Financial Crises and Equilibrium Uniqueness in Global Games Models of Crises *

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Abstract

We uncover a novel interaction between strategic uncertainty in coordination games of incomplete information - such as currency crises, bank runs and financial crises - and the informativeness of rational expectations equilibrium prices in financial markets with risk averse traders: when the private information of players in the coordination game is increased, the information conveyed by financial prices falls. We use this property to show that, differently from what argued by Angeletos and Werning (2006), information transmission from prices in financial markets can be consistent with the emergence of a unique equilibrium in global games of regime change, exactly when the private information of players in the game is sufficiently precise. In this sense, the original equilibrium uniqueness result of Morris and Shin (1998) for global games is robust to the introduction of endogenous information from financial markets.

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1 Introduction

It is well known that economic environments with strong coordination motives, also known as coordination games, allow for the existence of multiple equilibria. Interesting macroeconomic applications known as "regime attack games" include currency regime attacks, depositors bank runs, and financial crises more in general. In a set of influential papers Morris and Shin[1] argued that multiple equilibria are the consequence of perfect common knowledge which is usually assumed in these settings. In particular, using the global games approach pioneered by Carlsson and van Damme (1993) they show that games of incomplete information "nearby" the original complete information game admit a unique equilibrium, even at the limit as the incompleteness of private information is made arbitrarily small. Uniqueness of equilibrium is in general desirable because it makes both the positive and the normative predictions of the model a function of only the fundamentals of the economy, without having to resort to sunspot shocks or un-modeled shifts in sentiments.

The starkness of the uniqueness result sparked a literature that aimed at understanding whether uniqueness is robust to the introduction of richer and more realistic features of the economic environment. Atkeson (2000), in discussing Morris and Shin (2000), forcefully raised the concern that prices, by aggregating dispersed information, can weaken the uniqueness result and re-instate multiplicity. Indeed, in a very influential paper published in this journal, Angeletos and Werning (2006) have shown that the uniqueness result of Morris and Shin is not robust to the introduction of endogenous public information arising from the equilibrium price of a financial market for an asset whose return is a function of the outcome of the coordination game. In particular, they show that when the private information of agents is made arbitrarily precise, the price of the financial market becomes a precise signal

[1] See Morris and Shin (1998, 2000, 2003) in particular.

of the fundamentals and, being it common knowledge, reinstates the possibility of multiple equilibria, thus completely reversing the limit result of Morris and Shin (1998).[2]

In this paper we show that the result of Angeletos and Werning (2006) holds under the very special circumstance in which the players of the coordination game and the traders in the financial market are the same pool of agents. While this assumption can certainly be valid in some applications, it is generally not warranted. For instance, imagine the case of a bank that is subjected to a potential run by depositors and a financial market where credit default swaps on the liabilities of the bank are traded. It seems reasonable in this setting to assume that the pool of depositors does not coincide with the pool of traders taking positions in the financial market. It would seem in fact more appropriate to assume that the former is included in the latter and has, possibly, a negligible relative size. Similarly, consider the situation of a potential run on the short term debt of a financial institution and the derivative market on collateralized debt obligations of such institution. Once again, the pool of creditors with respect to which the institution has liabilities can be very different from the pool of traders on the derivative market on the very same liabilities.

To understand the separate role of the information precision of players in the regime attack game and traders in the financial market we modify the setup of Angeletos and Werning (2006) by allowing players and traders to be distinct. The distinction turns out to be crucial for multiplicity and uniqueness. On the one hand, when the private information of *traders* in the financial market is made arbitrarily precise, the price becomes a very informative signal about the fundamentals. Players

[2] Hellwig, Mukherji, and Tsyvinski (2006) also show that multiplicity re-emerges in currency crises settings where the interest rate is market determined. We focus on the setting of Angeletos and Werning (2006) to abstract from the specific structure of the currency market and central bank reserves policies that is critical for the results of Hellwig, Mukherji, and Tsyvinski (2006). Our analysis can, however, be applied to their setting as well.

in the coordination game observe the price, which is a public signal, and disregard their private information, as this becomes relatively noisy when the price is very informative. As a consequence, players have almost perfect common knowledge and multiple equilibria are possible. This case agrees with Angeletos and Werning (2006).

On the other hand, when the private information of *players* is made more precise, the price in the financial market becomes less informative. This perhaps surprising result originates from the interaction between the strategic uncertainty in the coordination game and the risk aversion of traders in the financial market. When their private information is very precise, players tend to discount heavily any public signal, so that each individual player takes action only based on her own private information. Because such information is private, the prediction of the aggregate behavior of players to an outside observer, i.e. a market trader, becomes more difficult as it depends sensibly on the exact value of the fundamentals and no longer on the public signal. Traders in the financial market recognize the heightened strategic uncertainty in the game and perceive the return on the asset - which is related to the outcome of the game - as riskier. Risk aversion instructs them to reduce the sensitivity of their net demand positions to their private information, but in so doing they reduce the informativeness of the equilibrium price, which in turn sustains the disregard of players for the public information coming from the price in the first place. In the limit, as the private information of players is made arbitrarily precise, the price in the financial market becomes completely uninformative and the Morris and Shin (1998) uniqueness result emerges again.

Before turning to the details of the analysis, we would like to point out that the inverse relationship between the precision of the private information of players in the regime attack game and the precision of the price in the financial market is, to the best of our knowledge, a novel result in the literature. We believe that such property is of independent interest, beyond the implications for equilibrium uniqueness that we purse in this paper.

