# Market Design @ Taiwan
# 市場設計：台灣國中會考

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# 志願難填 教團：學生陷賽局困境

(2014/6/9國語日報)國教行動聯盟昨天痛批，升學制度儼然變成**賭博式賽局**，學生想進理想學校，竟得**猜測別人的志願怎麼填**，陷入「**賽局理論**」困境。

- (國教行動聯盟理事長王立昇表示，志願序納入超額比序計分，填錯會被扣分，加上第一次免試分發後，基北區約有六千個學生可能放棄錄取考特招，所以**預測別人填哪些志願、會不會放棄一免，成了填寫志願的重要因素**
- 王立昇指出，「賽局理論」是**研究遊戲中個體預測對方和己方行為，所產生的影響，並分析最佳策略**。現在的十二年國教，已經讓學生面臨一樣的困擾。

# 填志願諜對諜　國教盟驚爆：學生想輕生

國中會考成績上周四公布後，家長學生茫然不知如何選填志願

國教行動聯盟今上午公開呼籲教育部，今年取消志願序計分或採3-7個志願為群組，差一個群組扣1分，以免學生陷入選填志願的博弈賽局中，填志願淪為諜對諜。


(2014/6/7蘋果日報)

# 填志願諜對諜　國教盟驚爆：學生想輕生

(2014/6/7蘋果日報)
國教行動聯盟理事長王立昇表示，...教育部應公布更多資訊並延長志願表繳交時間，讓學生有更充足資訊能錄取最理想的學校。他進一步表示，學生為了上好學校，同學間已互相猜忌，打探彼此第一志願是什麼做為自己選填志願的參考，陷入博弈賽局中，解決方法只有取消志願序計分，或擴大為群組計分，降低傷害。

# 制度變數多 教團憂入學如賽局 (2014/6/8)

- (中央社記者許秩維) 國教行動聯盟今天說，國教入學制度變數多，恐陷賽局理論，孩子得預測他人如何填志願，聯盟籲取消志願序計分。
- 國教行動聯盟舉行記者會，憂心國教入學制度陷入賽局理論的困境，讓學生和家長寢食難安。
- 國教行動聯盟理事長王立昇表示，目前國教入學制度面臨幾個問題，如志願序計分，由於不知別人如何填志願，要進入自己理想的學校就可能有很多變數，導致陷入賽局理論的困境，學生家長難以填志願。

# Taiwan High School Choice

- History School Choice in Taiwan
  - Old: Gale-Shapley Deferred Acceptance
  - New System in 2014
- Exam-exempt School Choice based on:
  - # of ABC from Joint Exam (會考)
  - Self-reported School Choice Rankings
  - Other factors (that all get the same score)
  - Chinese composition: Grade 1-6
  - A++, A+, A, A-, etc.

# Taiwan School Choice: A Simplified Model

- How can we analyze this?
  - Simplify to obtain a tractable model/example
  - Implement in the lab
- What are key elements of the situation?
- What are the key results to reproduce?
- Next: Run lab experiments to

1. Test the model
2. Try alternative institutions
3. Teach parents/policy makers

# Taiwan School Choice: A Simplified Model

- Three schools: $A$, $B$, $C$
- Three students: 1 & 2 are type $a$, 3 is type $c$
- Student Payoffs: $u(A) = h,\ \ u(B) = 1,\ \ u(C) = 0$
- School Payoffs: $v(a) = 1,\ \ v(c) = 0$
- Actions: Self-report School Choice Rankings

$$S = \{ABC, BAC, ACB, CAB, CBA, BCA\}$$

- Assign everyone to their first choice
  - Ties broken by student type/grade, then random
  - Remaining students assigned to remaining schools

# Taiwan School Choice: A Simplified Model

- This is manipulable (=not strategy-proof)
  - Truthful Reporting of Ranking is not BR!
- Suppose all students truthfully report $ABC$
- Outcome: Student 1, 2 go to schools $A$, $B$ (randomly) and student 3 goes to school $C$
  - Schools $ABC$ get students of type $aac$
- But: Student 3 could gain by misreporting!

$$U_3(\underline{\underline{BAC}}) = u(B) = 1 > u(C) = 0 = U_3(ABC)$$

# Taiwan School Choice: A Simplified Model

- What is the Nash Equilibrium of the game?

