# Experiments Games with Mixed Strategy Equilibrium (混合策略均衡實驗)

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Lecture 5, EE-BGT

# Games with MSE 有混合策略均衡的賽局

- Zero-Sum Games (零和賽局)
  - Rock-Scissor-Paper (剪刀石頭布)
  - Sports (PK, tennis serves, basketball drives, etc.)
    - 足球罰踢、網球發球、籃球切入或投籃
  - Military attack (軍事行動如登陸諾曼地或加萊)
- Deter Undesired Behavior (嚇阻「投機/不希望發生」的行為)
  - Searches of passengers after 9/11 (機場安檢、海關抓走私)
  - Randomizing across exam questions (老師隨機出題)
- But, there are interesting folk theories about these games... (但總有一些有趣的「理論」)

## 玩家公開猜拳遊戲必勝絕招：先出剪刀 中央社 2007-12-19

- 媒體報導，大多數人都知道，在猜拳遊戲中，石頭贏剪刀，剪刀贏布，布勝拳頭，但很少有人知道，如何贏得這個相當普遍的遊戲。現在死忠玩家透露了必殺秘技：先出剪刀。
- 英國「每日郵報」報導，研究顯示在這種快速擺出手部姿勢的猜拳遊戲中，石頭是三種猜拳手勢中玩家最喜歡出的一種。如果你的對手預期你會出石頭，他們就會選擇出布來贏過你，因此你要出剪刀才能贏，因為剪刀贏布。

L0

L1

L2

## 玩家公開猜拳遊戲必勝絕招：先出剪刀中央社 2007-12-19

- 報導說，這套剪刀策略讓拍賣商佳士得前年成功贏得一千萬英鎊的生意。一名有錢的日本藝術品收藏家，無法決定要讓哪家拍賣公司來拍賣自己收藏的印象派畫作，於是他要求佳士得與蘇富比兩家公司猜拳決定。
- 佳士得向員工討教猜拳策略，最後在一名主管十一歲的女兒的建議下決定出剪刀。這名女孩現在還在讀書，經常玩猜拳，她推論「所有人都以為你會出石頭」。這代表蘇富比會出布，想要打敗石頭，因此佳士得應該選擇出剪刀。
- 一如預期，蘇富比最後出布，輸給了佳士得的剪刀，拱手將生意讓給對方。

# Mixed-Strategy Equilibrium in RPS

- How do you play Rock-paper-scissors (RPS)?
  - 如果你來玩剪刀石頭布，你會出什麼？
- What is the MSE here? (剪刀石頭布賽局的均衡為何？)
- Mix with probabilities (1/3, 1/3, 1/3) (三者隨機)
- Would you really play this MSE in RPS?
  - News article suggests a level-k model...
  - (你真的會按均衡策略來玩嗎？新聞故事所反映的多層次思考模型預測為何？想知道更多請看課本第五章)
- Janken/RPS Robot with 100% winning rate:
  - http://www.youtube.com/watch?v=3nxjjztQKtY

# Advantages of Games with MSE (此種賽局的優點)

- Typically have unique equilibrium (有唯一均衡)
  - All games discussed have unique equilibrium
- Constant sum: No room for social preference
  - Not possible to help others without hurting self (總報酬為常數下通常無社會偏好，因為幫助別人一定傷到自己)
- Maximin leads to Nash in zero sum (避兇就是均衡)
  - Maximin is a simple rule: (對方就是要害我如何趨吉避兇)
  - "I want to maximize the worse case scenario..."
- A good place to test theory! (這是驗證理論的好地方)

