+1}), m) = \max_j U^S(\bar{y}(a_j, a_{j+1}), m) \quad \text{for all } m \in [a_i, a_{i+1}],$$

where the maximum in (13) is taken over $j = 0, \ldots, N-1$. To see this, note that because $U_{11}^S(\cdot) < 0$ and $\bar{y}(a_i, a_{i+1}) > \bar{y}(a_{i-1}, a_i)$, (A) implies (13) for $m = a_i$. Since $U_{12}^S(\cdot) > 0$ and $m \in [a_i, a_{i+1}]$,

(14) $$U^S(\bar{y}(a_i, a_{i+1}), m) - U^S(\bar{y}(a_k, a_{k+1}), m) \ge U^S(\bar{y}(a_i, a_{i+1}), a_i) - U^S(\bar{y}(a_k, a_{k+1}), a_i) \ge 0 \quad \text{and}$$

(15) $$U^S(\bar{y}(a_i, a_{i+1}), m) - U^S(\bar{y}(a_j, a_{j+1}), m) \ge U^S(\bar{y}(a_i, a_{i+1}), a_{i+1}) - U^S(\bar{y}(a_j, a_{j+1}), a_{i+1}) \ge 0,$$

(32) $$-V(c, a_1^x, a_2^x, b) \equiv U^S(\bar{y}(c, a_1^x), a_1^x, b) - U^S(\bar{y}(a_1^x, a_2^x), a_1^x, b) > 0 \quad \text{for all } x \in [a_{N-1}(N), a_N(N+1)) \text{ and } c \in [0, a_1^x].$$

Now $EU^R(x)$ is given by

(33) $$EU^R(x) \equiv \sum_{j=1}^{N+1} \int_{a_{j-1}^x}^{a_j^x} U^R(\bar{y}(a_{j-1}^x, a_j^x), m) f(m)\, dm.$$

Since $\bar{y}(a_{j-1}^x, a_j^x)$, defined in (9) as R's best response to a signal in the step $[a_{j-1}^x, a_j^x]$, maximizes the jth term in the sum and since $a_{N+1}^x \equiv 1$, the Envelope Theorem yields

(34) $$\frac{dEU^R(x)}{dx} \equiv \sum_{j=1}^{N} f(a_j^x) \frac{da_j^x}{dx} \left[U^R(\bar{y}(a_{j-1}^x, a_j^x), a_j^x) - U^R(\bar{y}(a_j^x, a_{j+1}^x), a_j^x) \right].$$

Assumption (M) guarantees that $da_j^x / dx > 0$ for all $j = 1, \ldots, N$, and

(35) $$U^R(\bar{y}(a_{j-1}^x, a_j^x), a_j^x) - U^R(\bar{y}(a_j^x, a_{j+1}^x), a_j^x) \ge U^S(\bar{y}(a_{j-1}^x, a_j^x), a_j^x, b) - U^S(\bar{y}(a_j^x, a_{j+1}^x), a_j^x, b) \ge 0$$

Let $a^x \equiv (a_0^x, a_1^x, \ldots, a_{N+1}^x)$ be the partition that satisfies (A) for $i = 2, \ldots, N$ with $a_0^x = 0$, $a_N^x = x$, and $a_{N+1}^x = 1$. If $x = a_{N-1}(N)$ then $a_1^x = 0$, and if $x = a_N(N+1)$ then $a^x = a(N+1)$ and (A) is satisfied for all $i = 1, \ldots, N$. When $x \in [a_{N-1}(N), a_N(N+1)]$, which is a nondegenerate interval by Lemma 3, $EU^R(x)$ is strictly increasing in x. To see this, note first that $V(c, a_1^x, a_2^x, b) \neq 0$ for all $c \in [0, a_1^x]$ if $x \in [a_{N-1}(N), a_N(N+1))$. This follows because $(a_{N+1}(N+1), a_{N+1}(N), \ldots, a_{N+1}(1), a_{N+1}(0))$ is a backward solution of (A) of length $N+1$, and (M′) guarantees that any other backward solution of (A), a, of length $N+1$ with $a_0 = 1$ and $a_1 = x$ must satisfy $x > a_{N+1}(N)$. Moreover $V(0, a_1(N+1), a_2(N+1), b) = 0$ by the definition of $a(N+1)$, and hence $-V(c, a_1(N+1), a_2(N+1), b) > 0$ for all $c \in (0, a_1(N+1)]$ by Lemma 2. It follows

