Riskier Compound Lotteries

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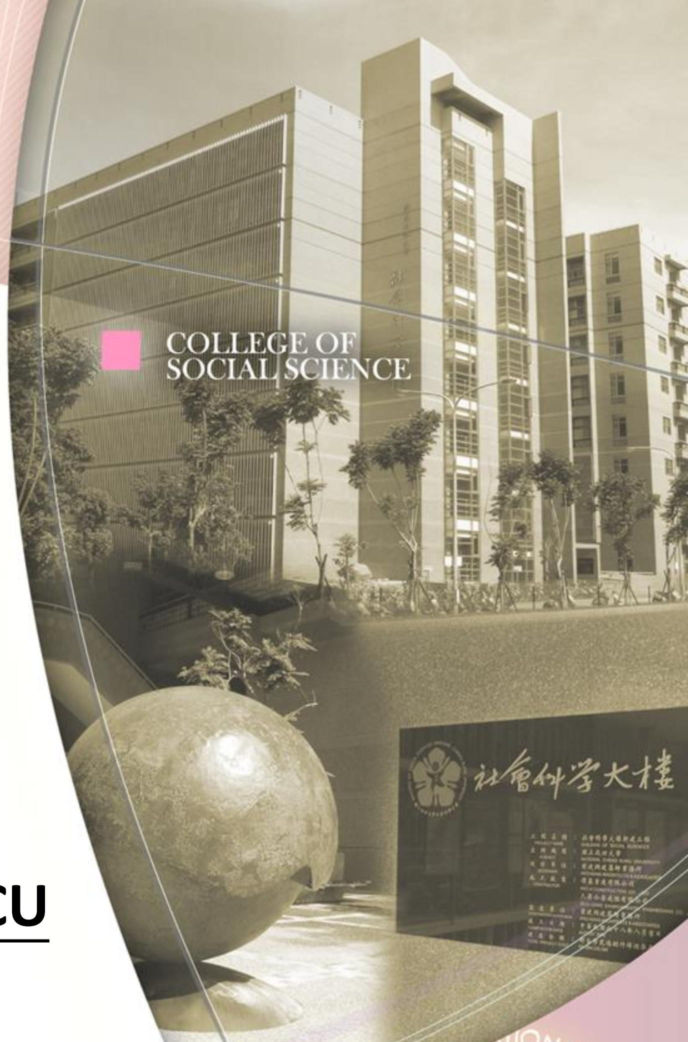
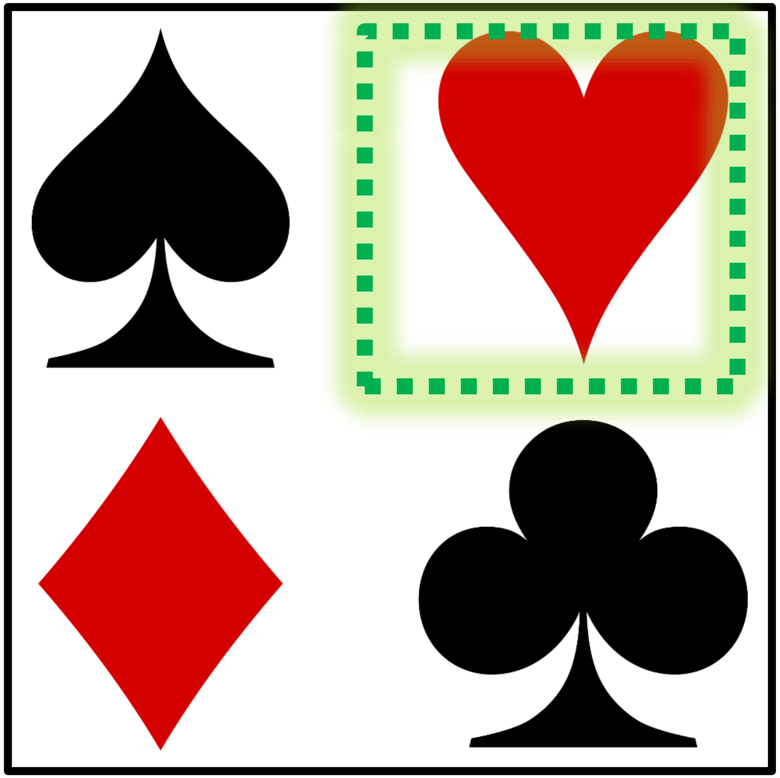


Pick a way to win $100 NTD?

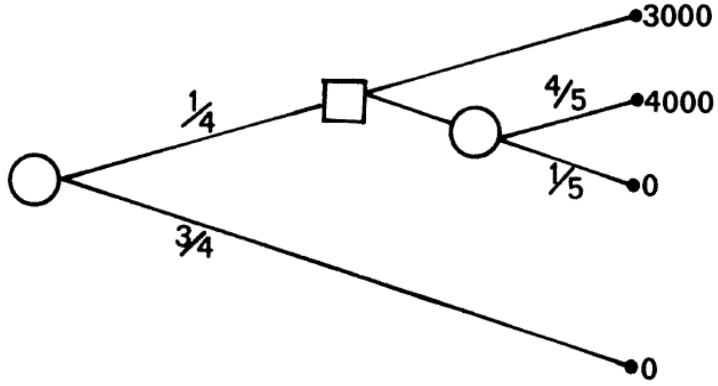


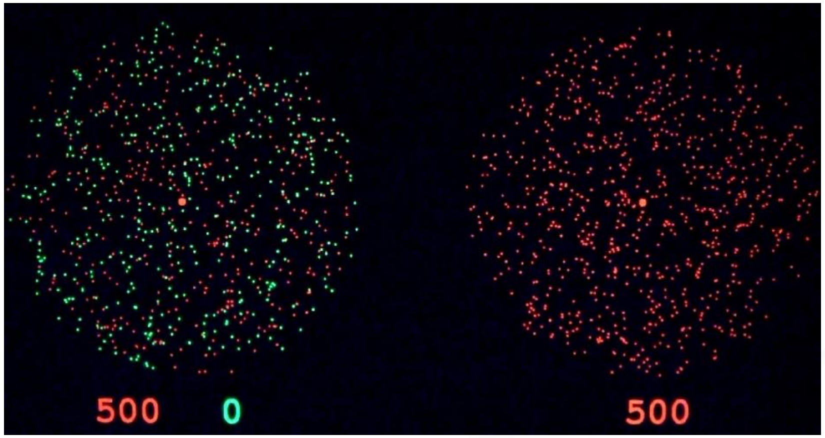
2 Straight Heads

Introduction

- Reduction Principle of compound lotteries (vs. simple lotteries)
  - von Neumann and Morgenstern (1944), Luce and Raiffa (1957)
  - Cornerstone to the expected utility theorem
  - 1/4 vs. 1/2・ 1/2

- Related studies
  - Kahneman & Tversky(1979)
  - Keller (1985)
  - Liu et al (2015)





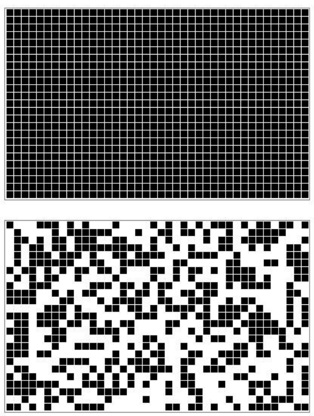
Method

- Participants
  - 31 (15 females), 1 excluded
  - age 19-37, (avg 23.07±3.27,)
  - paid $445NTD in average
    - $300 participation fee
    - ½ of won prize in a randomly-drawn trial ($0-$900).
- fMRI experiments conducted in Mind Research and Imaging Center at NCKU with GE 750, 3T, TR 2000ms, TE 33ms.