# Maximin in Matching Pennies (黑白猜下避兇)

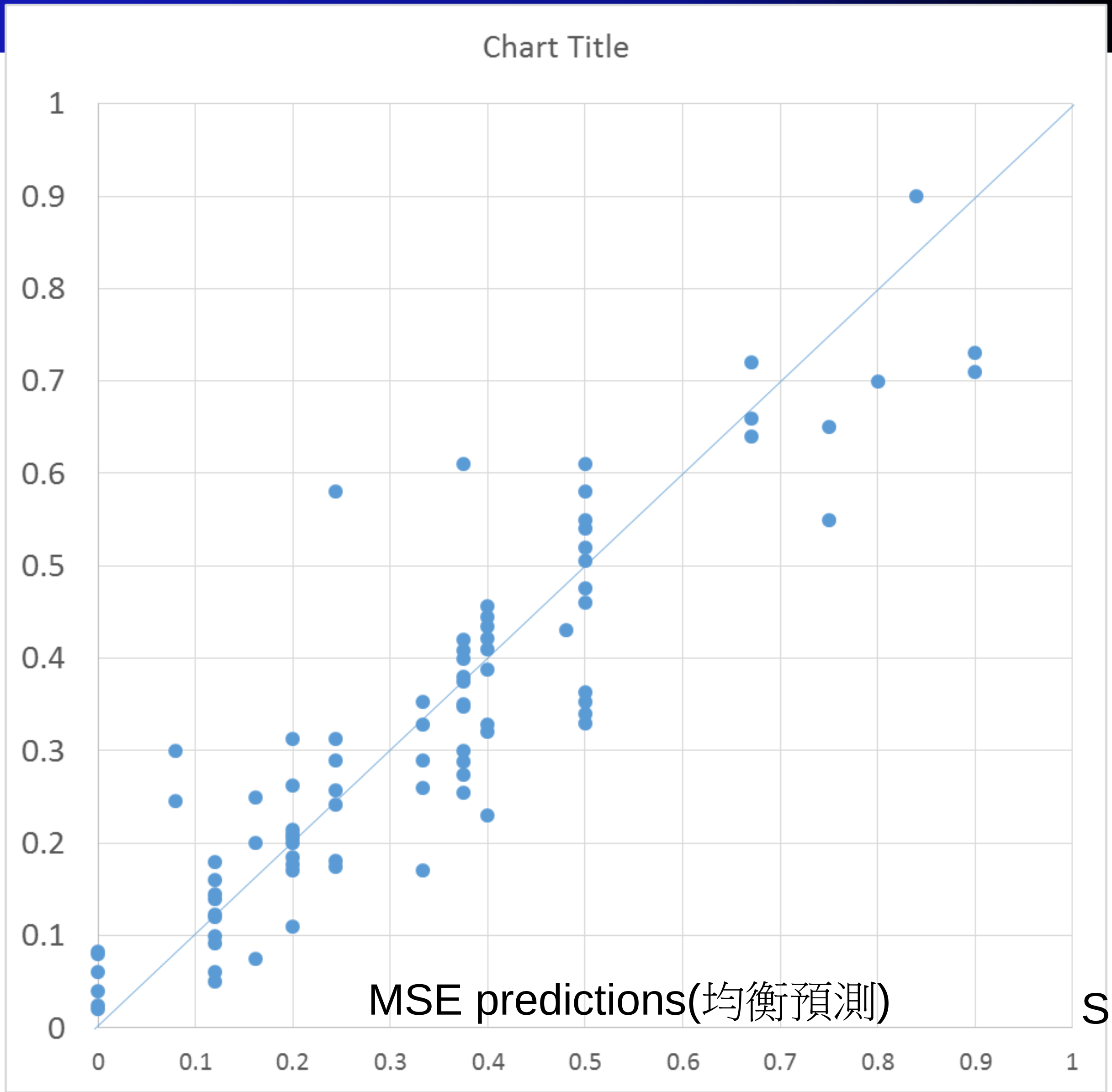
| | H | T |
|---|---|---|
| H | 1 | -1 |
| T | -1 | 1 |

- Rowena thinks: (列子認為)
- Play H: Worse case -1
- Play T: Worse case -1
- (1/2, 1/2): Worse case 0*
  (出正面最慘是對手選反面，出反面最慘是對手選正面，一半一半至少不賺不賠)
- Same for Colin (行家所見略同)
  - This is the MSE!
    (這正好是此賽局的混合策略均衡！)