The rest of the paper is organized as follows. Section 2 introduces the model economy and restates the uniqueness result of Morris and Shin. Section 3 considers the joint equilibrium of the regime attack game and the financial market and presents the novel result about information precision of players and prices. Section 4 shows that uniqueness is obtained once again when the private information of players is precise enough. Section 5 concludes.[3]

2 The Model

The structure of the model is similar to the framework used by Angeletos and Werning (2006). Time is static but there are two stages: a financial market stage and a regime attack game stage. There are two types of economic agents: traders, who trade at the financial market stage, and players, who take actions in the regime attack game.

Regime Attack Game. The individual players in the regime attack game are denoted by i and are distributed on the unit interval, $i \in [0,1]$. Player i's action a_i can be either attack the status-quo ($a_i = 1$) or not attack ($a_i = 0$). The mass of agents attacking the status-quo is given by $A \equiv \int_0^1 a_i di$, with $A \in [0,1]$, and it is referred to as the aggregate action of players. The status-quo is abandoned if the aggregate action is greater than $\theta \in \mathbb{R}$, a variable that captures the fundamental strength of the regime's status-quo.

Under perfect and common knowledge of the fundamental θ the regime attack game has always two equilibria whenever $\theta \in (0,1)$: nobody attacks ($A = 0$), and

[3] In the main text we state the results that are strictly necessary to convey our argument. We provide a complete description of the framework and the detailed proofs of our results in the Online Appendix section.

everybody attacks ($A = 1$). It is assumed that information about θ is imperfect. Before the game is played nature draws θ from a distribution that is assumed to be players' common prior about θ. The distribution is assumed to be an improper uniform over $\mathbb{R}$. Player i receives two signals about θ. The first signal is a private signal x given by

$$x = \theta + \sigma_x \varepsilon_x, \tag{2.1}$$

where $\sigma_x > 0$ and $\varepsilon_x \sim \mathcal{N}(0,1)$ and i.i.d. across players. In what follows we will make frequent use of the notion of the precision of a signal, which should be thought of being the squared inverse of the noise of a signal, σ_x^{-2} in this case. In addition to the private signal, players also receive a public signal p, which is common knowledge across players, and it is given by

$$p = \theta - \sigma_p \varepsilon, \tag{2.2}$$

where $\sigma_p > 0$ and $\varepsilon \sim \mathcal{N}(0,1)$.

Morris-Shin Equilibrium Analysis. As in most of the literature on global games of regime change we consider only equilibria that belong to the class of symmetric monotone equilibria. Monotone equilibria are perfect Bayesian equilibria of the incomplete information game where the optimal strategy of the players conditional on the realization of the public signal p is to attack if and only if their private signal x is below a threshold $x^*(p)$. The equilibrium is symmetric when the threshold is the same across all players. In a monotone equilibrium with threshold $x^*(p)$ the mass of players attacking the regime is

$$A(\theta, p) = \Pr\big(x < x^*(p) |\ \theta\ \big) = \Phi\big(\sigma_x^{-1}(x^*(p) - \theta)\big), \tag{2.3}$$

where Φ is the cumulative distribution function of the standard normal. The attack is successful in changing the regime from the status-quo if and only if the strength of the regime θ is smaller than the threshold $\theta^*(p)$, where the threshold is the value of the regime strength that makes the attack just enough to change the status quo, formally $A\big(\theta, p\big) = \theta^*(p)$.

Proposition 1 (Morris-Shin)**.** *For any realization of the public signal p, the regime attack game has a unique equilibrium $\{\theta^*(p), x^*(p)\}$ if and only if $0 < \sigma_x \leq \sigma_p^2\sqrt{2\pi}$. In the limit as $\sigma_x \to 0$ the unique equilibrium regime outcome $A(\theta, p)$ does not depend on p.*

To understand how incentives and information interact to shape the uniqueness result it is useful to separate the uncertainty affecting the choice of individual agents in the regime attack game into two types. On the one hand, there is the uncertainty about θ, the *fundamental* uncertainty, which is reduced as either signal x or p become more precise. On the other hand, there is the uncertainty about what other players are going to do, the *strategic* uncertainty, which matters because the individual payoff from attacking is a function of the aggregate action. Strategic uncertainty is reduced when p becomes more precise relative to x: when the public signal is more informative it will receive more weight in the signal extraction problem and, being this common knowledge, everybody's action will become more predictable from the individual player point of view. Strategic uncertainty is instead increased when x becomes more precise relative to p: when the private signal is more informative it will receive more weight in the signal extraction problem, but being such signal not common knowledge, other players' actions will become less predictable for the individual player. In the limit, as the private signal is arbitrarily precise each player will rely only on such signal, completely disregarding the public signal, making strategic uncertainty maximal.[4]

We conclude this section by formulating a thought exercise which will be useful later in the paper. Suppose that in the equilibrium of Proposition 1 there is an outside observer of the attack game that would like to make a prediction about the final regime outcome conditional on the public signal p. How would the precision of such prediction change with the precision of the private information of the players in the game? The public signal p is especially useful in formulating the outcome prediction for an outside observer insofar as it is used as a signal by players in the game. We know from Proposition 1 that the signal p becomes less relevant for players when the precision of their private signal is increased. It follows that precisely when players become privately more informed, the accuracy of the outcome prediction of an outside observer deteriorates! This mechanism is magnified when the signal p is generated by a financial market with risk averse traders, to which analysis we now turn.