Experiment Stimulus

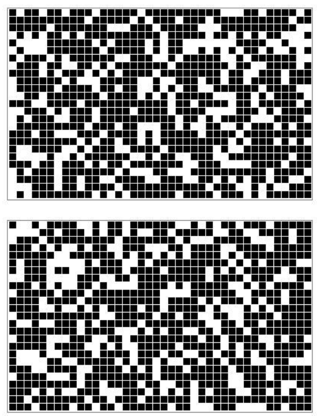
Type 1

800



0.49

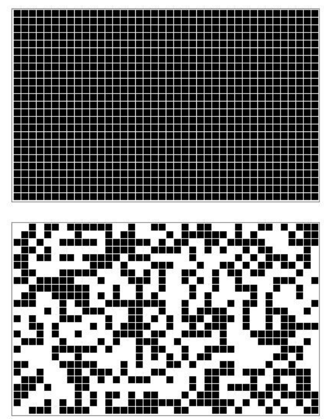
800



$0.7 \cdot 0.7$

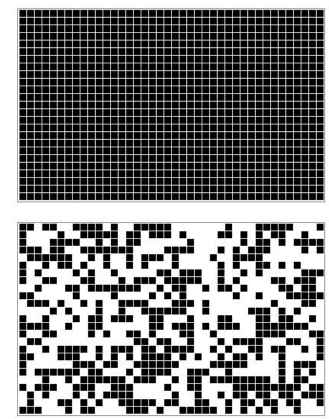
Type 2

800



0.49

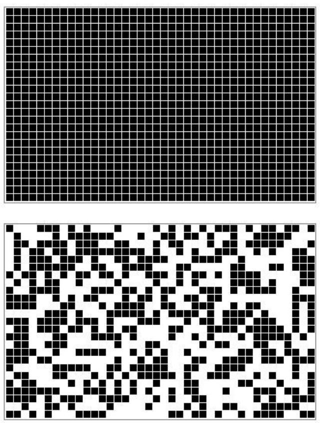
800



0.49

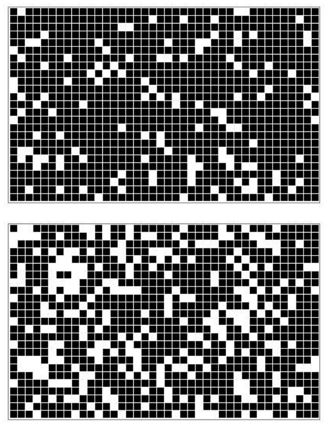
Type 3

900



0.51

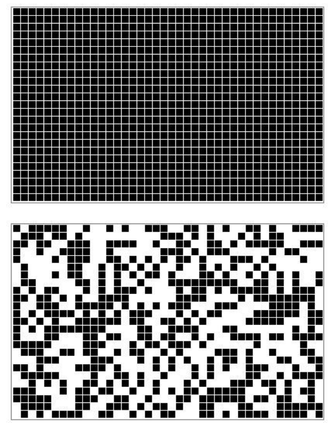
600



0.88 · 0.73

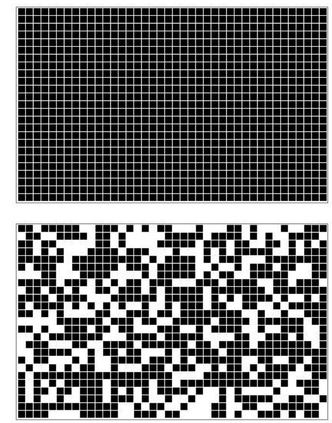
Type 4

900



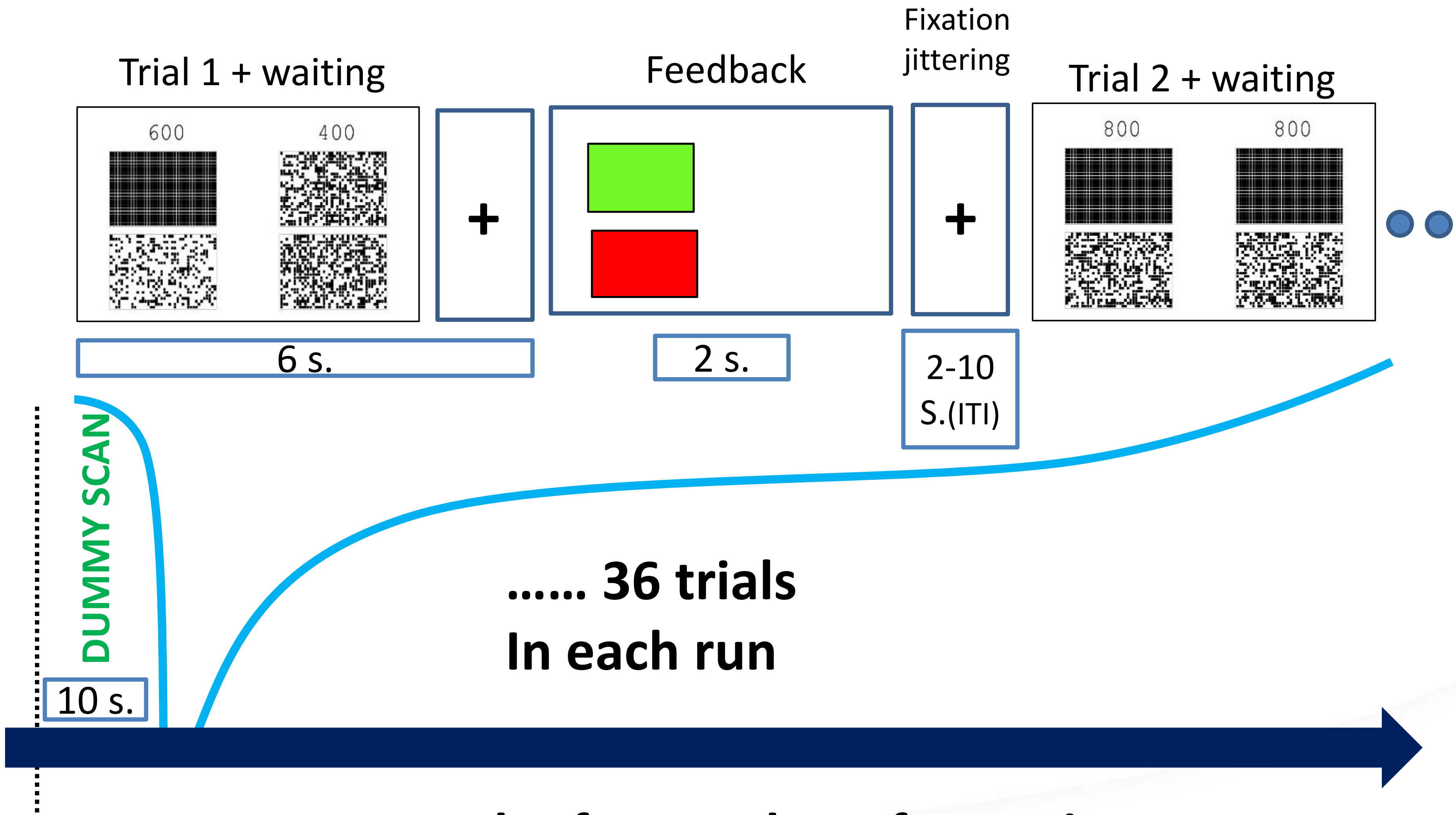
0.51

600



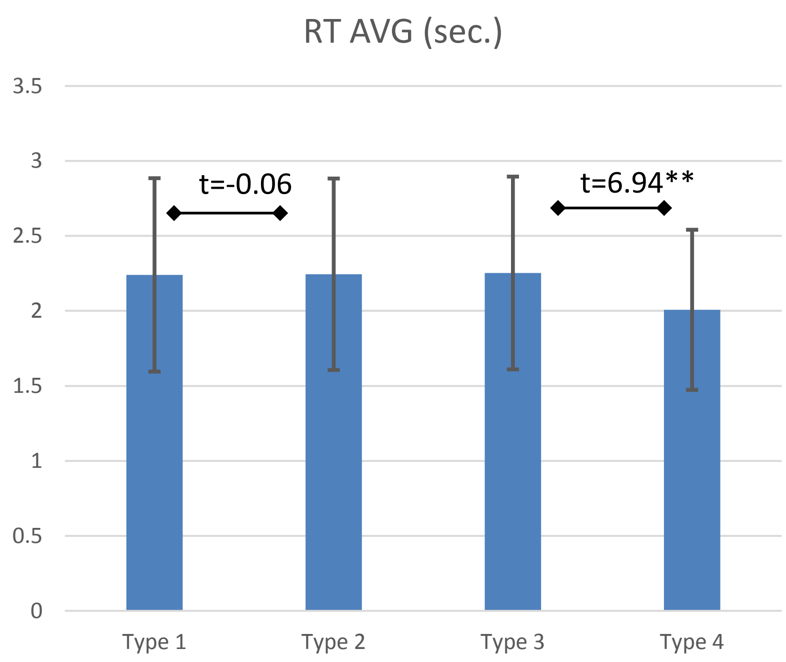
0.64

Experiment Sequence



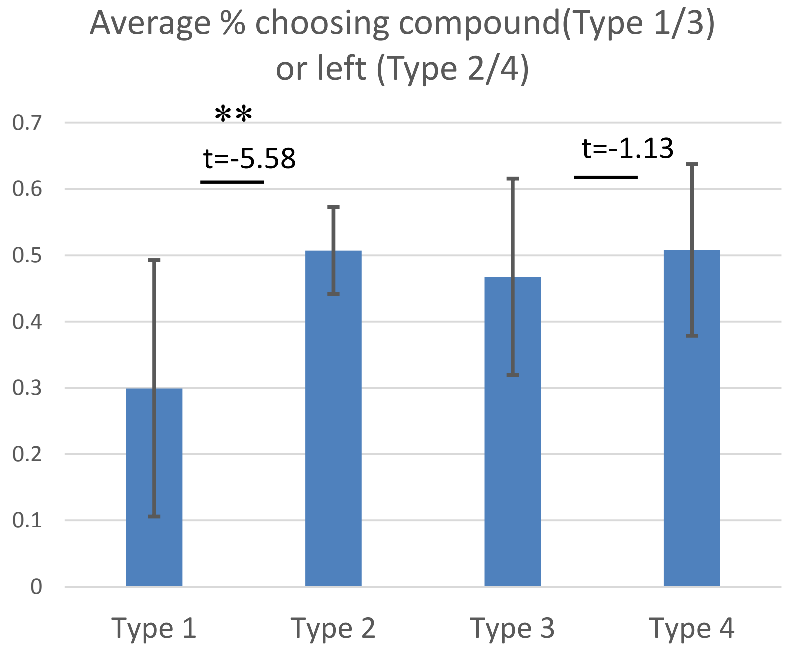
Each of 5 runs lasts for 7 min

Average RT across types



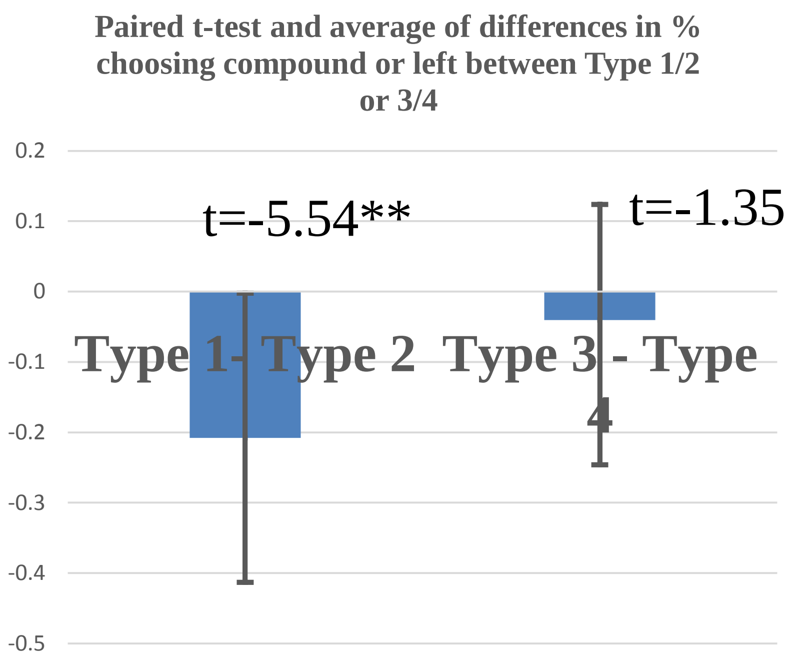
	SS	df	MS	F	P>F
Columns	1.2819	3	0.4273	1.12	0.3429
Error	44.153	116	0.3806		
Total	45.435	119			

% of Cmpd/left choices between types



Type 1/2	SS	df	MS	F	P>F
Columns	0.6478	1	0.6478	31.11	0.0000
Error	1.2078	58	0.0208		
Total	1.8556	59			

% Cmpd/left paired t-tests



Type 3/4	SS	df	MS	F	P>F
Columns	0.0247	1	0.0247	1.28	0.2629
Error	1.1223	58	0.0194		
Total	1.1471	59			

Determinants of Choices in type 1/2 ; 3/4

$\tilde{y}_i = \alpha + \beta cmp_i + \varepsilon_i$ $\tilde{y}_i$: propensity for left choice (1/2) cmp_i: left is compound lottery	$\sum L$	-1758.8
	D.F.	2697
	Prdct. Acc	0.6039

	Coefficient	t
α	0.0282	0.5173
β	-0.8790	-10.9042

$\tilde{y}_i = \alpha + \beta_1 cmp_i + \beta_2 \Delta p_i + \beta_3 \Delta m_i + \varepsilon_i$ $\tilde{y}_i$: propensity for left choice (3/4) cmp_i: left is compound lottery Δp_i: difference in probabilities Δm_i: difference in monetary rewards	$\sum L$	-1664.3
	D.F.	2690
	Prdct. Acc	0.6451