*We assume preferences satisfy axioms for EU... (假設偏好滿足期望效用公理)

# Challenges of Games with MSE (對理論的挑戰)

- Epistemic Foundation (認知基礎: 須清楚知道對手的策略)
  - Requires precise knowledge of opponent strategy
- Learning Dynamics may not work (動態學習不見得好)
  - Gradient processes spiral away (梯度逼近會螺旋脫離均衡)
  - No incentive to mix properly at MSE (均衡時亂做沒差)
- Randomization can be unnatural
  - Especially in repeated play (重複做的話, 隨機亂選不太自然)
- Purification (純化: 個體可做不同單純策略, 整體看起來「混合」即可)
  - MSE can occur at population level, not individually




Actual Data
（實驗資料）

Source(資料來源)：
BGT, Ch 3

# Joker Game: O'Neill (1987) (出鬼牌賽局)

- Earlier studies: Play between MSE & random
  - But had computerized opponents and/or low incentives, so hard to interpret the results... (早期實驗結果介於MSE和亂選之間，但通常對手是電腦且不見得有誘因)
- First "Modern" Studies: O' Neill (1987)
- Good Design Trick: (很棒的實驗設計技巧！)
  - Risk aversion plays no role when there are only two possible outcomes
  - (當實驗結果只有兩種可能時，風險偏好不會影響受試者的決定)

# Joker Game: O'Neill (1987) (出鬼牌賽局)

| | 1 | 2 | 3 | J | MSE | Actual | QRE |
|---|---|---|---|---|---|---|---|
| 1 | -5 | 5 | 5 | -5 | 0.2 | 0.221 | 0.213 |
| 2 | 5 | -5 | 5 | -5 | 0.2 | 0.215 | 0.213 |
| 3 | 5 | 5 | -5 | -5 | 0.2 | 0.203 | 0.213 |
| J | -5 | -5 | -5 | 5 | 0.4 | 0.362 | 0.360 |
| MSE | 0.2 | 0.2 | 0.2 | 0.4 | | | |
| Actual | 0.226 | 0.179 | 0.169 | 0.426 | | | |
| QRE | 0.191 | 0.191 | 0.191 | 0.427 | | | |

實際的出牌頻率跟MSE預測很接近

QRE的預測更接近，但無法解釋「不平均」

- Actual frequencies are quite close to MSE
- QRE better, but cannot get "imbalances"

# Quantal Response Equilibrium (手滑反應均衡)

- QRE - McKelvey and Palfrey (1995)
- Better response, not best response (更適/非最適)
- Logit payoff response function: (常用logit報酬反應函數)

$$P(s_i) = \frac{e^{\lambda \cdot \left[ \sum_{s_{-i}} P(s_{-i}) u_i(s_i, s_{-i}) \right]}}{\sum_{s_k} e^{\lambda \cdot \left[ \sum_{s_{-i}} P(s_{-i}) u_i(s_k, s_{-i}) \right]}}$$

# Quantal Response Equilibrium (QRE)

- $\lambda = 0$ : Noise (do not respond to payoffs)
  (對報酬無反應)
- $\lambda = \infty$ : Nash (perfectly respond to payoffs)
  (完全反應)

$$P(s_i) = \frac{e^{\lambda \cdot \left[\sum_{s_{-i}} P(s_{-i}) u_i(s_i, s_{-i})\right]}}{\sum_{s_k} e^{\lambda \cdot \left[\sum_{s_{-i}} P(s_{-i}) u_i(s_k, s_{-i})\right]}}$$

# Response to O’ Neill (1987)

- Brown and Rosenthal (1990) criticize O’Neill:
  - Overly support MSE (太過支持混合策略均衡)
  - Aggregate tests not good enough (只有總體檢定不夠)
- They run (temporal dependence):

(應該檢定跨期相關性)

$$J_{t+1} = a_0 + a_1 J_t + a_2 J_{t-1}$$
$$b_0 J^*_{t+1} + b_1 J^*_t + b_2 J^*_{t-1}$$
$$c_1 J_t J^*_t + c_2 J_{t-1} J^*_{t-1} + \epsilon$$
$$J_t = \text{Own Choice}; J^*_t = \text{Other's Choice};$$