Financial Market. We take the modeling of the financial market from Angeletos and Werning (2006) which is a version of Grossman and Stiglitz (1980). The traders in the model participate to the market for an asset that pays an uncertain dividend. Traders are endowed with initial wealth w_0 and decide how much to invest into a risky asset with price q and dividend f. The utility of the trader j is $V(w_j) = -e^{-\gamma w_j}$ with $w_j = w_0 + (f - q)k_j^d$, where k_j^d is the net demand for the asset from trader j. The total demand for the asset is obtained by aggregating the individual demands of traders so that $K^d(\theta, q) = \int_0^1 k_j^d dj$. The supply of the asset is assumed to be uncertain and not observable and is given by $K^s(\varepsilon) = \sigma_\varepsilon \varepsilon$, where

[4] To measure strategic uncertainty one can evaluate the beliefs of an individual agent about the fraction of the other agents that are believed to attack the regime, i.e. that choose $a_i = 1$, rather than $a_i = 0$. It is possible to show that when private information becomes arbitrarily precise, the beliefs of the marginal agent become uniform over the unit interval, implying that the uncertainty about the fraction of agents that will attack is maximal. Morris and Shin (2003) refer to this case as one of "Laplacian" beliefs.

$\varepsilon \sim \mathcal{N}(0,1)$ and $\sigma_\varepsilon > 0$. This last assumption ensures that the equilibrium price is not perfectly informative about the fundamental θ so that a rational expectations equilibrium always exists. In setting their demand for the asset, traders condition on the equilibrium price q and on a private signal y that is given by

$$y = \theta + \sigma_y \varepsilon_y \tag{2.4}$$

where $\sigma_y > 0$, $\varepsilon_y \sim N(0,1)$ and i.i.d. across traders. The individual asset demand for a trader takes the usual mean-variance form

$$k(y,q) = \frac{\mathbb{E}[f|y,q] - q}{\gamma \mathbb{V}[f|y,q]}. \tag{2.5}$$

where $\mathbb{E}(\cdot|\cdot)$ and $\mathbb{V}(\cdot|\cdot)$ are, respectively, the conditional expectation and the conditional variance functions. The trader has a positive demand for the asset when the expected dividend is higher than the price of the asset. When the opposite is true, the demand is negative and the trader sells the asset.[5] The variance term affects the sensitivity of the net demand of the trader to the size of the difference between the expected dividend and the price. When the perceived variance of the dividend is high, the risk averse trader will take relatively smaller positions on the asset, being them positive or negative.

The equilibrium in the financial market is finally given by the price $q(\theta, \varepsilon)$ that ensures that the aggregate net demand is equal to the aggregate net supply, i.e. $K^d(\theta, q) = \sigma_\varepsilon \varepsilon$. The price of the asset q affects the individual demand through two channels. The first channel is the direct substitution effect due to a higher cost of purchasing the asset: a higher price will reduce the net demand for the asset, and eventually turn it into a negative demand. The second channel is an

[5] To keep things simple we follow Angeletos and Werning (2006) and abstract completely from issues of borrowing constraint or limits to the short selling of an asset.

information aggregation effect: the price is a signal about the market beliefs about the dividend and as a signal it affects the conditional expectation of the traders and thus their demand, creating the information equilibrium feedback typical of rational expectations.

3 Equilibrium and Information Precisions

In this section we study the link between the financial market stage and the regime attack stage when the price of the asset in the financial market is a public signal for the regime attack game. To ensure that the price is potentially informative it is necessary to assume that the dividend of the asset is a function of the fundamental θ. We assume that the dividend paid by the asset is a function of the aggregate action at the regime attack game stage, which means that the mapping between dividends and fundamentals is itself an equilibrium object.[6] We follow Angeletos and Werning (2006) and specify $f = f(A) = -\Phi^{-1}\big(A\big)$ so that the dividend is decreasing in the mass of players that attack the status-quo and the normality of the information structure is preserved.[7]

To ensure tractability of the equilibrium signal extraction problem it is convenient to operate with signals that are linear functions of the fundamental θ and the noise. Since the aggregate action is $A(\theta, q) = \Phi\big(\sigma_x^{-1}\big(\theta - x^*(q)\big)\big)$, the dividend becomes $f = \sigma_x^{-1}\big(\theta - x^*(q)\big)$. We consider equilibria where the observation of the

[6] The specific form taken by the function that relates dividends to fundamental plays a very important role in the effect of information precisions on equilibrium uniqueness. One possibility is to assume that the function is exogenous to the equilibrium of the regime attack game and so anything that happens at the game stage does not feed back into the financial market equilibrium. In particular, whether the precision of the private information of the players is high or low is irrelevant for the precision of the price in the financial market. This case is studied in Angeletos and Werning (2006) as propaedeutic to the more realistic and interesting case of a two-way feedback, which we consider here.

[7] This could capture, for example, the case of a credit default swap contract on liabilities of a bank that may be subjected to a run: the stronger the size of the withdrawing depositors, the lower the probability that the bank will be able to meet her liabilities, the higher the payout due on the swap by the counterpart holding the default risk event.

price q is equivalent to the observation of the signal p where

$$p = \mathcal{P}\big(q\big) \equiv q\sigma_x + x^*(q), \tag{3.1}$$

which means that the function $\mathcal{P}$ relating q to p is a one-to-one mapping. With this transformation, while the equilibrium price $q(\theta, \varepsilon)$ is a nonlinear function, the transformation p is linear and is given by

$$p = \theta - \sigma_p \varepsilon. \tag{3.2}$$

The following proposition characterizes the precision of the signal p in equilibrium.

Proposition 2. *In a symmetric monotone perfect Bayesian equilibrium of the financial market and regime attack economy, the noise of the price in the financial market σ_p is given by*

$$\sigma_p = \gamma \sigma_\varepsilon \frac{\sigma_y^2}{\sigma_x}. \tag{3.3}$$

The precision of the public signal p generated by the financial market equilibrium is increasing in the precision of the market traders' private signal y, and decreasing in the precision of the regime attack game players' private signal x.

