A Modified Monty Hall Problem

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Abstract

We conduct a laboratory experiment using the Monty Hall Problem (MHP) to study how simplified examples improve learning behavior and correct irrational choices in probabilistic situations. In particular, we show that after experiencing a simplified version of the MHP (the 100-door version), subjects perform better in the MHP (the 3-door version), compared to the control group who only experienced the 3-door version. Our results suggest that simplified examples strongly induces learning.

Keywords: Bayesian Updating; Problem of Three Prisoners; Laboratory Experiments

JEL codes: C91, D81, D83

1 Introduction

First introduced by Selvin et al. [1975], the Monty Hall Problem (MHP) is a famous choice anomaly, which is notoriously difficult for people to learn the optimal action. Inspired by a famous game show, the MHP involves a contestant choosing between three doors, in which one of them contains a big prize. After selecting one door, the host opens one

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of the remaining two doors revealing that it is empty. Then, the contestant has to decide whether to switch to the other opaque door, or keep the initial choice. Assuming the host always randomly opens one empty door (among all empty doors not chosen), Bayes' rule says that the contestant should always switch because the probability of winning the prize is 2/3 (instead of 1/3 if one does not switch). However, most people's think this is a 50-50 gamble, and choose to stick with their initial choice [Friedman, 1998]. Even after several repetitions of the MHP with feedback, the overall switching rate is far from perfect [Tubau et al., 2015].

Given this difficulty, we consider a simplified version, MHP with 100 doors, to improve learning. This idea of simplifying the MHP was first proposed by vos Savant [1997], who claimed that, "Suppose there are a million doors, and you pick door #1. Then the host, who knows what is behind the doors and will always avoid the one with the prize, opens them all except door #777,777. You'd switch to that door pretty fast, wouldn't you?" Past research has shown that people switch more often in the MHP with more than 3 doors compared to the standard MHP [Page, 1998, Granberg and Dorr, 1998, Franco-Watkins et al., 2003, Brokaw and Merz, 2004, Stibel et al., 2009]. In other words, the 100-door version is indeed a simplified example of the original MHP.

To see if this simplified example can help subjects learn the optimal strategy in the original 3-door MHP, we conduct the following between-subject, two-stage experiment: For the treatment group, subjects play the 100-door MHP in the first stage and the 3-door MHP in the second stage. For the control group, subjects play the 3-door MHP in both stages. Our goal is to see if subjects can gain insight regarding the optimal strategy in the 100-door game, and transfer that knowledge to the second stage. We find that the average switching rate of the treatment group is significantly higher than that of the control group. In fact, subjects achieved a switching rate of 86% in the 3-door MHP, among the highest ever been reported.

Note that we are not the first paper to introduce the idea of using a MHP with more than 3 doors to teach subjects, but previous studies have either only let subjects experience the easier version once [Page, 1998], or evaluate subjects' performance in a one-shot 3-door MHP [Franco-Watkins et al., 2003]. Our experimental design let subjects play multiple rounds of both the 100-door and 3-door versions, allowing more learning behaviors to be observed in both cases. Similar ideas have been implemented in learning across games [Mengel, 2012] to teach subjects to play the game of 21 [Dufwenberg et al., 2010] and the race game [Gneezy et al., 2010].

The remaining of this paper is organized as follow: Section 2 introduces the game. Section 3 describes the experimental design, and the experimental results are in section 4. Section 5 provides some concluding remarks.

2 The K-door Monty Hall Problem

Here are the rules of the K-door MHP: There are K unopened doors, one of which contains the prize, and the other $K-1$ doors are empty. After the subject chooses a door, the computer opens all but one of the $K-1$ unchosen doors. If one of the unchosen doors contains the prize, the computer avoids it and opens the remaining $K-2$ empty doors. If the chosen door contains the prize, the computer randomly selects one of the unchosen doors to avoid and opens the other $K-2$ empty doors.

For any K, switching is the optimal strategy by the Bayes' rule, and the winning probability of switching is $\frac{K-1}{K}$.