- MSE implies only $a_0$ is not zero (均衡：只有 $a_0$ 不是0)

# Brown & Rosenthal (1990) Results

| Effect | Coefficient | % Players s.t. $p<0.05$ |
|---|---|---|
| Guessing | $b_0$ | 8% |
| Previous opp. choices | $b_1$, $b_2$ | 30% |
| Previous outcomes | $c_1$, $c_2$ | 38% |
| Previous choices & outcome | $b_1$, $b_2$, $c_1$, $c_2$ | 44% |
| Previous own choices | $a_1$, $a_2$ | 48% |
| All effects | | 62% |

Source: Table 3.4, BGT.

# Response to O' Neill (1987) (後續討論)

- Run: 2 JJJJ 1 2 33 (連發太短)
- Too Short runs: play J twice too rarely (鮮有連續J)
- Subjects react to what they see/do (對歷史有反應)
  - But most cannot use temporal dependence to guess opponent current action (無法用跨期相關性猜中對方這次行動)
- Equilibrium-in-beliefs somewhat supported (信念)
  - Each player may deviate from MSE (每人各自可能偏離)
  - But these deviations cannot be detected (卻沒有被破解)
- Purification interpretation of MSE (純化的MSE)
  - Equilibrium in beliefs, not in mixtures (信念非策略)

# Response to O' Neill (1987) (後續討論)

- Other similar studies (相關延伸研究)
  - Rapoport and Boebel (1992) [BGT, Table 3.5]
  - Mookerjhee and Sopher (1997) [BGT, Table 3.6-3.7]
  - Tang (1996abc, 2001) [BGT, Table 3.8]
  - Binmore, Swierzbinski, and Proulx (2001) [BGT, Table 3.9]
- Stylized Facts: (整體實驗結果)
  - Actual frequencies not far from MSE (出牌頻衡很接近MSE)
  - Deviations small but significant (跟MSE差距小但統計上顯著)
  - Temporal dependence at individual level (個人有跨期相關性)
- Can a theory explain these? (有何理論可以解釋這些實驗結果?)

# Psychology: Production Task (心理學：產生數列)

- Ask subjects generate random sequences (產生數列)

- Sequences resemble the underlying statistical process more closely than what short random sequences actually do (產生的比真正隨機數列還要更隨機)
  - Too balanced (太平衡)
  - Too few runs (連發太少)
  - Longest run is too short (最長的連發太短)
- Children do not learn this misconception until after 5th grade (小孩子在五年級之前沒有這個問題)
  - A learned mistake (這是一個後天學會的錯誤)

# Game Play (賽局實驗) vs. Production (產生數列)

- Rapoport and Budescu (1992, 1994, 1997)
  - Compare sequences from a production task to strategies in a constant-sum game (R&B, 1992)
  - 比較產生的數列和零和賽局實驗中的數列

Condition D: Matching pennies 150 times 1-by-1

- 150次逐次黑白猜

Condition S: Give sequence of 150 plays at once

- 一次給150回合黑白猜的決定

Condition R: Produce the outcome of tossing an unbiased coin 150 times

- 產生數列——丟銅板150次的結果

# Game Play (賽局實驗) vs. Production (產生數列)

- iid rejected for 40% (D), 65% (S), 80% (R) of the subjects in the three conditions
  - 三種分別有40%, 65% 和80%的受試者拒絕 iid 假設
  - Game play reduces deviations from randomness
  - 真的去玩會讓受試者比較隨機(降低偏離情形)
- Are subjects better motivated?
- Or, are their working memory interfered and randomize "memory-lessly"?
  - 這是因為受試者有更好的誘因，還是因為他們的腦部運作(工作記憶)受到干擾，以致於「忘記過去，努力面前」？