Proof. See the Online Appendix $\square$

The intuition for the result can be obtained by considering the equilibrium expression for the asset dividend $f = \sigma_x^{-1}(\theta - x^*(q))$ together with the individual trader demand for the financial asset

$$k(y, q) = \frac{\sigma_x}{\gamma \sigma_y^2}\big(y - p\big). \tag{3.4}$$

Since the individual trader observes q, any uncertainty about the dividend f is related to the term $\sigma_x^{-1}\theta$. Holding fixed σ_x, an increase in the precision of the signal y increases the precision with which the trader can predict dividends and so traders' individual demands will be more responsive to the signal y and, in turn, the market demand will be more responsive to the fundamental θ. In equilibrium the asset price will be more responsive to the fundamental and thus convey more information. Consider now the case where σ_y is held fixed and σ_x is decreased. An increase in the precision of the private signal for the attack game players increases the strategic uncertainty around the game and reduces the ability to predict the game outcome. The financial market traders look at the attack game as outside observers that try to predict the game outcome by conditioning on p and y, and taking into account the precision of x for the behavior of players in the game. As a result, an increase in strategic uncertainty corresponds to an increase in the perceived riskiness of the financial asset return by traders. Because traders are risk averse their demand will become less sensitive to the expected return, which will make the aggregate demand less sensitive to the fundamental θ, and thus reduce the information about θ conveyed by the financial asset price in equilibrium.

The novel insight offered by Proposition 2 is that traders in the financial market respond to the increase in the strategic uncertainty in the regime attack game as to an increase in the intrinsic riskiness of the asset return. Interestingly, a decrease in σ_x has the same effect on σ_p as an increase in the risk aversion parameter γ, or the noise in the asset supply σ_ε. In this sense, the financial market, because of risk aversion, prices the uncertainty of players about each others' actions as they discard the use of the public signal in favor of their private signal, thereby making the aggregate action less predictable. Such pricing reduces the informativeness of the public signal further, reinforcing the loss in precision of the public signal that increased the strategic uncertainty in the first place. Equation (3.3) characterizes the resting point of such self-fulfilling mechanism.

The intriguing equilibrium relationship between strategic uncertainty and the informativeness of prices in financial markets that is so starkly isolated in Proposition 2 is bound to extend to settings beyond the special framework considered in this paper, and it suggests a rich arrays of possible applications for the studying of crises and prices in financial markets.[8]

4 Information Precisions and Uniqueness

In this section we use Proposition 2 to study the robustness of the uniqueness result of Morris and Shin (1998) with respect to the introduction of endogenous public signals of the type considered by Angeletos and Werning (2006).

Angeletos and Werning. The multiplicity result in Angeletos and Werning (2006) can be obtained under a special assumption about the relationship between the precision of the private signals of the traders and the players in our setup.

Proposition 3 (Angeletos-Werning)**.** *Suppose that the noise in the private signals of the traders and of the players in the economy coincide, i.e.* $\sigma_y = \sigma_x$*, then, for any given realization of the endogenous financial market public signal* p*, the regime attack game has multiple equilibria if* $\sigma_x < 1/(\sigma_\varepsilon^2 \gamma^2 \sqrt{2\pi})$*. Taking the limit* $\sigma_x \to 0$ *the equilibria of the attack game converge to the equilibria under perfect common knowledge.*

The key to multiplicity in the attack game equilibrium is the multiplicity in the price function $q(\theta, \varepsilon)$. In the financial asset market multiple equilibria in q are possible because the aggregate demand can be increasing in the asset price for intermediate values of such price. The reason for this is that, in presence of a dividend that is a function of the outcome of the attack game, the asset price affects

[8] Two directions in which the analysis could be extended are the modeling of the choice of information precision by both players and traders and the modeling of the choice of participation to the financial market by heterogeneously informed agents. We take up such tasks in Rondina and Shim (2012).

the demand through a third channel, in addition to the substitution and information channels already discussed in Section 2. Such channel, which Angeletos and Werning (2006) term the price coordination channel, can be easily seen by looking at the dividend expression $f = \sigma_x^{-1}(\theta - x^*(q))$, which is increasing in q since the private signal threshold $x^*(q)$ is decreasing in q. A higher asset price might signal that the fundamental value θ is high, which makes the players attack the regime only if they observe a very low private signal. Therefore, a higher q increases the expected return of the asset and can increase the asset demand. For very high and very low values of q the substitution effect still dominates and so the demand remains downward sloping. Together with the continuity of the gaussian setup, the non-monotonic behavior of the demand implies multiple price equilibria. When multiple financial asset prices are possible, there is always a "most aggressive" price $\underline{q}$ and a "least aggressive" price $\overline{q}$. When $\underline{q}$ clears the market the associated private signal threshold $x^*(\underline{q})$ is the highest, which means that the mass of attacking agents is the biggest. Conversely, when $\overline{q}$ clears the market the associated private signal threshold $x^*(\overline{q})$ is the lowest, which results in the mass of attacking agents being the smallest. As the noise in the private signal σ_x is taken arbitrarily close to zero, under the most aggressive price equilibrium the probability of the regime collapsing converges to 1, while under the least aggressive price the same probability converges to 0.

We finally notice that from an informational point of view, the multiplicity in the financial market price q does not result in the multiplicity of the public signal p. In fact it can be showed (see the Online Appendix) that $\mathcal{P}(\underline{q}) = \mathcal{P}(\overline{q}) = p$ where p takes the form (2.2) with σ_p characterized by Proposition 2 under the assumption that $\sigma_x = \sigma_y$. When the precision of the signal p is high enough, the public signal serves as a coordination device that makes both an aggressive and a lenient equilibrium simultaneously possible in the regime change game, hence the multiplicity.