	Coefficient	t
α	0.0576	0.8939
β_1	-0.1908	-2.2929
β_2	5.3301	17.1660
β_3	4.0453	14.2938

Lotteries choices in type ¾ without compound factor considered

$\tilde{y}_i = \alpha + \beta_2 \Delta p_i + \beta_3 \Delta m_i + \varepsilon_i$ $\tilde{y}_i$: propensity for left choice (3/4) cmp_i: left is compound lottery Δp_i: difference in probabilities Δm_i: difference in monetary rewards	$\sum L$	-1666.9
	D.F.	2691
	Prdct. Acc	0.6518

	Coefficient	t
α	0.0377	-0.7665
β_2	5.3194	17.1530
β_3	4.0367	14.2814

Averting compound lotteries in type 1/2 & 3/4

Type 1/2

$$\tilde{y}_i = \alpha + \beta_1 cmp_i + \beta_2 pR_i + \beta_3 mR_i + \sum_{k=2}^{30} r_i^k D_i^k + \varepsilon_i$$

$\tilde{y}_i$: propensity for left choice (type 1/2)
cmp_i: left lottery is compound
pR_i: prob. of right lottery
mR_i: value of right lottery
D_i^k: dummy for participant k

$\sum L$	-1694.2
D.F.	2666
Prdc. Acc.	0.6187

	Coefficient	t
α	0.2753	1.1029
β_1	-0.9247	-11.1425
β_2	-0.0461	-0.2910
β_3	-0.4860	-3.3808

Type 3/4

$$\tilde{y}_i = \alpha + \beta_1 cmp_i + \beta_2 pR_i + \beta_3 mR_i + \beta_4 gapEV_i + \sum_{k=2}^{30} r_i^k D_i^k + \varepsilon_i$$

$\tilde{y}_i$: propensity for left choice (type 3/4)
cmp_i: left lottery is compound
pR_i: prob. of right lottery
mR_i: value of right lottery
$gapEV_i$: EV left- EV right
D_i^k: dummy for participant k

$\sum L$	-1581.4
D.F.	2660
Prdc. Acc.	0.6860

	Coefficient	t
α	0.1532	0.6003
β_1	-0.2024	-2.3593
β_2	0.3944	2.3302
β_3	-0.6437	-4.8254
β_4	10.1284	17.3398

Averting compound lotteries in type 1/3

$\tilde{y}_i = \alpha + \beta\, \boldsymbol{gapEV}_i + \varepsilon_i$ $\tilde{y}_i$: propensity for left choice (1/3) $gapEV_i$: EV left- EV right	$\sum L$	-1713.6
	D.F.	2694
	Prdct. Acc	0.6513

	Coefficient	t
$\boldsymbol{\alpha}$	-0.5422	-13.1114
$\boldsymbol{\beta}$	8.0392	11.0477

$\tilde{y}_i = \alpha + \beta_1 pL_i + \beta_2 mL_i + \beta_3 pR_i + \beta_4 mR_i + \varepsilon_i$ $\tilde{y}_i$: propensity for left choice (1/3) pL_i: probability of left lottery mL_i: value of left lottery pR_i: prob. of right lottery mR_i: value of right lottery	$\sum L$	-1713.4
	D.F.	2691
	Prdct. Acc	0.6517

	Coefficient	t
$\boldsymbol{\alpha}$	-1.0527	-7.9435
$\boldsymbol{\beta_1}$	4.5245	11.7778
$\boldsymbol{\beta_2}$	1.4332	4.058
$\boldsymbol{\beta_3}$	-4.5721	-11.8156
$\boldsymbol{\beta_4}$	-1.3999	-4.2595

Averting compound lotteries in type 1/3 with dummy for participants

$$\tilde{y}_i = \alpha + \beta gapEV_i + \sum_{k=2}^{30} r_i^k D_i^k + \varepsilon_i$$

$\tilde{y}_i$: propensity for left choice (1/3)
$gapEV_i$: EV left- EV right
D_i^k: dummy for participant k

$\sum L$	-1575.9
D.F.	2665
Prdc. Acc.	0.6973

	Coefficient	t
α	-0.6596	-2.8828
β	8.9037	11.5690

$$\tilde{y}_i = \alpha + \beta_1 pL_i + \beta_2 mL_i + \beta_3 pR_i + \beta_4 mR_i + \sum_{k=2}^{30} r_i^k D_i^k + \varepsilon_i$$

$\tilde{y}_i$: propensity for left choice (1/3)
pL_i: probability of left lottery
mL_i: value of left lottery
pR_i: prob. of right lottery
mR_i: value of right lottery
D_i^k: dummy for participant k

$\sum L$	-1575.4
D.F.	2662
Prdc. Acc.	0.6981

	Coefficient	t
α	-1.2357	-4.6483
β_1	5.041	12.3219
β_2	1.589	4.2653
β_3	-5.0955	-12.3587
β_4	-10.1284	-4.4684

Neutral choices in type 2/4

$\tilde{y}_i = \alpha + \beta gapEV_i + \varepsilon_i$

$\tilde{y}_i$: propensity for left choice (2/4)
$gapEV_i$: EV left- EV right

$\sum L$	-1744.2
D.F.	2695
Predict. Accuracy	0.6010

	Coefficient	t
α	-0.0168	-0.4179
β	11.0332	12.4324

$\tilde{y}_i = \alpha + \beta_1 pL_i + \beta_2 mL_i + \beta_3 pR_i + \beta_4 mR_i + \varepsilon_i$

$\tilde{y}_i$: propensity for left choice (2/4)
pL_i: probability of left lottery
mL_i: value of left lottery
pR_i: prob. of right lottery
mR_i: value of right lottery

$\sum L$	-1708.5
D.F.	2692
Predict. Accuracy	0.6077

	Coefficient	t
α	0.0654	0.5888
β_1	7.4037	14.2667
β_2	5.2654	11.9919
β_3	-7.3606	-14.1021
β_4	-5.3563	-12.5900

Neutral choices in type 2/4 with dummy for participants

$\tilde{y}_i = \alpha + \beta gapEV_i + \sum_{k=2}^{30} r_i^k D_i^k + \varepsilon_i$ $\tilde{y}_i$: propensity for left choice (2/4) $gapEV_i$: EV left- EV right D_i^k: dummy for participant k	$\sum L$	-1715.1
	D.F.	2666
	Predict. Accuracy	0.6118

