

# Experiments Games with Mixed Strategy Equilibrium

(混合策略均衡實驗)

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Lecture 5, EE-BGT

# Games with MSE 有混合策略均衡的賽局

- ▶ Zero-Sum Games (零和賽局)
  - ▶ Rock-Scissor-Paper (剪刀石頭布)
  - ▶ Sports (PK, tennis serves, basketball drives, etc.)
    - 足球罰踢、網球發球、籃球切入或投籃
  - ▶ Military attack (軍事行動如登陸諾曼地或加萊)
- ▶ Deter Undesired Behavior (嚇阻投機/不希望發生的行為)
  - ▶ Searches of passengers after 9/11 (機場安檢、海關抓走私)
  - ▶ Randomizing across exam questions (老師隨機出題)
- ▶ But, there are interesting folk theories about these games... (但總有一些有趣的「理論」)

# 玩家公開猜拳遊戲必勝絕招：先出剪刀 中央社 2007-12-19

▶ 媒體報導，大多數人都知道，在猜拳遊戲中，石頭贏剪刀，剪刀贏布，布勝拳頭，但很少有人知道，如何贏得這個相當普遍的遊戲。現在死忠玩家透露了必殺秘技：先出剪刀。

L0

▶ 英國「每日郵報」報導，研究顯示在這種快速擺出手部姿勢的猜拳遊戲中，石頭是三種猜拳手勢中玩家最喜歡出的一種。如果你的對手預

L1

期你會出石頭，他們就會選擇出布來贏過你，因此你要出剪刀才能贏，因為剪刀贏布。

L2

## 玩家公開猜拳遊戲必勝絕招：先出剪刀 中央社 2007-12-19

- ▶ 報導說，這套剪刀策略讓拍賣商佳士得前年成功贏得一千萬英鎊的生意。一名有錢的日本藝術品收藏家，無法決定要讓哪家拍賣公司來拍賣自己收藏的印象派畫作，於是 he 要求佳士得與蘇富比兩家公司猜拳決定。
- ▶ 佳士得向員工討教猜拳策略，最後在一名主管十一歲的女兒的建議下決定出剪刀。這名女孩現在還在讀書，經常玩猜拳，她推論「所有人都以為你會出石頭」。這代表蘇富比會出布，想要打敗石頭，因此佳士得應該選擇出剪刀。
- ▶ 一如預期，蘇富比最後出布，輸給了佳士得的剪刀，拱手將生意讓給對方。

# Mixed-Strategy Equilibrium in RPS

- ▶ How do you play Rock-paper-scissors (RPS)?
  - 如果你來玩剪刀石頭布，你會出什麼？
- ▶ What is the MSE here? (剪刀石頭布賽局的均衡為何?)
- ▶ Mix with probabilities  $(1/3, 1/3, 1/3)$  (三者隨機)
- ▶ Would you really play **this MSE** in RPS?
  - ▶ News article suggests a level-k model...
  - ▶ (你真的會按均衡策略來玩嗎？新聞故事所反映的多層次思考模型預測為何？想知道更多請看課本第五章)
- ▶ Janken/RPS Robot with 100% winning rate:
  - ▶ <http://www.youtube.com/watch?v=3nxjjztQKtY>

# Advantages of Games with MSE (此種賽局的優點)

- ▶ Typically have unique equilibrium (有唯一均衡)
  - ▶ All games discussed have unique equilibrium
- ▶ Constant sum: No room for social preference
  - ▶ Not possible to help others without hurting self (總報酬為常數下通常無社會偏好，因為幫助別人一定傷到自己)
- ▶ Maximin leads to Nash in zero sum (避兇就是均衡)
  - ▶ Maximin is a simple rule: (對方就是要害我如何趨吉避兇)
  - ▶ "I want to maximize the worse case scenario..."
- ▶ A good place to test theory! (這是驗證理論的好地方)

# Maximin in Matching Pennies (黑白猜下避兇)

|   |    |    |
|---|----|----|
|   | H  | T  |
| H | 1  | -1 |
| T | -1 | 1  |

- ▶ Rowena thinks: (列子認為)
- ▶ Play H: Worse case -1
- ▶ Play T: Worse case -1
- ▶  $(1/2, 1/2)$ : Worse case  $0^*$   
(出正面最慘是對手選反面，出反面最慘是對手選正面，一半一半至少不賺不賠)
- ▶ Same for Colin (行家所見略同)
- ▶ This is the MSE!  
(這正好是此賽局的混合策略均衡!)