With the help of Figure 1 we can summarize the results of both Proposition 1 and

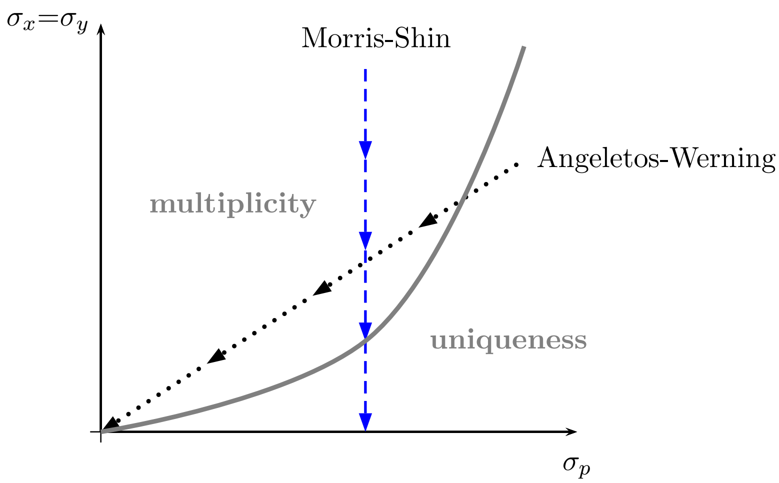

Figure 1: Information Precision, Uniqueness and Multiplicity

Proposition 3. The gray line represents the demarcation between the combinations of (σ_x, σ_p) that ensure multiplicity (above the line) and uniqueness (below the line). For any value of $\sigma_p > 0$ there is a σ_x such that multiple equilibria are possible. For the case considered by Morris and Shin (1998), σ_x and σ_p move independently. In particular, as the noise in private information is reduced (downward pointing dashed blue line), one eventually enters the uniqueness region and a unique equilibrium is obtained. In the case considered by Angeletos and Werning (2006), on the other hand, σ_x and σ_p are interrelated because of the financial market and the additional assumption that $\sigma_y = \sigma_x$. As the noise in private information is reduced, the noise in the public signal is reduced as well according to the relationship $\sigma_p = \gamma\sigma_\varepsilon\sigma_x$, which is represented as the straight dotted black line. The line eventually enters the multiplicity region and it remains in that region as the noise gets arbitrarily close to zero.

Traders and Players Information Precisions

There is no a-priori reason for the precision in the private information of traders and players to be equal and move one-by-one. For example, if one were to endogenize the choice of information precision of players and traders, they would respond to different types of incentives as they participate in different types of markets and thereby choose different precisions.[9] As a consequence, the equilibrium uniqueness robustness exercise should be undertaken while keeping the information precisions of traders and players as distinct and independent from one another. Once this is done, the relationship between information precisions and uniqueness of the equilibrium becomes much richer than the one summarized by Figure 1. The following proposition builds on Proposition 2 and characterizes the robustness of a unique equilibrium in the regime attack game to the introduction of information from financial markets.

Proposition 4. *For any given realization of the endogenous financial market public signal p, the regime attack game has a unique equilibrium if and only if $\sigma_y^4/\sigma_x^3 > 1/(\sigma_\varepsilon^2\gamma^2\sqrt{2\pi})$. Taking the limit $\sigma_x \to 0$ while $\sigma_y > 0$ the equilibrium of the regime attack game is unique and converges the limit equilibrium of Proposition 1. Taking the limit $\sigma_y \to 0$ while $\sigma_x > 0$ the equilibria of the regime attack game are multiple and converge to the limit equilibria of Proposition 3.*

Proposition 4 shows that a unique equilibrium in the regime attack game can be robust to the introduction of information from financial markets as long as the precision of the private information of players is sufficiently high. In particular, uniqueness can still result even when the precision of players' private information is smaller than the precision of traders' private information, as long as the noise in the asset supply σ_ε is high enough, or the risk aversion of market traders γ is strong enough.

The key to uniqueness in the regime attack game is the uniqueness of the equilibrium price in the financial market. When the private information of players in the attack game is very noisy, any movement in the asset price q acts as a pow-

[9] For a model of endogenous informational choice with such features see Rondina and Shim (2012).

erful coordination device across players and it affects the individual strategies of the players through the private signal threshold $x^*(q)$. Conditional on q the asset return is more predictable because players attach little relevance to their private signals. The market demand for the asset can therefore display the non-monotonic behavior described earlier as the price of the asset goes from low to high. Suppose now that the private information of players is made more precise, everything else equal. The increase in the use of the private signal by players reduces the relevance of the asset price in the individual threshold strategies. This in turn reduces the coordination channel in the dividend return, and the market asset demand eventually becomes monotonic, providing only one market clearing price, which is, by equilibrium result, very uninformative. A critical role in making the financial market price uninformative is played by the risk averse traders, who, in presence of a heightened strategic uncertainty, reduce their sensitivity to both the public and private signal, subtracting even further information from the financial market price. As already remarked, this is the novel effect identified in Proposition 2, and it is the key economic explanation behind Proposition 4.

A further significant implication of the proposition is that the limit uniqueness result of global games of Morris and Shin (1998) is robust to the introduction of endogenous public information via financial markets. In the limit, as the noise in the private signal σ_x approaches zero, the asset demands of traders become unresponsive to both the price and the traders' private signals! This is the magnification of strategic uncertainty operated by the financial market, and it results in the equilibrium price carrying no information about the fundamentals. This limit is equivalent to the limit of the Morris-Shin setup, and so the discontinuity between the limit of the incomplete information game and the common knowledge game, which disappeared in the limit considered by Angeletos and Werning (2006), re-emerges, once again.