3 Experimental Design

Subjects in the experiment were National Taiwan University (NTU) undergraduate and graduate students ($n = 40$) recruited through the Taiwan Social Sciences Experimental Laboratory (TASSEL) website in July 2017. They were assigned randomly to one of the two groups: Control group (n= 20, July 18) and Treatment group (n= 20, July 11).

Each subject individually played the MHP for 30 rounds. In the control group, subjects played the 3-door MHP for 15 rounds and repeated the same game for another 15 rounds. In the treatment group, subjects first played the 100-door MHP for 15 rounds and then played the 3-door version for 15 rounds. Feedback was given at the end of each round. Subjects were paid a show-up fee of 50 NT dollars (approx. USD \$1.50 at the time of the experiment), and earned 10 NT dollars (approx. USD \$0.30) each time they chose the door with the prize.

Note that although the monetary payoff of winning in each round is kept constant, when subjects were playing the 100-door MHP, the expected payoff of switching ($\frac{99}{10}$) is much higher than that of the 3-door MHP ($\frac{20}{3}$).

4 Results

Figure 1 shows the round-by-round switching rates for both treatments. In the first-half of the experiment, the treatment group has a 90% switching rate in the first round, and quickly converges to switching 100% of the time within three rounds. On the other hand, the switching rate of the control group starts at 60% and only converges to around 80%. In fact, the treatment group has a higher switching rate in all 15 rounds. A naive binomial test yields a p-value < 0.0001 on a two-tail test. Moreover, a logistic regression that predicts the dummy variable *Switch* (= 1 if and only if the subject chooses to switch) using the dummy variable *Treatment* (= 1 if and only if the subject is in the treatment group), with standard error clustered at each round, also shows that the treatment effect is significantly positive ($p < 0.001$). These results show that the 100-door MHP is truly easier than the 3-door version.