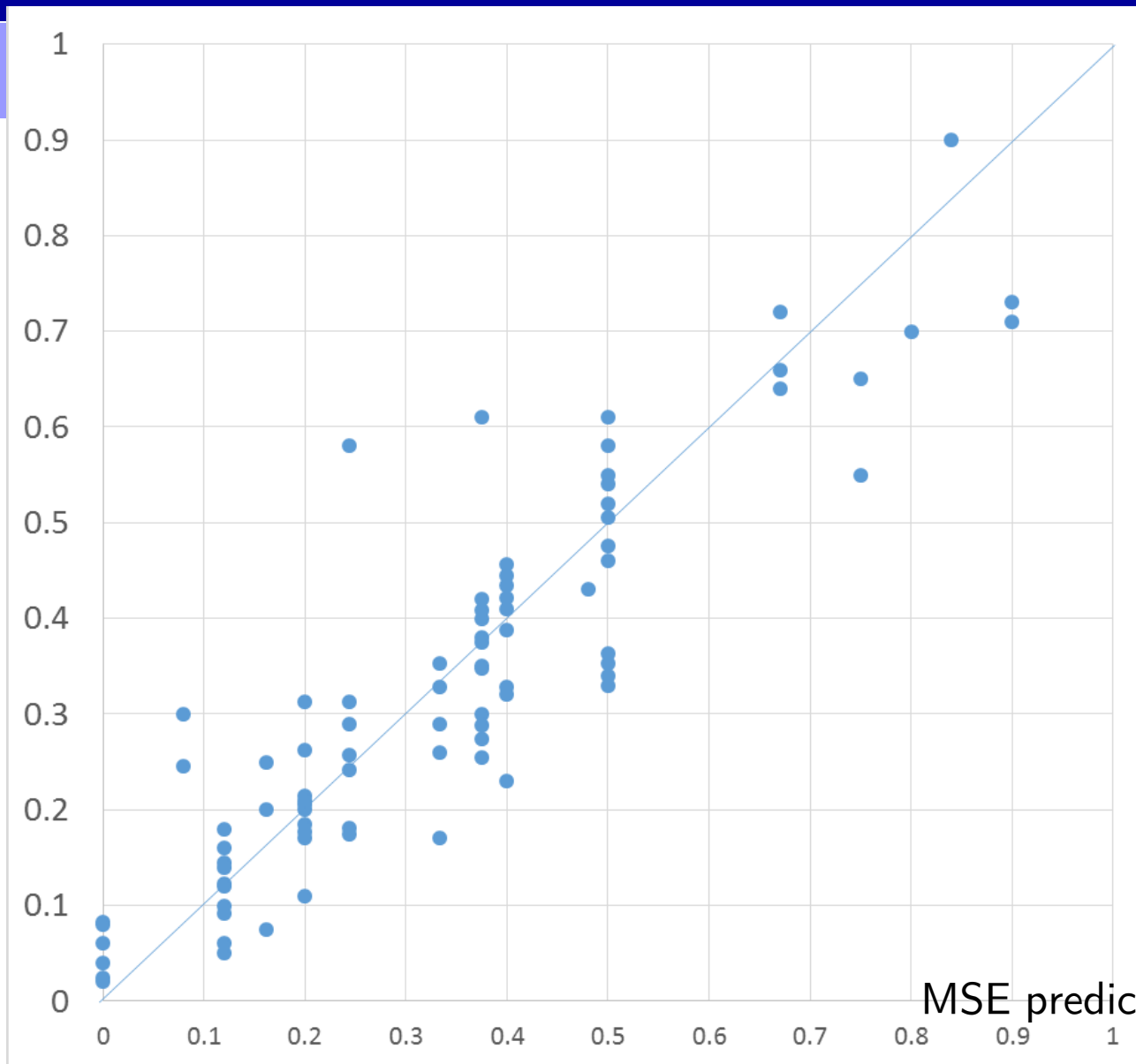
\*We assume preferences satisfy axioms for EU... (假設偏好滿足期望效用公理)

# Challenges of Games with MSE (對理論的挑戰)

- ▶ Epistemic Foundation (認知基礎: 須清楚知道對手的策略)
  - ▶ Requires precise knowledge of opponent strategy
- ▶ Learning Dynamics may not work (動態學習不見得好)
  - ▶ Gradient processes spiral away (梯度逼近會螺旋脫離均衡)
  - ▶ No incentive to mix properly at MSE (均衡時亂做沒差)
- ▶ Randomization can be unnatural
  - ▶ Especially in repeated play (重複做的話, 隨機亂選不太自然)
- ▶ Purification (純化: 個體可做不同單純策略, 整體看起來「混合」即可)
  - ▶ MSE can occur at population level, not individually