The model

- Random state “m” is observed by player 1 (sender)
- Sender sends signal (noise?) to player 2 (receiver)
- Receiver takes action
- b is difference in preferences

$$y^S(m,b) \equiv \arg\max U^S(y,m,b)$$

$$y^R(m) \equiv \arg\max U^R(y,m),$$

Equilibrium

- Define

- $y^S(m, b) \equiv argmax\ U^s(y, m, b)$

- $y^R(m) \equiv argmax\ U^R(y, m)$

THEOREM 1: *Suppose* b *is such that* $y^S(m,b) \neq y^R(m)$ *for all* m. *Then there exists a positive integer* $N(b)$ *such that, for every* N *with* $1 \leq N \leq N(b)$, *there exists at least one equilibrium* $(y(n), q(n \mid m))$, *where* $q(n \mid m)$ *is uniform, supported on* $[a_i, a_{i+1}]$ *if* $m \in (a_i, a_{i+1})$,

$$(A) \qquad U^S(\bar{y}(a_i, a_{i+1}), a_i, b) - U^S(\bar{y}(a_{i-1}, a_i), a_i, b) = 0 \qquad (i = 1, \ldots, N-1),$$

$$(10) \qquad y(n) = \bar{y}(a_i, a_{i+1}) \qquad \text{for all} \quad n \in (a_i, a_{i+1}),$$

$$(11) \qquad a_0 = 0, \qquad \text{and}$$

$$(12) \qquad a_N = 1.$$

Further, any equilibrium is essentially[3] *equivalent to one in this class, for some value of* N *with* $1 \leq N \leq N(b)$.