1. Student 3 reports $BAC$
2. Student 1 & 2 report $ABC$ with prob. $p$,
   report $BAC$ with prob. $(1-p)$

- Outcome:
- $p^2$ : School $ABC$ get students of type $aca$
  - When both Student 1 & 2 report ABC...
- $1-p^2$: School $ABC$ get students of type $aac$

# Taiwan School Choice: A Simplified Model

3 reports $BAC$; 1,2 report $ABC$/ $BAC$ with $(p, 1-p)$

- For Student 1 (and 2) to mix, need: $\boxed{1+p=h}$

$$U_1(ABC) = p\left(\frac{1}{2}\cdot \underline{\underline{u(A)}} + \frac{1}{2}\cdot \underline{\underline{u(C)}}\right) + (1-p)\cdot \underline{\underline{u(A)}}$$

$$= p\left(\frac{1}{2}\cdot \underline{\underline{h}} + \frac{1}{2}\cdot \underline{\underline{0}}\right) + (1-p)\cdot \underline{\underline{h}} = \left(1-\frac{p}{2}\right)h$$

$$U_1(BAC) = p\cdot \underline{\underline{u(B)}} + (1-p)\left(\frac{1}{2}\cdot \underline{\underline{u(B)}} + \frac{1}{2}\cdot \underline{\underline{u(A)}}\right)$$

$$= p\cdot \underline{\underline{1}} + (1-p)\left(\frac{1}{2}\cdot \underline{\underline{1}} + \frac{1}{2}\cdot \underline{\underline{h}}\right) = \frac{1+p}{2} + \frac{1-p}{2}\cdot h$$

# Taiwan School Choice: A Simplified Model

- Why is this a Nash Equilibrium?
  - Student 1 & 2 report $ABC$ with prob. $p = h - 1$
- For Student 3, we need $p > 0.555(0.55496)$

$$f(p) = U_3(BAC) - U_3(ABC) \geq 0$$
$$= p^2 \cdot 1 - (1-p)^2 \cdot h$$
$$= p^2 - (1-p) \cdot (1-p^2)$$

- Since $f'(p) = 2p + (1-p^2) + 2p(1-p) > 0$

$f(p)$ increasing $\Rightarrow 1 + p = h > 1.555(0.55496)$

# Conclusion (for the Example) 結論

- Nash Equilibrium of this 3-student game:

1. Student 3 untruthfully reports $BAC$
2. Student 1 & 2 mix between truthful & untruthful reports $ABC$/ $BCA$, $(p, 1-p)$

- Outcome:
- $p^2$ : School $ABC$ get students of type $aca$
  - When both Student 1 & 2 report ABC…
- $1-p^2$: School $ABC$ get students of type $aac$

# Possible Extensions:

1. Is Cardinal Utility Required?
   - Ordinal preferences is fine if exists p so that

$$\left(\frac{p}{2}\right) \cdot C + \left(1 - \frac{p}{2}\right) \cdot A \sim \left(\frac{1+p}{2}\right) \cdot B + \left(\frac{1-p}{2}\right) \cdot A$$

2. What if students have different preferences?
   - Different Risk Attitudes?
3. What if there are more students/schools?
4. What if schools can also act strategically?
5. What is a Good Alternative Mechanism?

# A Simple Theory of Matching (R-S, Ch.2)

- Gale & Shapley (1962); Roth & Sotomayor (1990)
- Finite Set of Students $S$ and Schools $C$
- 1-1 Matching, Strict (Ordinal) Preferences:
  - $c \succ_s \tilde{c}$ : Student $s$ prefers School $c$ to $\tilde{c}$
  - $s \succ_c \tilde{s}$ : School $c$ prefers Student $s$ to $\tilde{s}$
  - $i \succ_j \emptyset$ : $i$ is acceptable to $j$
- A matching is $\mu : S \cup C \rightarrow S \cup C \cup \{\emptyset\}$

$$\mu(s) = c_{\in C \cup \{\emptyset\}} \quad \Leftrightarrow \quad \mu(c) = s_{\in S \cup \{\emptyset\}}$$

# A Simple Theory of Matching (R-S, Ch.2)

- Matching $\mu$ blocked by individual $i$ if $\emptyset \succ_i \mu(i)$
- Matching $\mu$ blocked by pair $s$, $c$ if
  - $c \succ_s \mu(s)$ and $s \succ_c \mu(c)$
- Matching is stable if it is blocked by neither
  - Core = Set of all stable matchings
  - A stable matching is Pareto efficient
- Theorem (Gale-Shapley, R-S Theorem 2.8)
  - Exists a stable matching in any 1-1 matching market

# Deferred Acceptance Algorithm

- Step 1: Students apply to their first choices
  - Schools tentatively hold most preferred student and reject all others
- Step $t$ (2 and above): Students rejected in Step $t - 1$ apply to next highest choice
  - Schools tentatively hold most preferred student (new or held) and reject all others
- Stop when no more new applications
  - Happens in finite time!