Actual Data  
(實驗資料)



Source  
(資料來源):  
Behavioral  
Game Theory  
Ch 3

# Joker Game: O'Neill (1987) (出鬼牌賽局)

- ▶ Earlier studies: Play between MSE & random
  - ▶ But had computerized opponents and/or low incentives, so hard to interpret the results...  
(早期實驗結果介於MSE和亂選之間，但通常對手是電腦且不見得有誘因)
- ▶ First "Modern" Studies: O'Neill (1987)
- ▶ **Good Design Trick:** (很棒的實驗設計技巧! )
  - ▶ Risk aversion plays no role when there are only two possible outcomes  
(當實驗結果只有兩種可能時，風險偏好不會影響受試者的決定)

# Joker Game: O'Neill (1987) (出鬼牌賽局)

|        | 1     | 2     | 3     | J     | MSE | Actual | QRE   |
|--------|-------|-------|-------|-------|-----|--------|-------|
| 1      | -5    | 5     | 5     | -5    | 0.2 | 0.221  | 0.213 |
| 2      | 5     | -5    | 5     | -5    | 0.2 | 0.215  | 0.213 |
| 3      | 5     | 5     | -5    | -5    | 0.2 | 0.203  | 0.213 |
| J      | -5    | -5    | -5    | 5     | 0.4 | 0.362  | 0.360 |
| MSE    | 0.2   | 0.2   | 0.2   | 0.4   |     |        |       |
| Actual | 0.226 | 0.179 | 0.169 | 0.426 |     |        |       |
| QRE    | 0.191 | 0.191 | 0.191 | 0.427 |     |        |       |

- ▶ 實際的出牌頻率跟MSE預測很接近
- ▶ QRE的預測更接近，但無法解釋「不平衡」

- ▶ Actual frequencies are quite close to MSE
- ▶ QRE better, but cannot get "imbalances"

# Quantal Response Equilibrium (手滑反應均衡)

- ▶ QRE - McKelvey and Palfrey (1995)
- ▶ Better response, not best response (更適/非最適)
- ▶ Logit payoff response function: (常用logit報酬反應函數)

$$P(s_i) = \frac{e^{\lambda \cdot \left[ \sum_{s_{-i}} P(s_{-i}) u_i(s_i, s_{-i}) \right]}}{\sum_{s_k} e^{\lambda \cdot \left[ \sum_{s_{-i}} P(s_{-i}) u_i(s_k, s_{-i}) \right]}}$$

# Quantal Response Equilibrium (QRE)

- ▶  $\lambda = 0$ : Noise (do not respond to payoffs)
  - ▶ (對報酬無反應)
- ▶  $\lambda = \infty$ : Nash (perfectly respond to payoffs)
  - ▶ (完全反應)

$$P(s_i) = \frac{e^{\lambda \cdot \left[ \sum_{s_{-i}} P(s_{-i}) u_i(s_i, s_{-i}) \right]}}{\sum_{s_k} e^{\lambda \cdot \left[ \sum_{s_{-i}} P(s_{-i}) u_i(s_k, s_{-i}) \right]}}$$

# Response to O'Neill (1987) (後續討論)

- ▶ Brown and Rosenthal (1990) criticize O'Neill:

- ▶ Overly support MSE (太過支持混合策略均衡)

- ▶ Aggregate tests not good enough (只有總體檢定不夠)

- ▶ They run (temporal dependence):

$$J_{t+1} = a_0 + a_1 J_t + a_2 J_{t-1} \quad (\text{應該檢定跨期相關性})$$

$$b_0 J_{t+1}^* + b_1 J_t^* + b_2 J_{t-1}^*$$

$$c_1 J_t J_t^* + c_2 J_{t-1} J_{t-1}^* + \epsilon$$

$J_t$  = Own Choice;  $J_t^*$  = Other's Choice;

- ▶ MSE implies only  $a_0$  is not zero (均衡: 只有  $a_0$  不是0)

# Brown & Rosenthal (1990) Results

| Effect                      | Coefficient          | % Players<br>s.t. $p < 0.05$ |
|-----------------------------|----------------------|------------------------------|
| Guessing                    | $b_0$                | 8%                           |
| Previous opp. choices       | $b_1, b_2$           | 30%                          |
| Previous outcomes           | $c_1, c_2$           | 38%                          |
| Previous choices & outcomes | $b_1, b_2, c_1, c_2$ | 44%                          |
| Previous own choices        | $a_1, a_2$           | 48%                          |
| All effects                 |                      | 62%                          |

Source: Table 3.4, BGT.