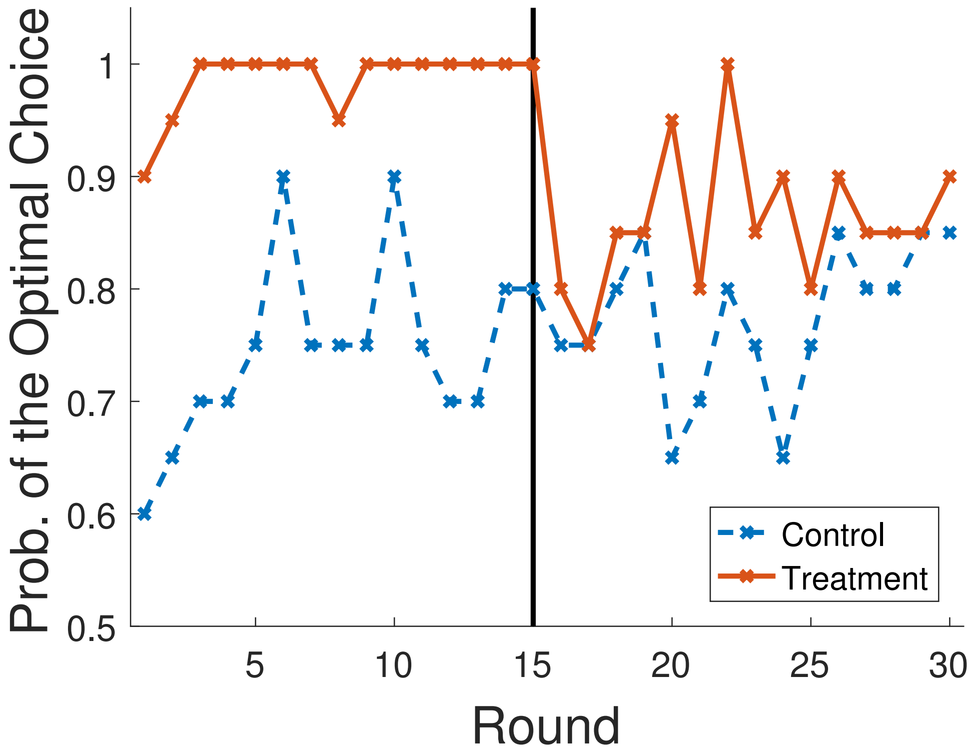

Figure 1: Round-by-round switching rates for the two treatments

In the second-half of the experiment, where both groups played the 3-door MHP, the switching rate in the treatment group is 86%, which is higher than 77% of the control group. In fact, the treatment group has a higher switching rate in 12 out of 15 rounds (equal otherwise). A naive binomial test yields a p-value of 0.018 on a one-tail test and 0.035 on a two-tail test. Moreover, a logistic regression that predicts the dummy variable *Switch* using the dummy variable *Treatment*, with standard error clustered at each round, also shows that the treatment effect is significantly positive ($p < 0.001$).

One possible explanations for our results is the effect of incentives. After all, the expected payoff of switching in the 100-door MHP is higher than that of the 3-door MHP. This difference itself could trigger learning, as high stakes may induce behavior change [Holt and Laury, 2002].

Another possible explanation is experience. In the past few decades, the MHP has became more famous due to the massive exposure in the public media, such as the movie "21" and the TV series "Brooklyn 99." Therefore, it is possible that a lot of subjects have seen the MHP before. In fact, in the questionnaire after the experiment, when asked whether they have seen games that are similar to the MHP before, seventy-five percent of our subjects report "Yes." For these subjects, even if they cannot recall that switching is optimal, they could rely on, say, small samples of past experience to deduce it.

Suppose subjects initially behave randomly, and later choose the strategy that yields the highest average payoff from a small sample of past history. Erev and Roth [2014] show that a random sample with replacement from all past history explains the empirical speed of learning in a variety of games and individual decision problems when the sample size is 5. With the same sample size of 5, one could in principle achieve a switching rate of 79.01% in the 3-door MHP, and a switching rate of 99.999% in the 100-door MHP. If our subjects follow Chen et al. [2011] and Barron and Erev [2003], in which recent outcomes are overweighted, and use the last few rounds as the small sample to evaluate each strategy, they would achieve a switching rate of 70.42% in the 3-door MHP and a switching rate of 97.92% in the 100-door MHP when the small sample has a sample size of 5.

Panel regression analysis also confirms our findings. In particular, we conducts probit regressions with random effects predicting switching behavior (the probability that the dummy variable *Switch* equals to 1) with a constant term, the round number (correlated with how many rounds the subject have played the relevant case), and two variables that represent past experience: *Switch_bonus* (the cumulative earning difference between always switching and always remaining) and *Switch_won* (a dummy variable which equals to 1 if switching would have won the prize in the most recent round). The same model was used by Friedman [1998] to show that subjects were using reinforcement learning to learn the optimal strategy. Table 1 shows that in round 16 to 30 of the treatment group, consistent with Friedman [1998], the variable *Switch* is positively correlated with *Switch_bonus* (p-value < 0.001), but not with *Switch_won* (p-value > 0.05). In other words, from the aggregate data, we find evidence of learning only in the treatment group when they were playing the 3-door MHP.

Data	Control 1-15	Control 16-30	Treatment 1-15	Treatment 16-30
	Switch (= 1 if subject switches, = 0 otherwise)			
Switch_bonus	0.002	0.001	-0.180	0.016**
	(0.004)	(0.003)	(27.140)	(0.004)
Switch_won	-0.356	0.125	-0.340	-0.370
	(0.210)	(0.192)	(1,886.646)	(0.219)
Round	0.015	0.017	1.885	-0.020
	(0.025)	(0.024)	(271.405)	(0.025)
Constant	0.791**	0.242	0.343	1.586**
	(0.239)	(0.558)	(1,981.851)	(0.605)

Standard errors in parentheses; ** $p < 0.01$, * $p < 0.05$

Table 1: Probit regression used in Friedman [1998].

5 Conclusions

We investigate whether a simplified example (the 100-door MHP) can improve learning of the optimal strategy in the MHP. We conduct a laboratory experiment to test our hypothesis and find that indeed our design could teach subjects to achieve a switching rate of 86% (compared to the 77% in the control group), which is higher than any of the previous study that had 15 repetitions or less.