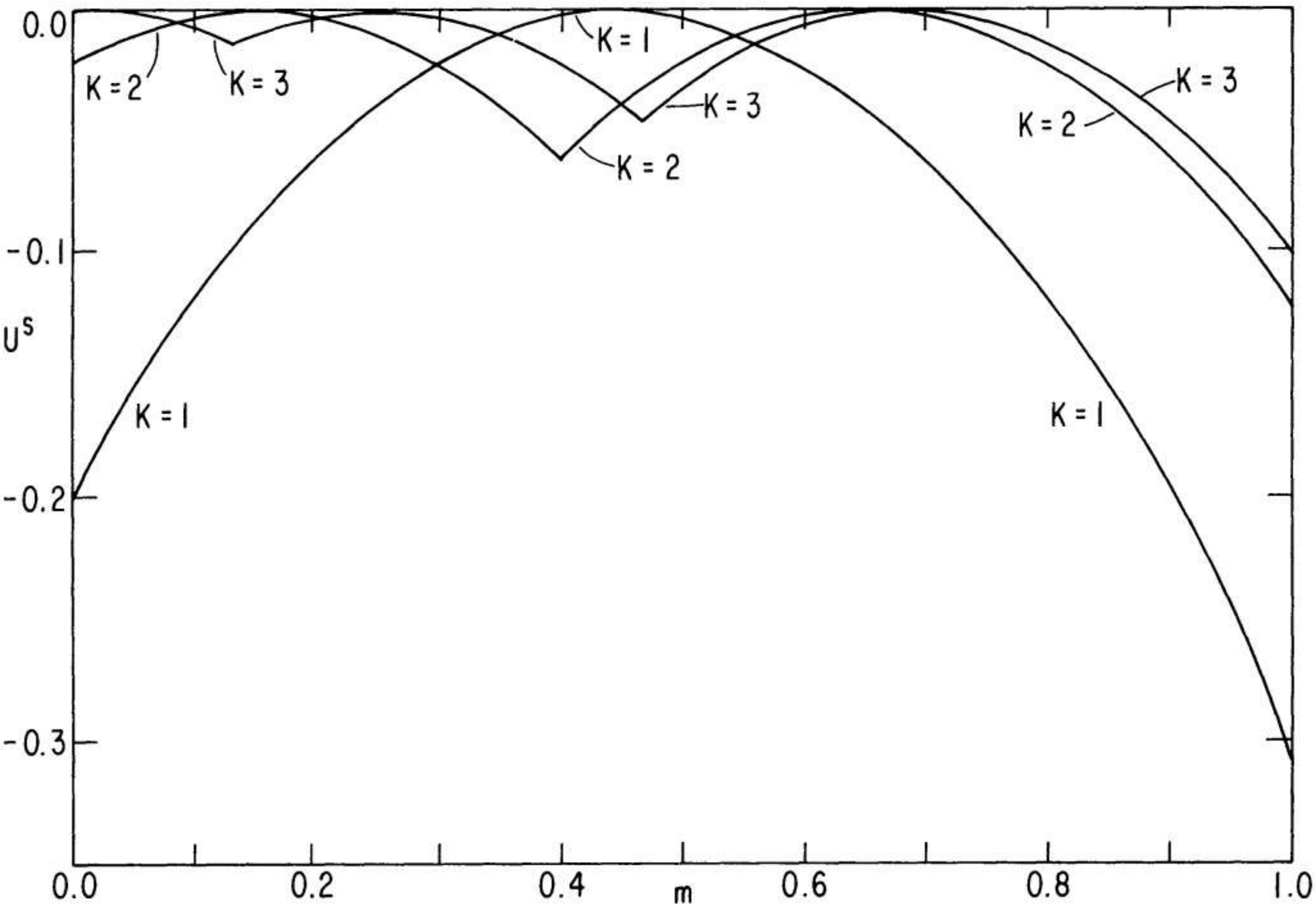
COROLLARY 1: *If* $V(0, a, 1, b) > 0$ *for all* $a \in [0, 1]$, *then* $N(b) = 1$; *that is, the only equilibrium is uninformative.*

b=0	b>0 V(0,a,1,b)=0 has solution	V(0,a,1,b)>0
Partition equilibria & Truth telling equilibrium	Partition equilibria	Uninformative equilibria

Which equilibrium will be chosen?

Example

- $U^S(y, m, b) \equiv -(y - (m + b))^2$
- $U^R(y, m) \equiv -(y - m)^2$
- $b = \frac{1}{20}$



0.0
-0.1
-0.2
-0.3
U^s
K = 1
K = 2
K = 3
K = 1
K = 2
K = 3
K = 1
K = 2
K = 3
0.0
0.2
0.4
m
0.6
0.8
1.0

More general result

(M) For a given value of b, if $\hat{a}$ and $\tilde{a}$ are two forward solutions of (A) with $\hat{a}_0 = \tilde{a}_0$ and $\hat{a}_1 > \tilde{a}_1$, then $\hat{a}_i > \tilde{a}_i$ for all $i \geq 2$.

- This is only a sufficient condition.

THEOREM 3: *For given preferences (i.e., b), R always strictly prefers equilibrium partitions with more steps (larger N's).*

THEOREM 4: *For a given number of steps (i.e., N), R always prefers the equilibrium partition associated with more similar preferences (i.e., a smaller value of b).*

THEOREM 5: *For given preferences (i.e., b), S always strictly prefers ex ante (that is, before learning his type) equilibrium partitions with more steps (larger N's).*

Conclusion

- Direct communication is more likely to play in important role, the more closely related at agents’ goals
- Perfect communication never happens (except b=0, interests coincide)
- Rational behavior can result in non-communication (b=…)