## Response to O'Neill (1987) (後續討論)

- ▶ **Run:** 2 JJJJ 1 2 33 (連發太短)
- ▶ **Too Short runs:** play J twice too rarely (鮮有連續J)
- ▶ **Subjects react to what they see/do** (對歷史有反應)
  - ▶ But most cannot use temporal dependence to guess opponent current action
    - ▶ (無法用跨期相關性猜中對方這次行動)
- ▶ **Equilibrium-in-beliefs** somewhat supported
  - ▶ (證據支持信念上的均衡)
- ▶ Each player may deviate from MSE (每人各自可能偏離)
- ▶ But these deviations cannot be detected (卻未被破解)



## Response to O'Neill (1987) (後續討論)

- ▶ **Purification interpretation of MSE** (純化的MSE)
  - ▶ Equilibrium in beliefs, not in mixtures (信念非策略)
- ▶ **Other similar studies** (相關延伸研究)
  - ▶ Rapoport and Boebel (1992) [BGT, Table 3.5]
  - ▶ Mookerjee and Sopher (1997)
    - ▶ [BGT, Table 3.6-3.7]
  - ▶ Tang (1996abc, 2001)
    - ▶ [BGT, Table 3.8]
  - ▶ Binmore, Swierzbinski and Proulx (2001)
    - ▶ [BGT, Table 3.9]

# Response to O'Neill (1987) (後續討論)

- ▶ Stylized Facts: (整體實驗結果)
  - ▶ Actual frequencies not far from MSE
    - ▶ (出牌頻衡很接近MSE)
  - ▶ Deviations small but significant
    - ▶ (跟MSE差距小但統計上顯著)
  - ▶ Temporal dependence at individual level
    - ▶ (個人有跨期相關性)
- ▶ Can a theory explain these?
  - ▶ (有何理論可以解釋這些實驗結果?)

## Response to O'Neill (1987) (後續討論)

- ▶ Ask subjects generate random sequences (產生數列)
- ▶ Sequences resemble the underlying statistical process **more closely than** what short random sequences actually do (產生的比真正隨機數列還要**更隨機**)
  - ▶ Too balanced (太平衡)
  - ▶ Too few runs (連發太少)
  - ▶ Longest run is too short (最長的連發太短)
- ▶ Children do not learn this misconception until after 5th grade (小孩子在五年級之前沒有這個問題)
  - ▶ A learned mistake (這是一個後天學會的錯誤)

# Game Play (賽局實驗) vs. Production (產生數列)

- ▶ Rapoport and Budescu (1992, 1994, 1997)
  - ▶ Compare sequences from a production task to strategies in a constant-sum game (R&B, 1992)
    - 比較產生的數列和零和賽局實驗中的數列
- ▶ **Condition D:** Matching pennies 150 times 1-by-1
  - ▶ 150次逐次黑白猜
- ▶ **Condition S:** Give sequence of 150 plays at once
  - ▶ 一次給150回合黑白猜的決定
- ▶ **Condition R:** Produce the outcome of tossing an unbiased coin 150 times
  - ▶ 產生數列——丟銅板150次的結果

## Game Play (賽局實驗) vs. Production (產生數列)

- ▶ iid rejected for 40% (D), 65% (S), 80% (R) of the subjects in the three conditions
  - ▶ 三種分別有40%, 65% 和80%的受試者拒絕 iid 假設
  - ▶ **Game play** reduces deviations from randomness
    - ▶ 真的去玩會讓受試者比較隨機(降低偏離情形)
- ▶ Are subjects better motivated?
- ▶ Or, are their working memory interfered and randomize "memory-lessly"?
  - ▶ 這是因為受試者有更好的誘因，還是因為他們的腦部運作(工作記憶)受到干擾，以致於「忘記過去，努力面前」？

# 3-action Matching Pennies

|   | 1  | 2  | 3  | MSE |
|---|----|----|----|-----|
| 1 | 2  | -1 | -1 | 1/3 |
| 2 | -1 | 2  | -1 | 1/3 |
| 3 | -1 | -1 | 2  | 1/3 |

| MSE | 1/3 | 1/3 | 1/3 |
|-----|-----|-----|-----|
|-----|-----|-----|-----|

- ▶ Rapoport and Budescu (1994)

