Applications of the Completeness Axiom

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- The completeness axiom of real numbers is usually used to prove properties about **existences**.
- Here, we will prove the existence of absolute extreme values of a continuous function on a bounded and closed interval.

Applications of the Completeness Axiom

- Theorem:
- For any bounded sequence $\{a_n\}$, $\lim_{n\to\infty}(\sup_{i\geq n} a_i)$ and $\lim_{n\to\infty}(\inf_{i\geq n} a_i)$ exist.
- Notation:
- Denote $\lim_{n\to\infty}(\sup_{i\geq n} a_i)$ by $\limsup a_n$ and denote $\lim_{n\to\infty}(\inf_{i\geq n} a_i)$ by $\liminf a_n$.

Applications of the Completeness Axiom

- Theorem:
- For any bounded sequence $\{a_n\}$, there exists a convergent subsequence $\{a_{n_k}\}$.
- Theorem:
- A continuous function $f(x)$ on a bounded and closed interval $[a,b]$ must be bounded. Hence there are constants m, M such that $m \leq f(x) \leq M$ for all $x \in [a,b]$.

Applications of the Completeness Axiom

- Theorem:
- If $f(x)$ is continuous on a bounded and closed interval $[a,b]$, then $f(x)$ attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers $c, d \in [a,b]$.