However, the major limitation of this study is clearly the use of only an individual decision problem. Although past research [Dufwenberg et al., 2010, Gneezy et al., 2010] has shown similar results in the race games, it is still not clear how general our results are. In addition, it is not clear if there is a general principle for us to identify a "simplified example" for an otherwise difficult problem. Even for the MHP, we only identify one particularly successful way of teaching the subjects. Future studies should be undertaken to explore these questions. Despite these limitations, we recommend using the 100-door version as an example to teach people the optimal strategy of the MHP.

References

G. Barron and I. Erev. Small feedback-based decisions and their limited correspondence to description-based decisions. *Journal of Behavioral Decision Making*, 16(3):215–233, 2003. doi: 10.1002/bdm.443. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/bdm.443.

A. J. Brokaw and T. E. Merz. Active learning with monty hall in a game theory class. *The Journal of Economic Education*, 35(3):259–268, 2004.

W. Chen, S.-Y. Liu, C.-H. Chen, and Y.-S. Lee. Bounded memory, inertia, sampling and weighting model for market entry games. *Games*, 2(1):187–199, Mar 2011. ISSN 2073-4336. doi: 10.3390/g2010187. URL http://dx.doi.org/10.3390/g2010187.

Dufwenberg, Martin, Sundaram, Ramya, and D. J. Butler. Epiphany in the game of 21. *Journal of Economic Behavior & Organization*, 75(2):132–143, August 2010.

I. Erev and A. E. Roth. Maximization, learning, and economic behavior. *Proceedings of the National Academy of Sciences*, 111(Supplement 3):10818–10825, 2014. ISSN 0027-8424. doi: 10.1073/pnas.1402846111. URL https://www.pnas.org/content/111/Supplement_3/10818.

A. Franco-Watkins, P. Derks, and M. Dougherty. Reasoning in the monty hall problem: Examining choice behaviour and probability judgements. *Thinking & Reasoning*, 9(1):67–90, 2003.

D. Friedman. Monty hall's three doors: Construction and deconstruction of a choice anomaly. *American Economic Review*, 88(4):933–946, 1998.

U. Gneezy, A. Rustichini, and A. Vostroknutov. Experience and insight in the race game. *Journal of economic behavior & organization*, 75(2):144–155, 2010.

D. Granberg and N. Dorr. Further exploration of two-stage decision making in the monty hall dilemma. *The American journal of psychology*, 111(4):561, 1998.

C. A. Holt and S. K. Laury. Risk aversion and incentive effects. *American Economic Review*, 92(5):1644–1655, December 2002. doi: 10.1257/000282802762024700. URL http://www.aeaweb.org/articles?id=10.1257/000282802762024700.

F. Mengel. Learning across games. *Games and Economic Behavior*, 74(2):601 – 619, 2012. ISSN 0899-8256. doi: https://doi.org/10.1016/j.geb.2011.08.020. URL http://www.sciencedirect.com/science/article/pii/S0899825611001552.

S. E. Page. Let's make a deal. *Economics Letters*, 61(2):175–180, 1998.

S. Selvin, M. Bloxham, A. I. Khuri, M. Moore, R. Coleman, G. R. Bryce, J. A. Hagans, T. C. Chalmers, E. A. Maxwell, and G. N. Smith. Letters to the editor. *The American Statistician*, 29(1):67–71, 1975. ISSN 00031305. URL http://www.jstor.org/stable/2683689.

J. M. Stibel, I. E. Dror, and T. Ben-Zeev. The collapsing choice theory: Dissociating choice and judgment in decision making. *Theory and Decision*, 66(2):149–179, 2009.

E. Tubau, D. Aguilar-Lleyda, and E. D. Johnson. Reasoning and choice in the monty hall dilemma (mhd): implications for improving bayesian reasoning. *Frontiers in psychology*, 6, 2015.

M. vos Savant. *The Power of Logical Thinking: Easy Lessons in the Art of Reasoning...and Hard Facts About Its Absence in Our Lives*. St. Martin's Griffin, 1997.