# Runs in 3-action Matching Pennies R&B '94

| Pattern | Game (%) | Production (%) | iid (%) |
|---------|----------|----------------|---------|
| xx      | 26.9%    | 27.2%          | 33.3%   |
| xxx     | 7.3%     | 6.3%           | 11.1%   |
| xyy     | 19.6%    | 20.9%          | 22.2%   |
| xyy     | 19.6%    | 21.0%          | 22.2%   |
| xxxx    | 2.0%     | 1.8%           | 3.7%    |
| xxxy    | 5.3%     | 4.5%           | 7.4%    |
| yxxx    | 5.4%     | 4.5%           | 7.4%    |
| xyxx    | 5.6%     | 3.5%           | 7.4%    |
| xyyx    | 5.8%     | 3.7%           | 7.4%    |

# Other Play in 3-action Matching Pennies

| Pattern | Game (%) | Production (%) | iid (%) |
|---------|----------|----------------|---------|
| xy      | 73.1%    | 72.8%          | 66.7%   |
| xyx     | 23.7%    | 16.0%          | 22.2%   |
| xyz     | 29.7%    | 35.9%          | 22.2%   |
| yxzx    | 9.6%     | 7.8%           | 7.4%    |
| xyxz    | 9.9%     | 7.9%           | 7.4%    |
| xyzx    | 12.1%    | 17.3%          | 7.4%    |

Source: Table 3.10, BGT.



# A Limited Memory Model (有限記憶模型)

- ▶ Subjects only remember last  $m$  elements (記得最後 $m$ 回合)
- ▶ Chose the  $(m+1)$ st to balance the number of H and T choices in the last  $(m+1)$  flips
  - ▶ 受試者第 $(m+1)$ 回合做決定來平衡正反面在 $(m+1)$ 次中出現的次數
- ▶ If  $m$  is small, alternate choices too frequently
  - ▶ 如果  $m$  很小，就會正反變換太頻繁
- ▶ Experimental Data: (Should all be 0.5 if iid)
  - ▶  $P(H|H)=0.42$  (實驗結果：如果iid的話應該都是0.5)
  - ▶  $P(H|HH)=0.32$
  - ▶  $P(H|HHH)=0.21$
- ▶ Requires  $m=7$  to generate this (Magic 7?) (才符合結果)

# Explicit Randomization (使用亂數產生器)

- ▶ Observe the randomization subjects want to play
  - ▶ 觀察人們會為亂數產生器設定何種機率來做
  - ▶ Bloomfield (1994), Ochs (1995b), Shachat (2002)
- ▶ Explicit Randomization: (使用亂數產生器)
  1. Allocate 100 choices to either strategies
  2. Choices are shuffled and computer selects one
    - ▶ 總共100張牌/選擇，決定兩邊各放幾張讓電腦隨機打一張出來...
- ▶ Deviations cannot be due to cognitive limit!
  - ▶ 如果還偏離均衡，就不是因為不能產生亂數！
- ▶ Result: Deviations from MSE small but significant
- ▶ About 10% purists (偏離MSE很小但顯著。10%「單純的人」)

# Explicit Randomization (使用亂數產生器)

- ▶ Ex: Ochs (1995b) - Matching Pennies (黑白猜)
  - ▶ Row player payoff of (H, H):  $1 \rightarrow 9 \rightarrow 4$  (改變列子報酬)
- ▶ MSE: Column MSE changes; row is same...
  - ▶ 行家的MSE會改變；列子的反而不會變
- ▶ Allocate 10 plays of H or T (分配十個選擇給正或反)
  - ▶ Becomes a 10-play sequence (變成「做十次的數列」)
- ▶ Note: Random draw without replacement
  - ▶ This is not exactly randomization of MSE...
  - ▶ 註：這是隨機抽取不放回，不是真的MSE...