# 3-action Matching Pennies

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 2 | -1 | -1 |
| 2 | -1 | 2 | -1 |
| 3 | -1 | -1 | 2 |

| MSE |
|---|
| 1/3 |
| 1/3 |
| 1/3 |

| | | | |
|---|---|---|---|
| MSE | 1/3 | 1/3 | 1/3 |

- Rapoport and Budescu (1994)

# Runs in 3-action Matching Pennies: R&B '94

| Pattern | Game Freq. | Production Freq. | iid Freq. |
|---|---|---|---|
| xx | 0.269 | 0.272 | 0.333 |
| xxx | 0.073 | 0.063 | 0.111 |
| xxy | 0.196 | 0.209 | 0.222 |
| xyy | 0.196 | 0.210 | 0.222 |
| xxxx | 0.020 | 0.018 | 0.037 |
| xxxy | 0.053 | 0.045 | 0.074 |
| yxxx | 0.054 | 0.045 | 0.074 |
| xyxx | 0.056 | 0.035 | 0.074 |
| xxyx | 0.058 | 0.037 | 0.074 |

# Other Play in 3-action Matching Pennies

| Pattern | Game Freq. | Production Freq. | iid Freq. |
|---|---|---|---|
| xy | 0.731 | 0.728 | 0.667 |
| xyx | 0.237 | 0.160 | 0.222 |
| xyz | 0.297 | 0.359 | 0.222 |
| yxzx | 0.096 | 0.078 | 0.074 |
| xyxz | 0.099 | 0.079 | 0.074 |
| xyzx | 0.121 | 0.173 | 0.074 |

Source: Table 3.10, BGT.

# A Limited Memory Model (有限記憶模型)

- Subjects only remember last $m$ elements (記得最後 $m$ 回合)
- Chose the $(m+1)$st to balance the number of H and T choices in the last $(m+1)$ flips
  - 受試者第$(m+1)$回合做決定來平衡正反面在$(m+1)$次中出現的次數
- If $m$ is small, alternate choices too frequently
  - 如果 $m$ 很小，就會正反變換太頻繁
- Experimental Data: (Should all be 0.5 if iid)
  - P(H|H)=0.42 (實驗結果：如果iid的話應該都是0.5)
  - P(H|HH)=0.32
  - P(H|HHH)=0.21
- Requires $m=7$ to generate this (Magic 7?) (才符合實驗結果)

# Explicit Randomization (使用亂數產生器)

- Observe the randomization subjects want to play
  - 觀察人們會為亂數產生器設定何種機率來做
  - Bloomfield (1994), Ochs (1995b), Shachat (2002)
- Explicit Randomization: (使用亂數產生器)

1. Allocate 100 choices to either strategies
2. Choices are shuffled and computer selects one
   - 總共100張牌/選擇，決定兩邊各放幾張讓電腦隨機打一張出來…

- Deviations cannot be due to cognitive limit!
  - 如果還偏離均衡，就不是因為不能產生亂數!
- Result: Deviations from MSE small but significant
- About 10% purists (偏離MSE很小但顯著。10%「單純的人」)

# Explicit Randomization (使用亂數產生器)

- Ex: Ochs (1995b) - Matching Pennies (黑白猜)
  - Row player payoff of (H, H): 1→9→4 (改變列子報酬)
- MSE: Column MSE changes; row is same...
  - 行家的MSE會改變；列子的反而不會變
- Allocate 10 plays of H or T (分配十個選擇給正或反)
  - Becomes a 10-play sequence (變成「做十次的數列」)
- Note: Random draw without replacement
  - This is not exactly randomization of MSE...
  - 註：這是隨機抽取不放回，不是真的MSE...