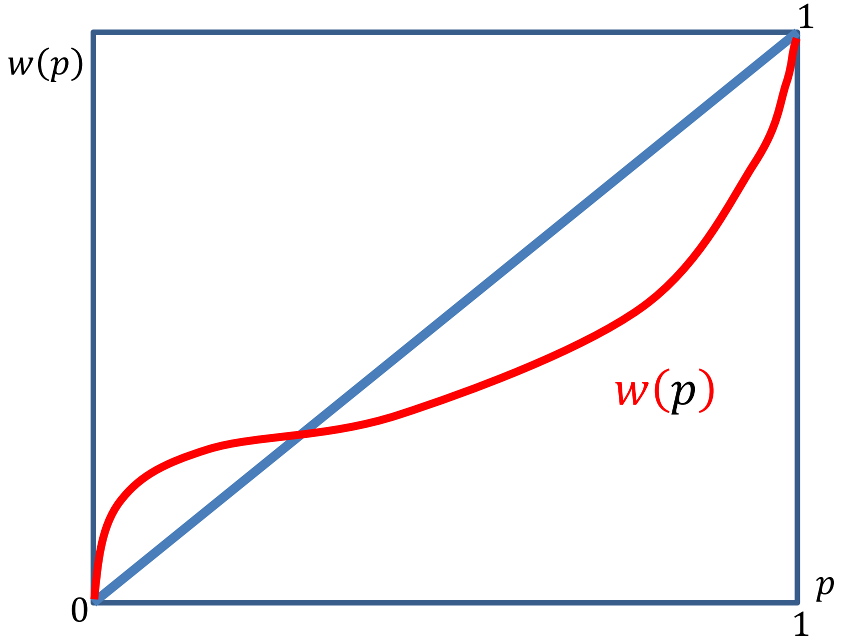
	Coefficient	t
α	-0.0025	-0.0112
β	11.2378	12.5422

$\tilde{y}_i = \alpha + \beta_1 pL_i + \beta_2 mL_i + \beta_3 pR_i + \beta_4 mR_i + \sum_{k=2}^{30} r_i^k D_i^k + \varepsilon_i$ $\tilde{y}_i$: propensity for left choice (2/4) pL_i: probability of left lottery mL_i: value of left lottery pR_i: prob. of right lottery mR_i: value of right lottery D_i^k: dummy for participant k	$\sum L$	-1678.5
	D.F.	2663
	Predict. Accuracy	0.6118

	Coefficient	t
α	0.0837	0.3390
β_1	7.5576	14.402
β_2	5.3765	12.1056
β_3	-7.5156	-14.2400
β_4	-5.4698	-12.7109

Implication of prospect theory

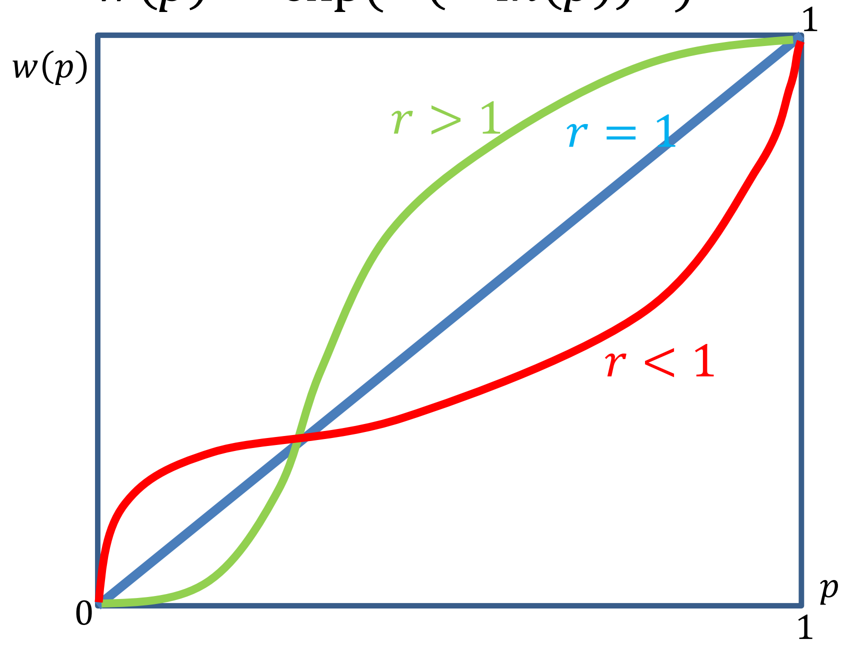
- Violation of RP was implied by decision weight in prospect theory (Kahneman & Tversky (1979))



Proposition

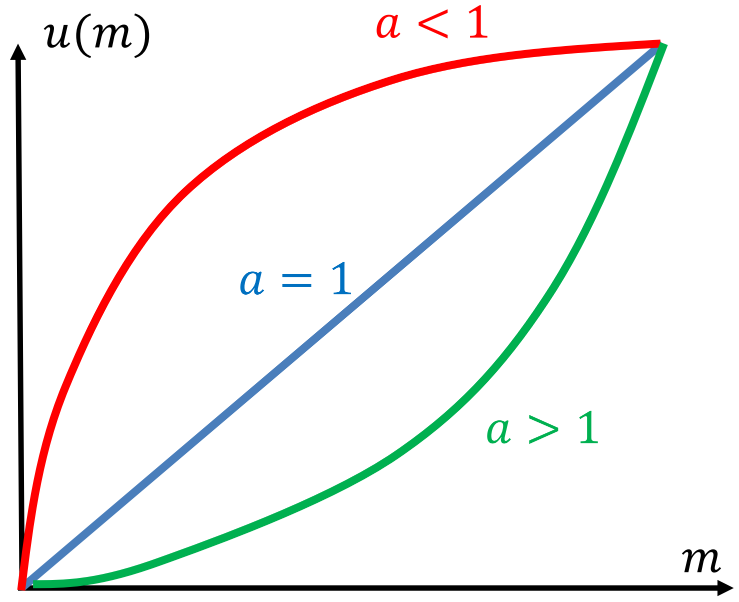
- *Weighting function (Prelec 1998)*
  - $w(p) = exp(-(-ln(p))^r)$
    - *simple lotteries*
  - $w(p) = w(p^c)w(p^{1-c})$
    - *compound lotteries with* $p = p^c p^{1-c}$
- *Compound lotteries are more (less) preferred to equivalent simple lotteries if* $r > 1$ *(*< 1*).*

Prelec (1998):
$w(p) = \exp(-(-\ln(p))^{r})$



MLE estimation

- $U = p \cdot u(m) + (1-p) \cdot u(0)$ for any lottery earning m or 0
- Simple lottery
  - $U_s = w(p,r) \cdot u(m)$
  - $u(m) = m^a$
  - $w_s(p,r) = exp(-(-ln(p))^r)$
- Compound lottery
  - $U_C = w_C(p_1,p_2,r) \cdot u(m)$
  - $w_C(p_1,p_2,r) = exp(-(-ln(p_1))^r) \cdot exp(-(-ln(p_2))^r)$



MLE estimation

- The probability participants choosing left over right is given by
  - $P(p_{11}, p_{12}, m_1, p_2, m_2, r, a, \lambda) = \frac{1}{1+\exp\{-\lambda(U_L - U_R)\}}$ (Hsu et al,2009)
    - λ: sensitivity parameter
  - $P(p_{11}, p_{12}, m_1, p_2, m_2, r, a, k) = 1 - \int_x^0 \frac{1}{\sqrt{2\pi\sigma}} e^{-(x-\mu)^2/(2\sigma)}\, dx$ (Wu et al,2009)
    - $\mu = U_L + \varepsilon_L - (U_R + \varepsilon_R)$
    - $\varepsilon_L \sim N(0, kU_L)$; $\varepsilon_R \sim N(0, kU_R)$;