# Matching Pennies (Baseline)

|   |      |      |
|---|------|------|
|   | H    | T    |
| H | 1, 0 | 0, 1 |
| T | 0, 1 | 1, 0 |

- ▶ MSE:
  - ▶ R: (0.500, 0.500)
  - ▶ C: (0.500, 0.500)
- ▶ Actual Frequency: (實際頻率)
  - ▶ R: (0.500, 0.500)
  - ▶ C: (0.480, 0.520)
- ▶ QRE:
  - ▶ R: (0.500, 0.500)
  - ▶ C: (0.500, 0.500)

# Matching Pennies (Game 2)

|   | H    | T    |
|---|------|------|
| H | 9, 0 | 0, 1 |
| T | 0, 1 | 1, 0 |

- ▶ MSE:
  - ▶ R: (0.500, 0.500)
  - ▶ C: (0.100, 0.900)
- ▶ Actual Frequency: (實際頻率)
  - ▶ R: (0.600, 0.400)
  - ▶ C: (0.300, 0.700)
- ▶ QRE:
  - ▶ R: (0.649, 0.351)
  - ▶ C: (0.254, 0.746)

# Matching Pennies (Game 3)

|   |      |      |
|---|------|------|
|   | H    | T    |
| H | 4, 0 | 0, 1 |
| T | 0, 1 | 1, 0 |

- ▶ MSE:
  - ▶ R: (0.500, 0.500)
  - ▶ C: (0.200, 0.800)
- ▶ Actual Frequency: (實際頻率)
  - ▶ R: (0.540, 0.460)
  - ▶ C: (0.340, 0.660)
- ▶ QRE:
  - ▶ R: (0.619, 0.381)
  - ▶ C: (0.331, 0.669)

# MSE in Field Context (實際現場的MSE)

- ▶ Rapoport and Almadoss (2000)
- ▶ Patent races games (競相專利賽局)
  - ▶ Two firms with endowment  $e$  (兩家廠商, 各有財產)
  - ▶ Invest  $1, 2, \dots, e$  (integer)
  - ▶ Win  $r$  if invest most
- ▶ **Unique MSE:** Invest  $e$  with prob.  $1-e/r$ , invest others with prob.  $1/r$  (not obvious)

# Patent Race Results (競相專利賽局實驗結果)

| (Table 3.14) | Game L:<br>$e = 5, r = 8$ |        | Game H:<br>$e = 5, r = 20$ |        |
|--------------|---------------------------|--------|----------------------------|--------|
|              | MSE                       | Actual | MSE                        | Actual |
| Investment   |                           |        |                            |        |
| 0            | 12.5%                     | 16.9%  | 5.0%                       | 14.1%  |
| 1            | 12.5%                     | 11.6%  | 5.0%                       | 5.5%   |
| 2            | 12.5%                     | 8.8%   | 5.0%                       | 5.3%   |
| 3            | 12.5%                     | 11.8%  | 5.0%                       | 5.3%   |
| 4            | 12.5%                     | 9.0%   | 5.0%                       | 6.9%   |
| 5            | 37.5%                     | 41.8%  | 75.0%                      | 62.8%  |