# Matching Pennies (Baseline)

| | H | T |
|---|---|---|
| H | 1,0 | 0,1 |
| T | 0,1 | 1,0 |

- MSE:
  - R: (0.500, 0.500)
  - C: (0.500, 0.500)
- Actual Frequency: (實際頻率)
  - R: (0.500, 0.500)
  - C: (0.480, 0.520)
- QRE:
  - R: (0.500, 0.500)
  - C: (0.500, 0.500)

# Matching Pennies (Game 2)

| | H | T |
|---|---|---|
| H | 9,0 | 0,1 |
| T | 0,1 | 1,0 |

- MSE:
  - R: (0.500, 0.500)
  - C: (0.100, 0.900)
- Actual Frequency:(實際頻率)
  - R: (0.600, 0.400)
  - C: (0.300, 0.700)
- QRE:
  - R: (0.649, 0.351)
  - C: (0.254, 0.746)

# Matching Pennies (Game 3)

| | H | T |
|---|---|---|
| H | 4,0 | 0,1 |
| T | 0,1 | 1,0 |

Source: Table 3.12, BGT.

- MSE:
  - R: (0.500, 0.500)
  - C: (0.200, 0.800)
- Actual Frequency:(實際頻率)
  - R: (0.540, 0.460)
  - C: (0.340, 0.660)
- QRE:
  - R: (0.619, 0.381)
  - C: (0.331, 0.669)

# MSE in Field Context (實際現場的MSE)

- Rapoport and Almadoss (2000)
- Patent races games (競相專利賽局)
  - Two firms with endowment $e$ (兩家廠商，各有財產)
  - Invest $1, 2, \ldots, e$ (integer)
  - Win $r$ if invest most
- Unique MSE: Invest $e$ with prob. $1\text{-}e/r$, invest others with prob. $1/r$ (not obvious)

# Patent Race Results (競相專利賽局實驗結果)

| (Table 3.14) | Game L: $e=5, r=8$ | | Game H: $e=5, r=20$ | |
|---|---|---|---|---|
| Investment | MSE | Actual | MSE | Actual |
| 0 | 0.125 | 0.169 | 0.050 | 0.141 |
| 1 | 0.125 | 0.116 | 0.050 | 0.055 |
| 2 | 0.125 | 0.088 | 0.050 | 0.053 |
| 3 | 0.125 | 0.118 | 0.050 | 0.053 |
| 4 | 0.125 | 0.090 | 0.050 | 0.069 |
| 5 | 0.375 | 0.418 | 0.750 | 0.628 |

# MSE in Field Context

- 3 Firm Hotelling: Collins and Sherstyuk (2000)
  - 2-Firm: Brown-Kruse, Cronshaw & Schenk (1993)
  - 4-Firm: Huck, Muller and Vreiend (2002)
- Location Games (3 Firm Hotelling Model)
  - Three firms simultaneously choose [0,100]
  - Consumers go to nearest firm
  - Profits proportional to units sold
- Unique MSE: Randomize uniformly [25,75]