MLE estimation

- MLE (Maximum Likelihood Estimator) of (a, r, λ) is defined as the maximizer of $\sum_{y_i} [y_i \ln\left(P_i(L)\right) + (1 - y_i)\ln(1 - P_i(L))]$
  - $y_i = 1$ if the left lottery is chosen
  - $P_i(L)$ is the probability that the left lottery is chosen in trial i

Out-of-sample prediction (a/r from type 3/4)

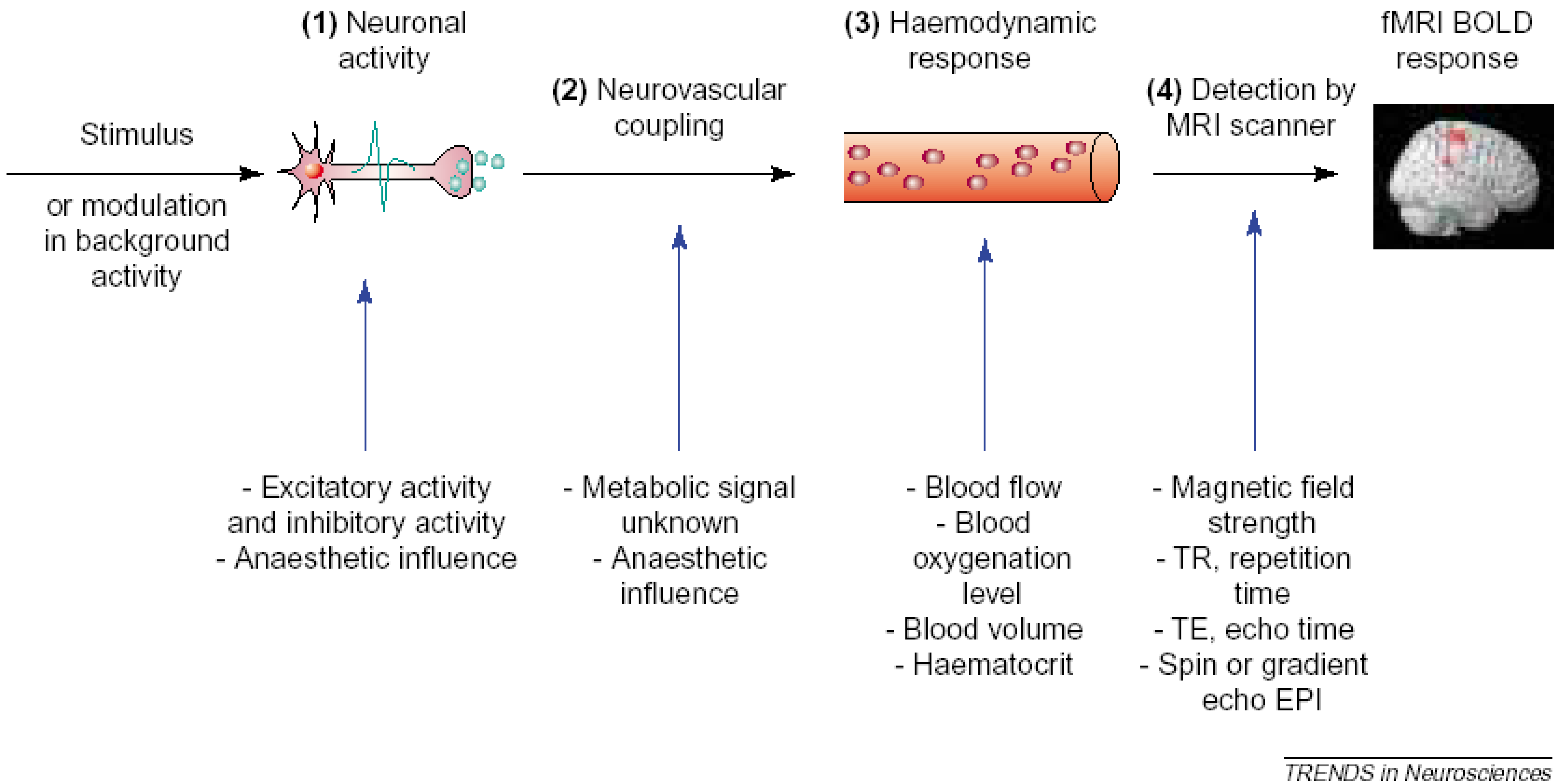
$\%compound(type1) = \beta_0 + \beta_1 r$

	Coef.	s.e.	t	p-value
constant	0.233333	0.038299	6.092352	1.43E-06
r>1	0.197778	0.066336	2.981433	0.005881
	R^2	0.241	Adj-R^2	0.214

$\%compound = \beta_0 + \beta_1 r + \beta_2 a$

	Coef.	s.e.	t	p-value
constant	0.219529	0.051548	4.258721	0.000223
r>1	0.206061	0.070341	2.929444	0.006826
a>1	0.027609	0.067686	0.407905	0.68656
	R^2	0.246	Adj-R^2	0.190

Stimulus to BOLD

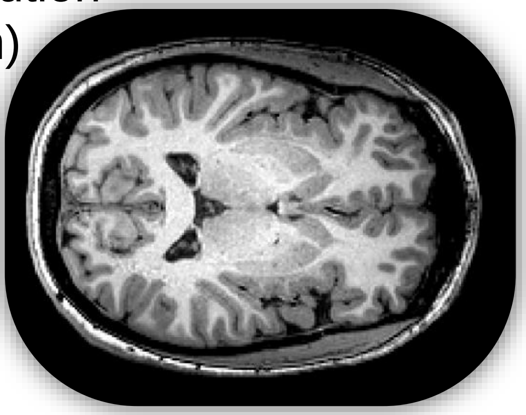


Source: Arthurs & Boniface, 2002, *Trends in Neurosciences*

MRI vs. fMRI

MRI

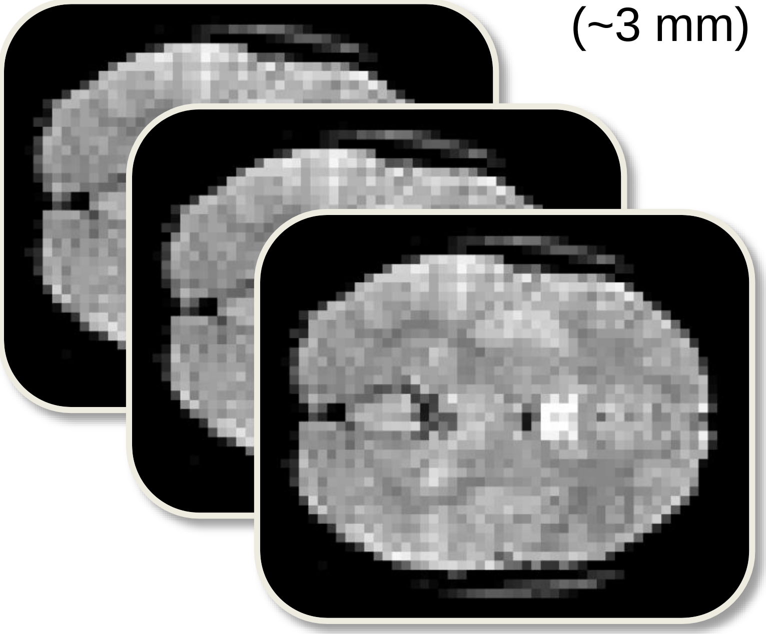
high resolution
(1 mm)