# DA Algorithm: Taiwan School Choice Model

- 3 schools: $A$, $B$, $C$; 3 students: $a$, $b$, $c$
  - Student Payoffs: $u(A) = h, \;\; u(B) = 1, \;\; u(C) = 0$
  - School Payoffs: $v(a) = 1, \;\; v(b) = 0.999, \;\; v(c) = 0$
- Step 1: All students apply to school $A$
  - School $A$ holds student $a$ and rejects $b$, $c$
- Step 2: Students $b$, $c$ apply to school $B$
  - School $B$ holds student $b$ and rejects $c$
- Step 3: Students $c$ applies to school $C$
  - School $C$ holds student $c$ and terminates DA!

# Deferred Acceptance Algorithm

- Proof of Theorem (Gale-Shapley)
  - DA gives matching where no student/school applies to/holds unacceptable schools/students
- Matching $\mu$ not blocked by any individual!
  - If $c \succ_s \mu(s) \neq c$, $s$ was rejected by $c$ before in DA
  - But in DA, $c$ rejects only if it sees better choice!
  - Hence, $\mu(c) \succ_c s$
- Matching $\mu$ not blocked by any pair!
- Resulting Matching $\mu$ of DA is stable. QED

# DA Algorithm: Taiwan School Choice Model

- What does stable mean in the field?!
- Roth (1984):
  - stable ones successfully used
  - continue to be used (unstable ones abandoned)
- Few complaints in Taiwan?!
- A student-proposing DA algorithm yields:
- Student-optimal stable matching
    - (superior to all other stable matching)
  - Proof of Theorem? See R-S Theorem 2.12

# DA Algorithm: Marriage Matching

- Male-optimal stable matching
  - (superior to all other stable matching)

= Female-pessimal
  - (inferior to all other stable matching)
- In contrast, A female-proposing DA leads to
  - Female-optimal/male-pessimal stable matching
- Why is proposing power less important school choice?
  - Student/School Preferences More Aligned?

# Rural Hospital Theorem (R-S Th'm 2.22)

- The same set of students/schools are left unmatched in all stable matching
- This means:
  - A loser is a loser in any stable matching (魯蛇到哪裡都是魯蛇)
  - Cannot expect any stable-matching mechanism to solve rural hospital problem (偏遠地區醫療)
- Proof?

# Proof of Rural Hospital Theorem

- Student-optimal stable matching $\overline{\mu}$
- Alternative stable matching $\mu$
- $\overline{\mu}$ is student-optimal:
  - Students matched in $\mu$ also matched in $\overline{\mu}$
- $\overline{\mu}$ is school-pessimal:
  - Schools matched in $\overline{\mu}$ also matched $\mu$
- # of matches are the same in any match
- Same set of students/schools matched in $\overline{\mu}, \mu$

# Truthful Reporting and Strategy-Proofness

- Main problem of the new system in Taiwan:
  - People want to misrepresent their preferences!
- Mechanism: Rule that yields a matching from (reported) preferences
- A mechanism is strategy-proof if reporting true preferences is a dominant strategy for everyone
  - The new system in Taiwan is not strategy-proof
  - Is DA strategy-proof?

# Truthful Reporting and Strategy-Proofness

- In fact, no stable mechanism is strategy-proof! (R-S Theorem 4.4)
  - But, by Dubins and Freedman 1981, Roth 1982:
- Theorem (R-S Theorem 4.7): The student-proposing DA is strategy-proof for students.
- Why DA (old system in Taiwan) is good:
  1. Stable
  2. Students prefer it to all other stable matching
  3. Strategy-proof for students

# Truthful Reporting and Strategy-Proofness

1. Strategy-proof → Manipulable
   - Degree of strategy-proofness (instead of Y/N)
2. 1-1 → Many-to-one
   - Schools can accept up to $q_c$ students (quota)
   - Existence of stable many-to-one matching market
   - X-proposing DA → X-optimal stable matching
   - Rural Hospital Theorem (fill same # of students)
   - Student-proposing DA strategy-proof for students
   - No stable mechanism strategy-proof for schools
3. Problem for Married Couples?!