We conclude by graphing the results of proposition 4 in Figure 2. The gray line

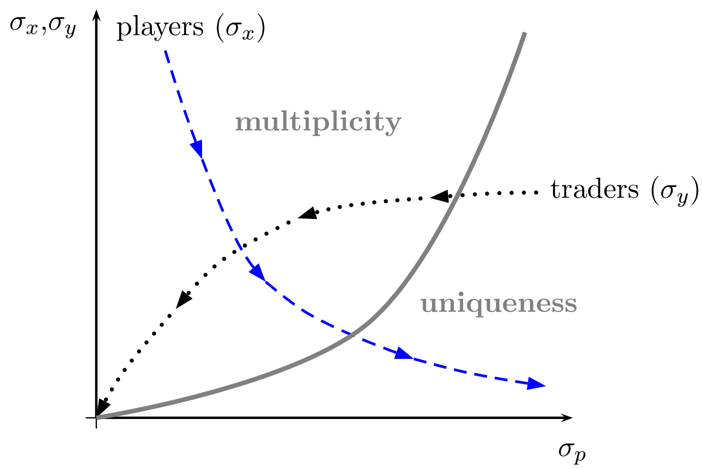

Figure 2: Players and Traders Information Precisions and Uniqueness

still parts the space into the multiplicity and uniqueness areas. The dashed blue line termed "players" represents the relationship between the noise in the private information of the regime attack game players and the noise in the financial market equilibrium price. Suppose we start from a point where the noise σ_x is high, while the noise in the endogenous public signal is low, so that multiplicity is possible. As the noise σ_x is reduced the public signal becomes less and less informative, as per Proposition 2, until the line finally enters the uniqueness region. The figure also reports the relationship between the noise in the private signals of traders, σ_y, and the noise in the endogenous public signal, the dotted line termed "traders". In this case, an increase in the precision of the signals of traders always reduces the noise in the endogenous public signal and so the line eventually enters in the multiplicity area.[10]

[10]The line that demarcates the multiplicity and uniqueness areas for the traders' information case would not correspond to the line that demarcates the analogue areas for the players' information case. This is due to the fact that the demarcation line is positioned depending on the value of σ_x as σ_y is changed, and viceversa. We report only one demarcation line for easiness of exposition. As the noise in the public signal becomes negligible when $\sigma_y \to 0$ for any value $\sigma_x > 0$, the line "traders" does eventually enter the appropriate multiplicity region.

5 Conclusion

We are very much sympathetic with the idea first raised by Atkeson (2000) and then elegantly formalized by Angeletos and Werning (2006) that financial market prices, being public information, are a powerful source of coordination that can undermine the unique equilibrium prediction of global games in macroeconomic settings. However, in light of the results presented in this paper, we argue that the way information precisions of distinct agents in the economy affects the information transmitted by prices, and as such it affects the uniqueness of the equilibrium outcome, depends strongly on the specific incentives and the particular role played by the agents in the model. Our results then call for a more sophisticated treatment of the interaction across information precisions of diverse agents in models of coordination games in presence of endogenous information from financial markets.

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A Online Appendix (not for publication)

Full Description of the Model

The individual players in the regime attack game are denoted by i and are distributed on a unit interval, $i \in [0,1]$. Player i's action a_i can be either attack the status-quo ($a_i = 1$) or not attack($a_i = 0$). The payoff from attacking is $1-c$ if the attack succeeds and is $-c$ otherwise, where $c \in (0,1)$ is the cost of attacking. The mass of agents attacking the status-quo is given by $A \equiv \int a_i di$, with $A \in [0,1]$, and it is referred to as the aggregate action of players. The status-quo is abandoned if the aggregate action is greater than θ, a variable that captures the fundamental strength of the regime's status-quo. The payoff of player i is therefore given by

$$U(a_i, A, \theta) = a_i \left(\mathbf{1}_{A>\theta} - c\right) \tag{A.1}$$

Note that this payoff structure assumes that the payoff from not attacking is normalized to zero. While this feature is not relevant for equilibrium behavior, which is the focus of this paper, it can be very important for welfare analysis [11]. In what follows we define the notation $\alpha_x \equiv \sigma_x^{-2}$, $\alpha_y \equiv \sigma_y^{-2}$ and $\alpha_p \equiv \sigma_p^{-2}$. In a monotone equilibrium with threshold $x^*(p)$ the mass of players attacking the regime is

$$A(\theta, p) = \Pr\big(x < x^*(p) | \; \theta \;\big) = \Phi\big(\sigma_x^{-1}(x^*(p) - \theta)\big), \tag{A.2}$$

where Φ is the cumulative distribution function of the standard normal. The attack is successful in changing the regime from the status-quo if and only if the strength of the regime θ is smaller than the threshold $\theta^*(p)$, where the threshold is the value of the regime strength that makes the attack just enough to change the status quo,

[11] See Morris and Shin (2000) and Angeletos and Werning (2006) for useful discussions about the structure of the regime attack game.

formally

$$\theta^*(p) : A\big(\theta, p\big) = \theta. \tag{A.3}$$

PROOF OF PROPOSITION 1

Proposition A1 (Morris-Shin)**.** *For any realization of the public signal p, the regime attack game has a unique equilibrium $\{\theta^*(p), x^*(p)\}$ if and only if $0 < \sigma_x \leq \sigma_p^2\sqrt{2\pi}$. In the limit as $\sigma_x \to 0$ the unique equilibrium regime outcome $A(\theta, p)$ does not depend on p.*

Using (A.2) the relationship between the private signal threshold and the fundamental threshold can be written as

$$x^*(p) = \theta^*(p) + \frac{1}{\sqrt{\alpha_x}}\Phi^{-1}\big(\theta^*(p)\big). \tag{A.4}$$