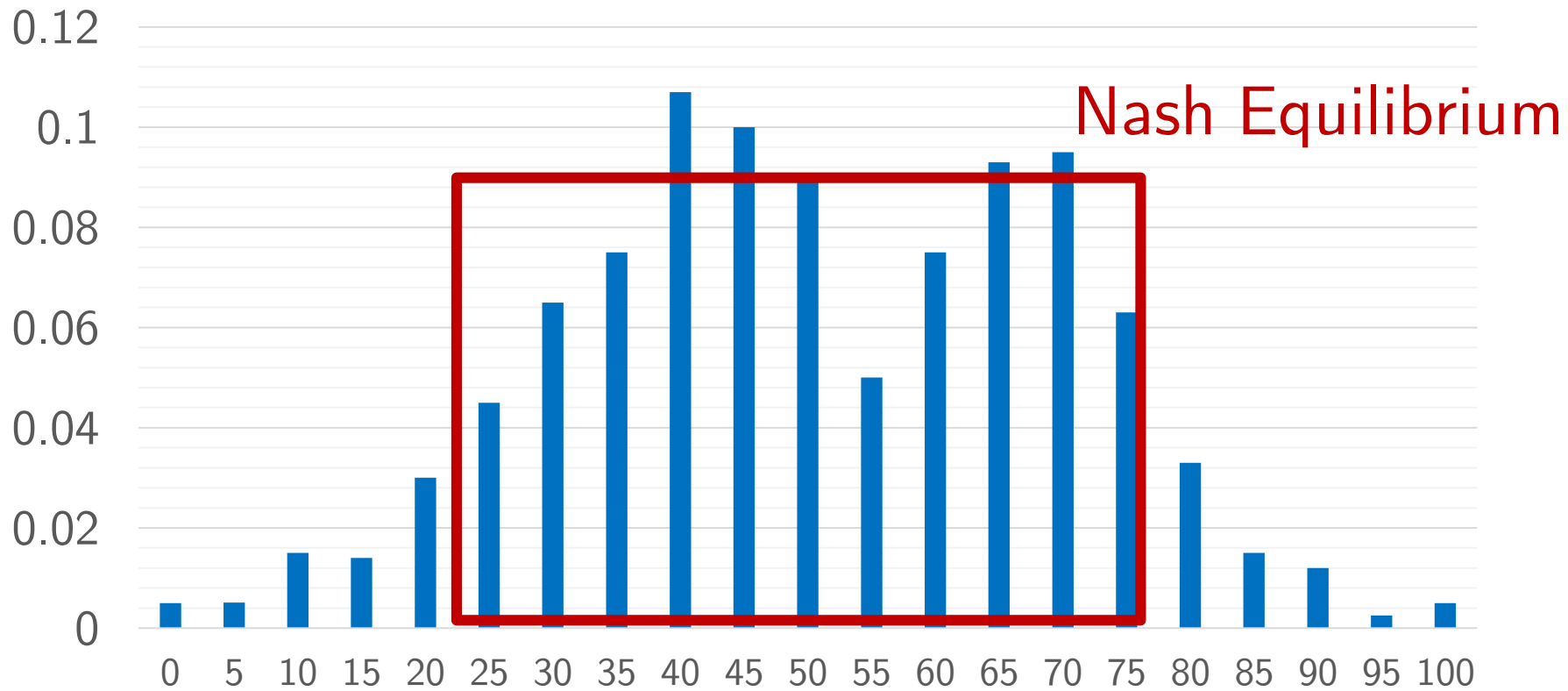
# MSE in Field Context (實際現場的MSE)

- ▶ **3 Firm Hotelling:** Collins and Sherstyuk (2000)
  - ▶ 2-Firm: Brown-Kruse, Cronshaw & Schenk (1993)
  - ▶ 4-Firm: Huck, Muller and Vreind (2002)
- ▶ **Location Games (3 Firm Hotelling Model)**
  - ▶ Three firms simultaneously choose  $[0,100]$
  - ▶ Consumers go to nearest firm
  - ▶ Profits proportional to units sold
- ▶ **Unique MSE:** Randomize uniformly  $[25,75]$

# MSE in Field Context (實際現場的MSE)

Frequency

Source: Figure 3.2, BGT; Based on Colins and Sherstyuk (2000).



Distribution of location choices

# Two Field Studies

- ▶ **Walker and Wooders (2001)**
  - ▶ serve decisions (L or R) of tennis players in 10 Grand Slam matches
- ▶ **Result:**
  - ▶ Win rates across two different directions are not statistically different ( $p < 0.10$  for only 2/40)
  - ▶ Players still exhibit some over-alteration in serve choices through temporal dependence ( $p < 0.10$  for 8/40) [weaker than lab subjects]