One 3D volume

fMRI

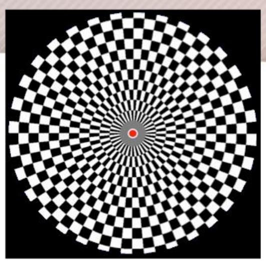
low resolution
(~3 mm)

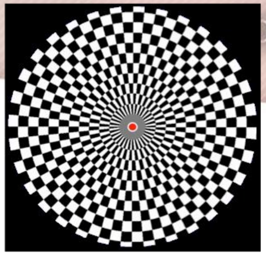


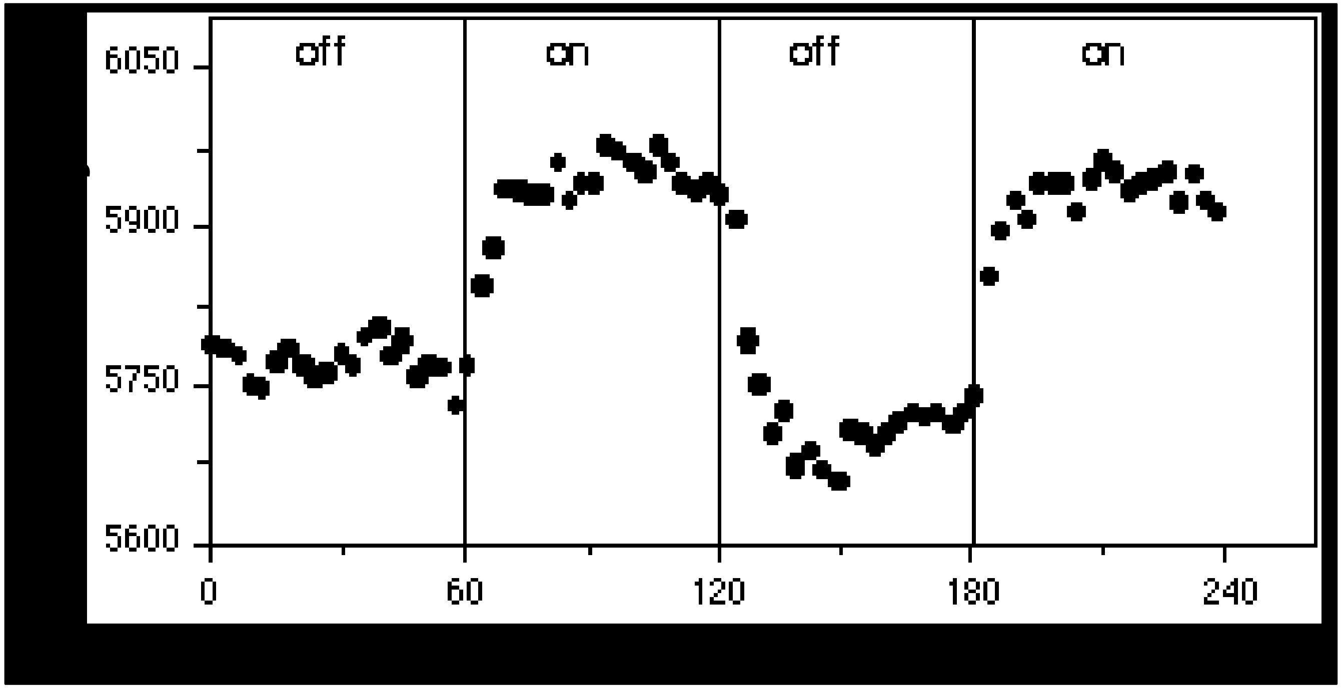
series of 3D volumes (i.e., 4D data)
(e.g., every 2 sec for 5 mins)