The threshold $x^*(p)$ is defined as the private signal realization that makes the player indifferent between attacking or not attacking when the probability of the regime change is given by $\Pr\big(\theta \leq \theta^*(p)|\ x, p\ \big)$; the marginal player is indifferent when this probability is equal to the cost of attacking c. The posteriors of θ conditional on x and p are

$$\theta|x, p \sim \mathcal{N}\left(\frac{\alpha_x}{\alpha_x + \alpha_p}x + \frac{\alpha_p}{\alpha_x + \alpha_p}p, \frac{1}{\alpha_x + \alpha_p}\right). \tag{A.5}$$

It follows that the indifference condition can be written as

$$\Phi\left(\sqrt{\alpha_x + \alpha_p}\left(\theta^*(p) - \frac{\alpha_x}{\alpha_x + \alpha_p}x^*(p) - \frac{\alpha_p}{\alpha_x + \alpha_p}p\right)\right) = c. \tag{A.6}$$

A solution $x^*(p), \theta^*(p)$ to (A.4) and (A.6) is a perfect Bayesian equilibrium of the incomplete information game. Substituting (A.4) into (A.6) one obtains one equation for $\theta^*(p)$ of the form

$$\sqrt{\alpha_x}\Phi^{-1}\big(\theta^*\big) - \alpha_p\theta^* = \sqrt{\alpha_x + \alpha_p}\Phi^{-1}(1-c) - \alpha_p p. \tag{A.7}$$

This equation always has a solution, and using the properties of the Gaussian density function one can show that such solution is unique for every p if and only if $\alpha_p/\sqrt{\alpha_x} \leq \sqrt{2\pi}$. The limit result also immediately follows.

Financial Market and Regime Attack Game Equilibrium Definition

The equilibrium of the financial market and the regime attack game is defined as follows.

Definition. *A symmetric monotone perfect Bayesian equilibrium of the financial market and regime attack economy is a price function $q(\theta, \varepsilon)$, a one-to-one mapping function $\mathcal{P}$, an aggregate asset demand function $K^d(\theta, q)$ and an aggregate attack action function $A(\theta, q)$ such that*

(i) given the price realization q, the public signal $p = \mathcal{P}(q)$ and the private signal y, individual traders maximize their expected utility

$$k(y,q) \in \arg\max_{k\in\mathbb{R}} \mathbb{E}\big[V\big(w_0 + \big(f(A) - q\big)k\big) \mid y, p\big], \tag{A.8}$$

(ii) aggregate asset demand is

$$K^d(\theta, q) = \int_{\mathbb{R}} k(y,q)d\Phi(\varepsilon_y), \tag{A.9}$$

(iii) the asset market clears

$$K^d(\theta, q) = \sigma_\varepsilon \varepsilon. \tag{A.10}$$

(iv) Given the price realization q, the public signal $p = \mathcal{P}(q)$ and the private signal

x, individual players maximize their expected utility

$$a(x,q) \in \arg\max_{a\in\{0,1\}} \mathbb{E}\big[U\big(a,A,\theta\big) \mid x,p\big], \tag{A.11}$$

(v) and the aggregate attack action is

$$A(\theta,q) = \int_{\mathbb{R}} a(x,q) d\Phi(\varepsilon_x). \tag{A.12}$$

To solve for an equilibrium it is then sufficient to characterize the thresholds $\theta^*(q)$ and $x^*(q)$, together with the precision of the signal p, which is given by $\alpha_p = \sigma_p^{-2}$, under the guess, to be verified, that the mapping $\mathcal{P}(q)$ results into (3.2) when the equilibrium price function $q(\theta,\varepsilon)$ is used.

Proof of Proposition 2

The proof of Proposition 2 is obtained by proving the following result.

Proposition A2. *A collection $\{\theta^*(q), x^*(q), \alpha_p\}$ is a symmetric monotone perfect Bayesian equilibrium of the financial market and regime attack economy if it satisfies the following conditions:*

(a) regime fundamental threshold

$$\theta^*(q) = \Phi\left(\sqrt{\frac{\alpha_x}{\alpha_x+\alpha_p}}\Phi^{-1}(1-c) - \frac{\alpha_p}{\alpha_x+\alpha_p}q\right), \tag{A.13}$$

(b) individual signal threshold

$$x^*(q) = \theta^*(q) + \frac{1}{\sqrt{\alpha_x}}\Phi^{-1}\left(\theta^*(q)\right), \tag{A.14}$$

(c) financial market price precision

$$\alpha_p = \frac{\alpha_y^2}{\gamma^2\sigma_\varepsilon^2\alpha_x}. \tag{A.15}$$

Proof. The individual threshold $x^*(q)$ and the fundamental threshold $\theta^*(q)$ are obtained from (A.4) and (A.6), where now p is specified as $\mathcal{P}(q)$. To see this first notice that (A.4) must hold if we substitute p for q, once the functional form $\mathcal{P}(q)$ is embedded into the functions $x^*(\cdot)$ and $\theta^*(\cdot)$, which results in (A.14). The same logic can be applied to (A.6), but now we would need to specify explicitly the function $\mathcal{P}(q)$ for the last term on the left hand side, this gives

$$\Phi\left(\sqrt{\frac{\alpha_x+\alpha_p}{\alpha_x}}\left(\theta^*(q)-x^*(q)-\frac{\alpha_p}{\alpha_x+\alpha_p}q\right)\right)=c. \tag{A.16}$$