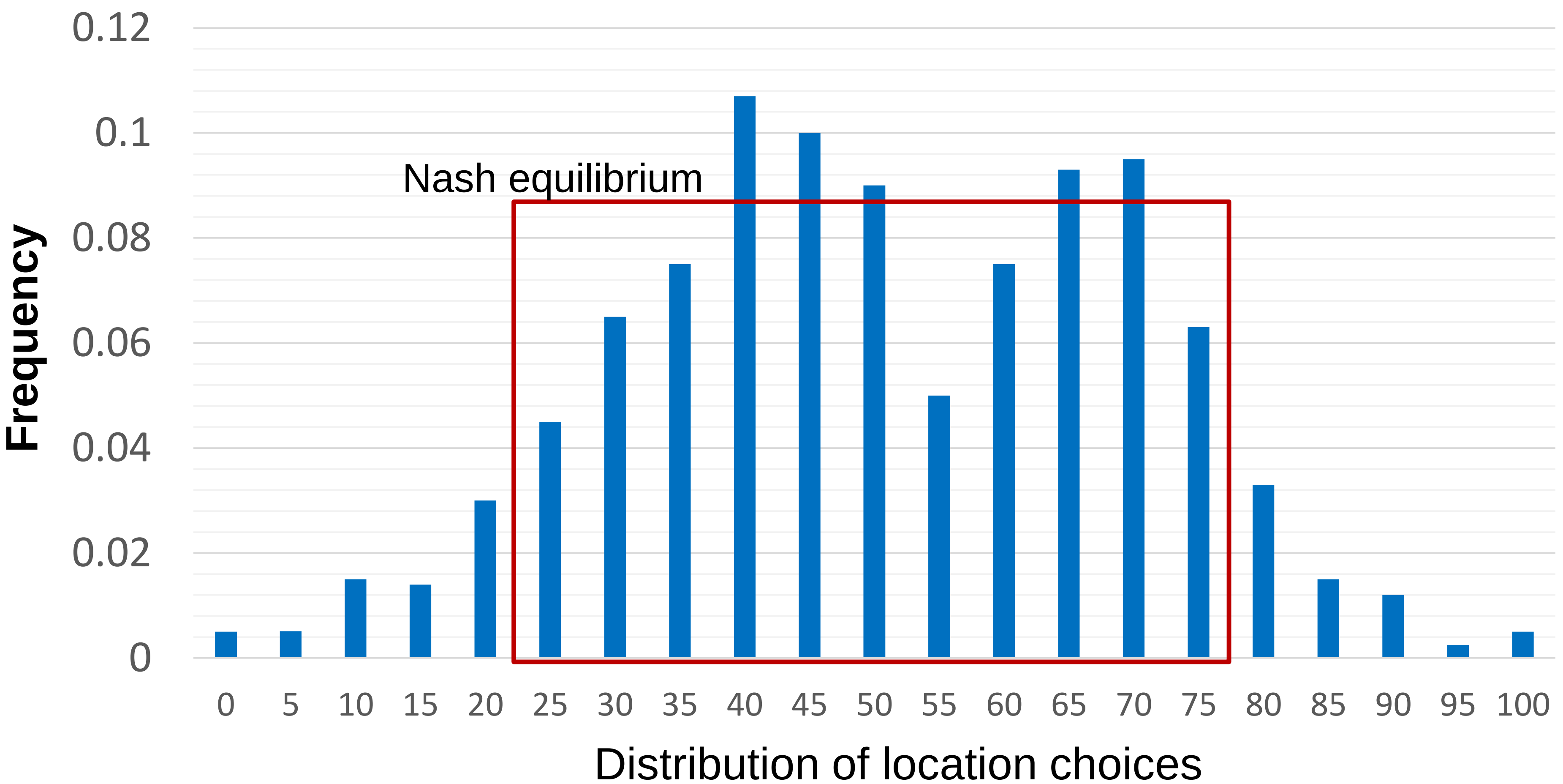



Figure 3.2 Behavioral Game Theory
Source: Based on Collins and Sherstyuk (2000)

# Two Field Studies

- Walker and Wooders (2001)
  - serve decisions (L or R) of tennis players in 10 Grand Slam matches
- Result:
  - Win rates across two different directions are not statistically different ($p<0.10$ for only $2/40$)
  - Players still exhibit some over-alteration in serve choices through temporal dependence ($p<0.10$ for $8/40$) [weaker than lab subjects]