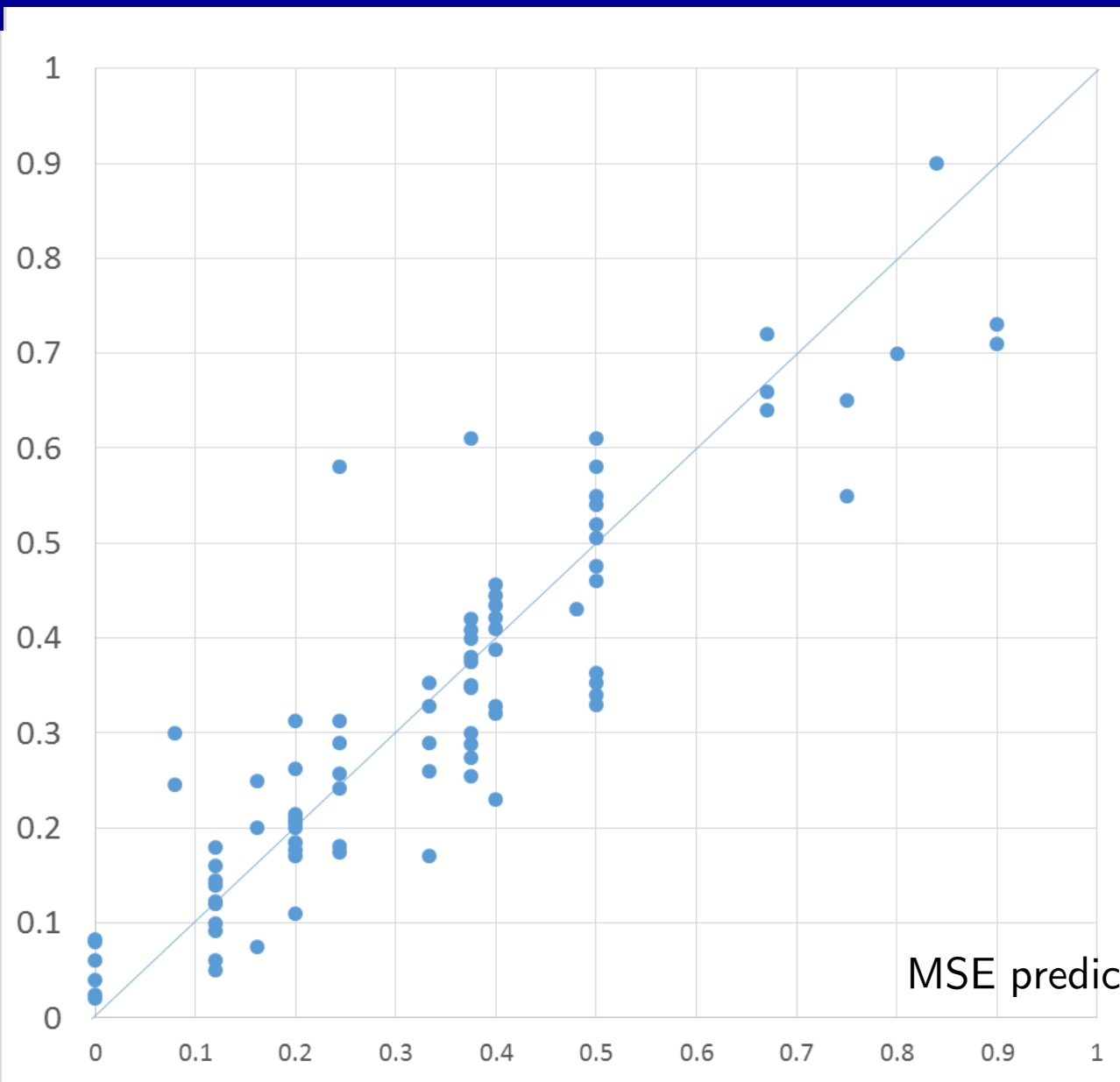
# Two Field Studies

- ▶ **Palacios-Huerta (2001): soccer penalty kicks**
  - ▶ Code both kicker and goalie's choices
  - ▶ No selection bias (look at all games)
- ▶ Win rates are equal; no serial dependence
  - ▶ Not surprising since penalty kicks are few and are often done by different players
- ▶ Recent: Huang, Hsu, and Tang (AER 2007)
  - ▶ Chen-Ying Huang (here at NTU)

# Conclusion

- ▶ **Take-home Message:**
- ▶ Aggregate frequencies of play are close to MSE but the deviations are statistically significant.
- ▶ **QRE seems to fit behaviors well.**
- ▶ Temporal dependence frequently observed

Actual Data  
(實驗資料)



Source  
(資料來源):  
Behavioral  
Game Theory  
Ch 3

MSE predictions(均衡預測)

ions(均衡預測)

# Conclusion

- ▶ With explicit randomization, the existence of purists hint on **equilibrium in beliefs**
  - ▶ Players cannot guess what opponents are doing
  - ▶ Beliefs about opponents are correct on average
  - ▶ But, they may not be randomizing themselves
- ▶ **Field-Lab-Theory:** Ostling, Wang, Chou and Camerer (2011), "Testing Game Theory in the Field: Evidence from Swedish Poisson LUPI Lottery Games," *American Economic Journal: Microeconomics*, 3(3), 1-33.

# Some Comments Regarding 20-Minute Presentations

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# Games with MSE 有混合策略均衡的賽局

- ▶ Rule 1: Don't use more than 20 slides
  - ▶ It takes on average 1 minute to go over 1 slide
- ▶ Rule 2: Don't use font sizes below 28
  - ▶ Try looking at your slides from far behind
  - ▶ Font sizes  $< 28$  means:  
You DON'T want people to see it
- ▶ Rule 3: This is a **teaser-trailer** of the movie
  - ▶ Show the experiment + a snapshot of the results
- ▶ Write down a script and NOT memorize it