Using the property of the standard normal that $1-\Phi(z)=\Phi(-z)$ one obtains (A.13). For the financial market equilibrium we guess that the signal $p=\mathcal{P}(q)$ takes the linear form $p=\theta-\frac{1}{\sqrt{\alpha_p}}\varepsilon$. If this is the case then the posterior of θ conditional on y and p is normally distributed with mean $\frac{\alpha_y}{\alpha_y+\alpha_p}y+\frac{\alpha_p}{\alpha_y+\alpha_p}p$ and variance $\frac{1}{\alpha_y+\alpha_p}$. In the attack game equilibrium the dividend of the asset is $f=\sqrt{\alpha_x}(\theta-x^*(q))$, which results in the individual demand

$$k(y,q)=\frac{\alpha_y}{\gamma\sqrt{\alpha_x}}\big(y-p\big). \tag{A.17}$$

The aggregate demand is then given by

$$K(\theta,q)=\frac{\alpha_y}{\gamma\sqrt{\alpha_x}}\big(\theta-p\big). \tag{A.18}$$

Imposing market clearing and solving for α_p one obtains (3.3). □

Proof of Proposition 3

Proposition A3 (Angeletos-Werning)**.** *Suppose that the noise in the private signals of the traders and of the players in the economy coincide, i.e.* $\sigma_y=\sigma_x$*, then, for any given realization of the endogenous financial market public signal* p*, the regime* *attack game has multiple equilibria if and only if $\sigma_x < 1/(\sigma_\varepsilon^2\gamma^2\sqrt{2\pi})$. Multiplicity emerges in the price function $q(\theta,\varepsilon)$ and carries over to the equilibrium thresholds $\theta^*(q)$ and $x^*(q)$. Taking the limit $\sigma_x \to 0$ the equilibria of the attack game converge to the equilibria under common knowledge ($\sigma_x = 0$).*

The key to multiplicity in the attack game equilibrium is the multiplicity in the price function $q(\theta,\varepsilon)$. Together with the continuity of the gaussian setup, the behavior of the demand makes multiple price equilibria possible if there is an upward sloping portion of the demand. Formally, the sign of the response of the aggregate demand to the asset price is given by

$$\operatorname{sign}\left(\frac{\partial K^d(\theta,q)}{\partial q}\right) = -\operatorname{sign}\left(\frac{\sqrt{\alpha_x}}{\alpha_p} - \phi\big(\Phi^{-1}(\theta^*(q))\big)\right) \tag{A.19}$$

where $\phi(\cdot)$ denotes the density of the standard normal. It follows that the aggregate demand is non-monotone if and only if $\frac{\sqrt{\alpha_x}}{\alpha_p} < \frac{1}{\sqrt{2\pi}}$. Using the equilibrium expression for the precision of the public signal when $\sigma_y = \sigma_x$, which is $\alpha_p = \frac{\alpha_x}{\gamma^2\sigma_\varepsilon^2}$, the non-monotonicity condition can be written as $\sigma_x < 1/(\gamma^2\sigma_\varepsilon^2\sqrt{2\pi})$, which proves the proposition.

PROOF THAT $\mathcal{P}(\underline{q}) = \mathcal{P}(\bar{q}) = p$.

As in the text, let $\bar{p} = \sigma_x\bar{q} + x^*(\bar{q})$ and $\underline{p} = \sigma_x\underline{q} + x^*(\underline{q})$. By assumption both $\bar{p}$ and $\underline{p}$ are linear in θ and ε and are given by $\bar{p} = \theta - \bar{\sigma}_p\varepsilon$ and $\underline{p} = \theta - \underline{\sigma}_p\varepsilon$. To prove the result we just need to show that $\bar{\sigma}_p = \underline{\sigma}_p$. Consider the high price, $\bar{q}$. Since the corresponding public signal is $\bar{p}$ and it has a normal distribution, the conditional distribution of θ given y and $\bar{p}$ is

$$\theta|y,\bar{p} \sim \mathcal{N}\left(\frac{\alpha_y}{\alpha_y+\bar{\alpha}_p}y + \frac{\bar{\alpha}_p}{\alpha_y+\bar{\alpha}_p}\bar{p}, \frac{1}{\alpha_y+\bar{\alpha}_p}\right) \tag{A.20}$$

The individual asset demand is thus given by $k(y,\bar{q}) = \frac{\alpha_y}{\gamma\sqrt{\alpha_x}}(y-\bar{p})$ which results in the aggregate market demand $\mathcal{K}^d(\theta, \bar{q}) = \frac{\alpha_y}{\gamma\sqrt{\alpha_x}}(\theta - \bar{p})$. Applying the market clearing condition yields $\bar{p} = \theta - \frac{\sigma_\varepsilon \gamma \sqrt{\alpha_x}}{\alpha_y}\varepsilon$. Proceeding in exactly the same way but using the low price $\underline{q}$ one can similarly show that $\underline{p} = \theta - \frac{\sigma_\varepsilon \gamma \sqrt{\alpha_x}}{\alpha_y}\varepsilon$, which proves the result.

Proof of Proposition 4

Proposition A4. *For any given realization of the endogenous financial market public signal p, the regime attack game has a unique equilibrium if and only if $\sigma_y^4/\sigma_x^3 > 1/(\sigma_\varepsilon^2\gamma^2\sqrt{2\pi})$. Taking the limit $\sigma_x \to 0$ while $\sigma_y > 0$ the equilibrium of the regime attack game is unique and converges the limit equilibrium of Proposition 1. Taking the limit $\sigma_y \to 0$ while $\sigma_x > 0$ the equilibria of the regime attack game are multiple and converge to the limit equilibria of Proposition 3.*

The proof of the proposition follows the same lines as the proof of proposition 3, where the condition for a non-monotonic asset demand is still given by $\frac{\sqrt{\alpha_x}}{\alpha_p} < \frac{1}{\sqrt{2\pi}}$ but now $\alpha_p = \frac{\alpha_y^2}{\gamma^2\sigma_\varepsilon^2\alpha_x}$. Straightforward algebra leads to the final result.