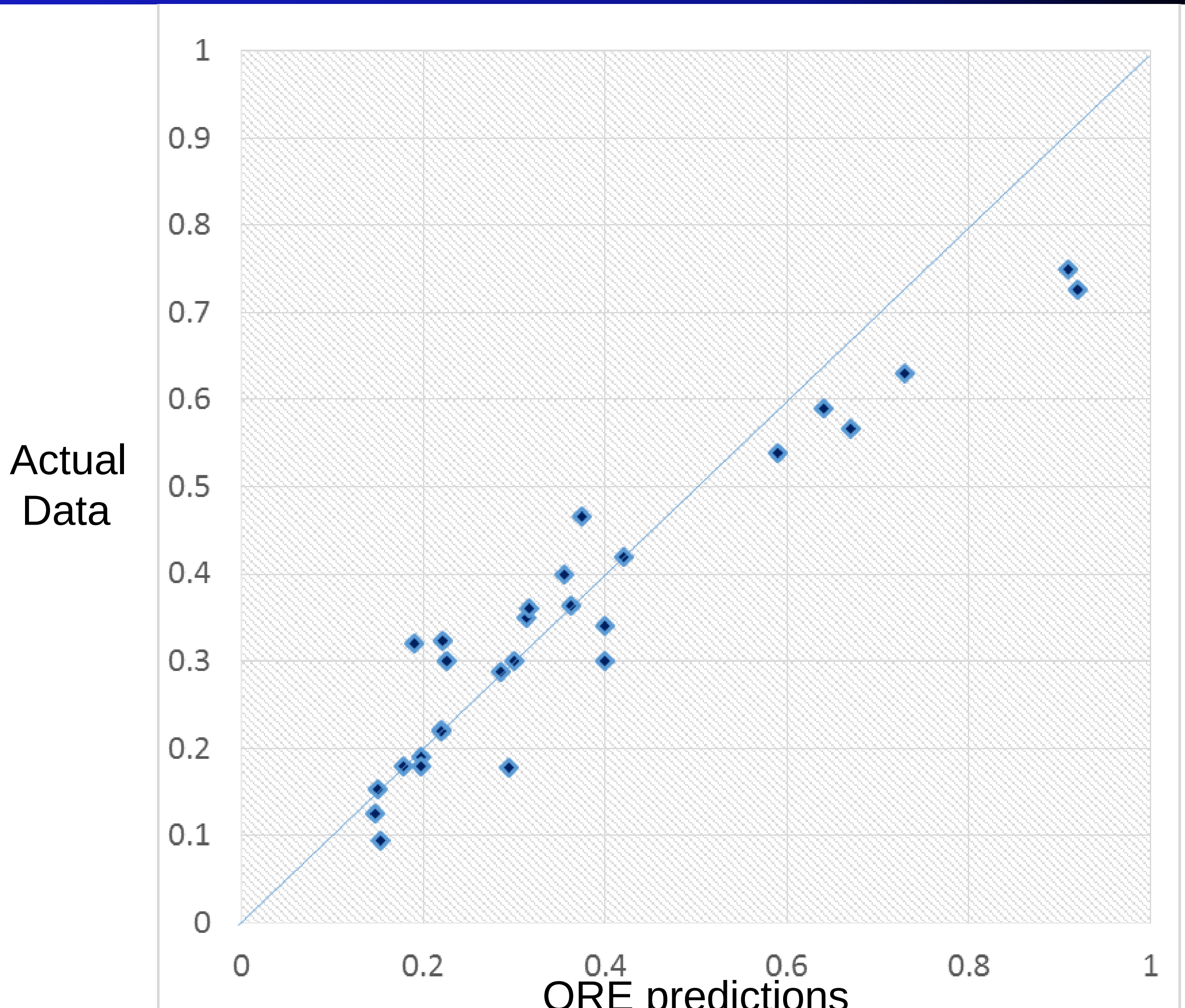
# Two Field Studies

- Palacios-Huerta (2001): soccer penalty kicks
  - Code both kicker and goalie’ s choices
  - No selection bias (look at all games)
- Win rates are equal; no serial dependence
  - Not surprising since penalty kicks are few and are often done by different players
- Recent: Huang, Hsu, and Tang (AER 2007)
  - Chen-Ying Huang (here at NTU)

# Conclusion

- Take-home Message:
- Aggregate frequencies of play are close to MSE but the deviations are statistically significant.

- QRE seems to fit behaviors well.
- Temporal dependence frequently observed




Source: BGT, Ch. 3

# Conclusion

- With explicit randomization, the existence of purists hint on equilibrium in beliefs
  - Players cannot guess what opponents are doing
  - Beliefs about opponents are correct on average
  - But, they may not be randomizing themselves
- Field-Lab-Theory: Ostling, Wang, Chou & Camerer (2011), "*Testing Game Theory in the Field: Evidence from Swedish Poisson LUPI Lottery Games*," American Economic Journal: Microeconomics, 3(3), 1-33.