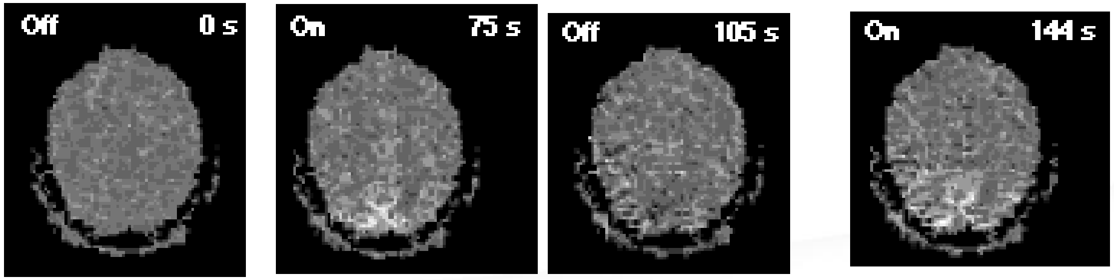
Source: fMRI4Newbies

First fMRI Experiment



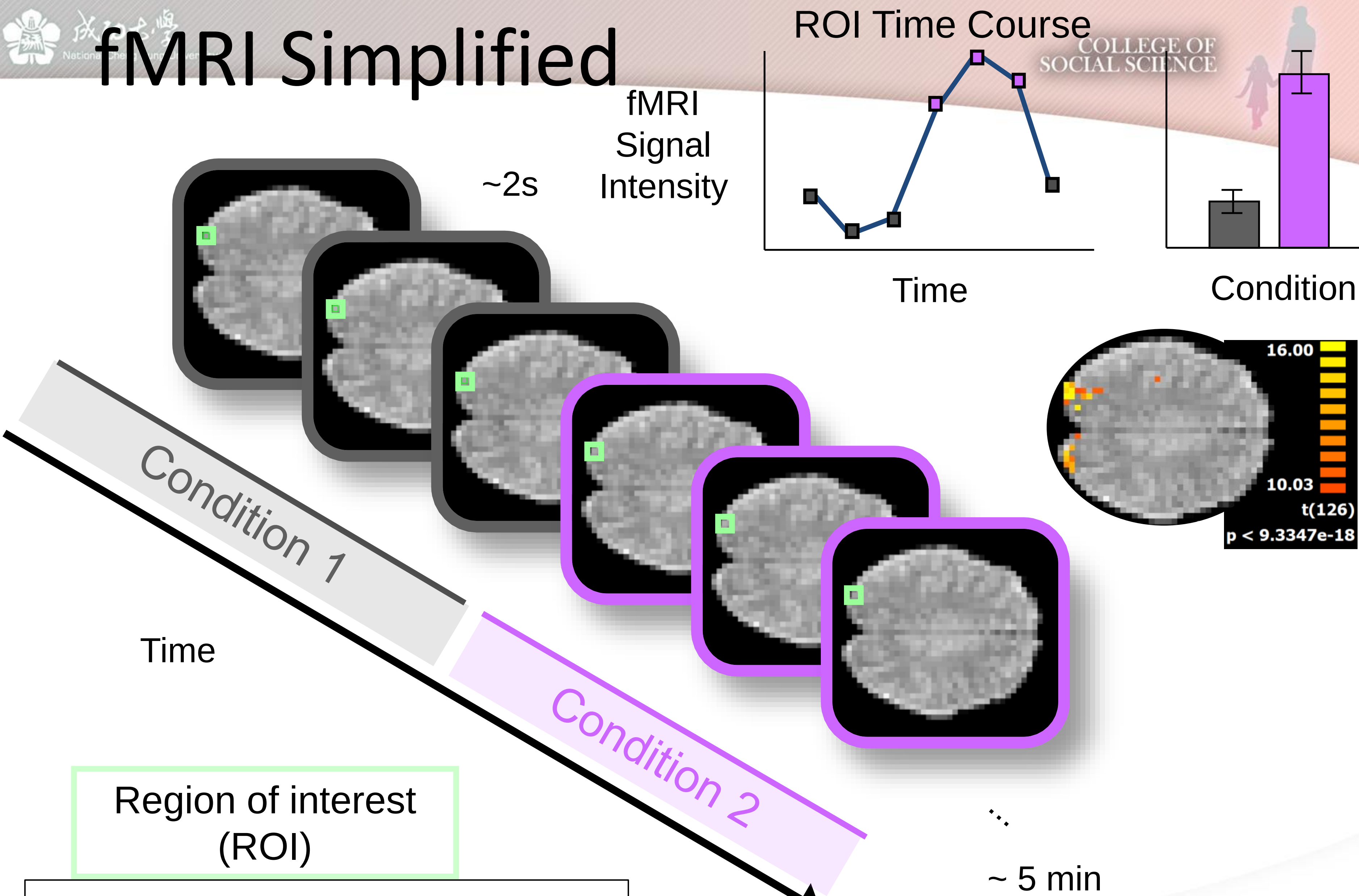






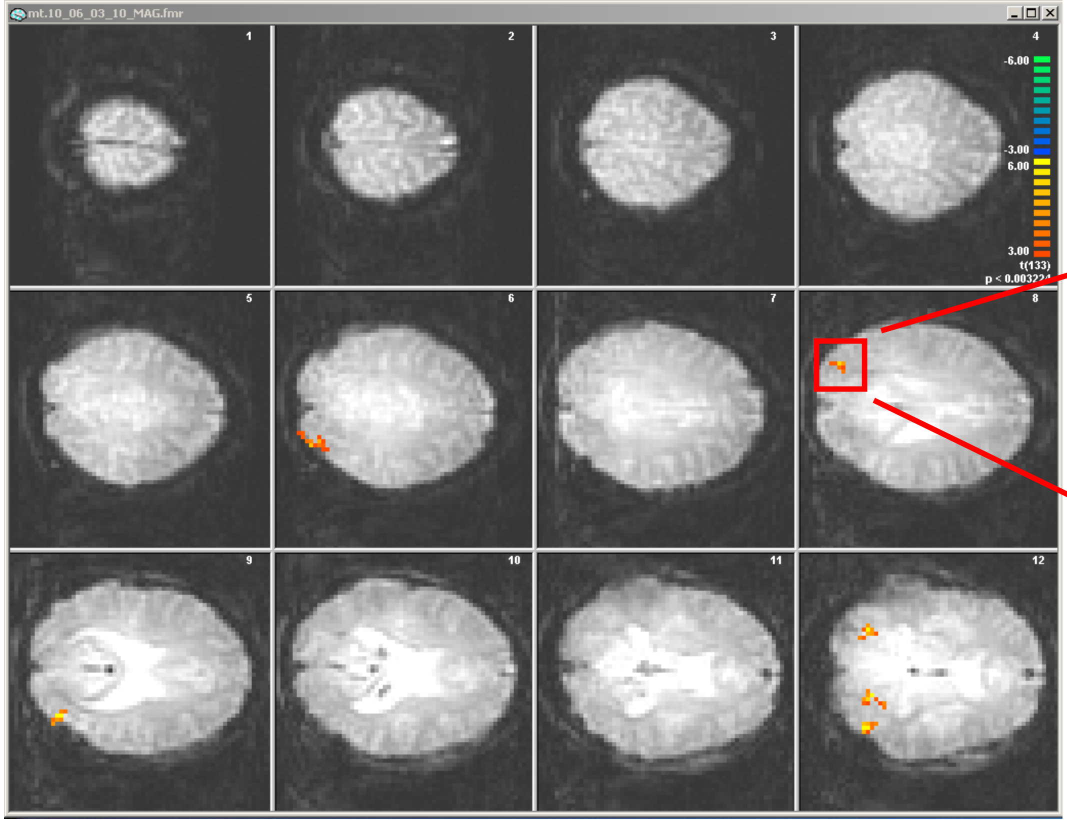
Source: Kwong et al., 1992

fMRI Simplified



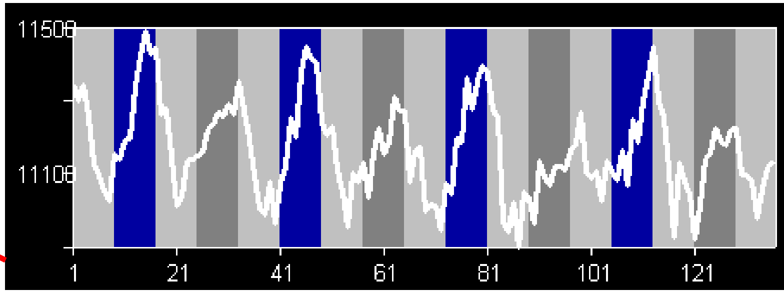
Source: fMRI4Newbies

Statistical Maps & Time Courses



Use stat maps to pick regions

Then extract the time course

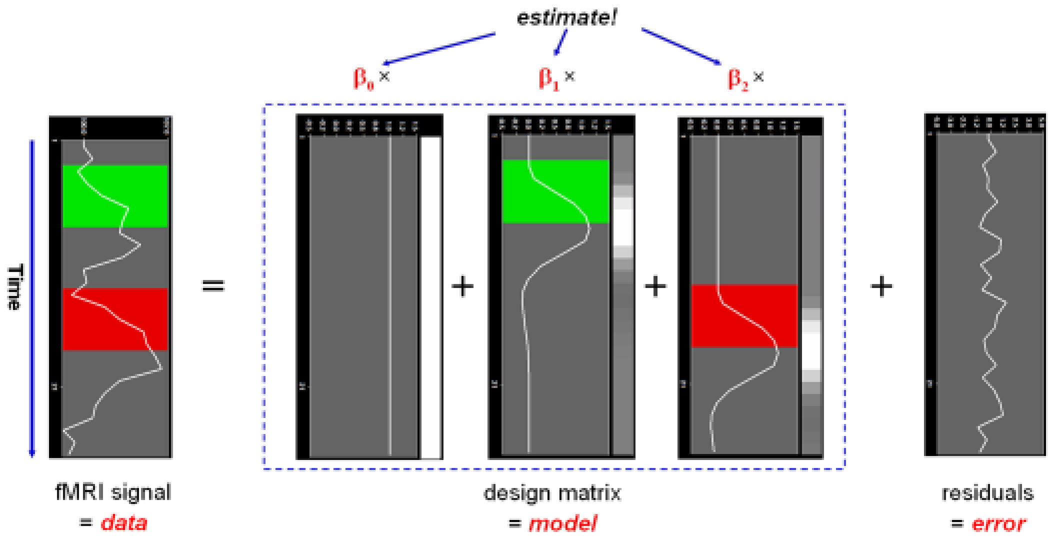


Source: fMRI4Newbies

Neural data analysis

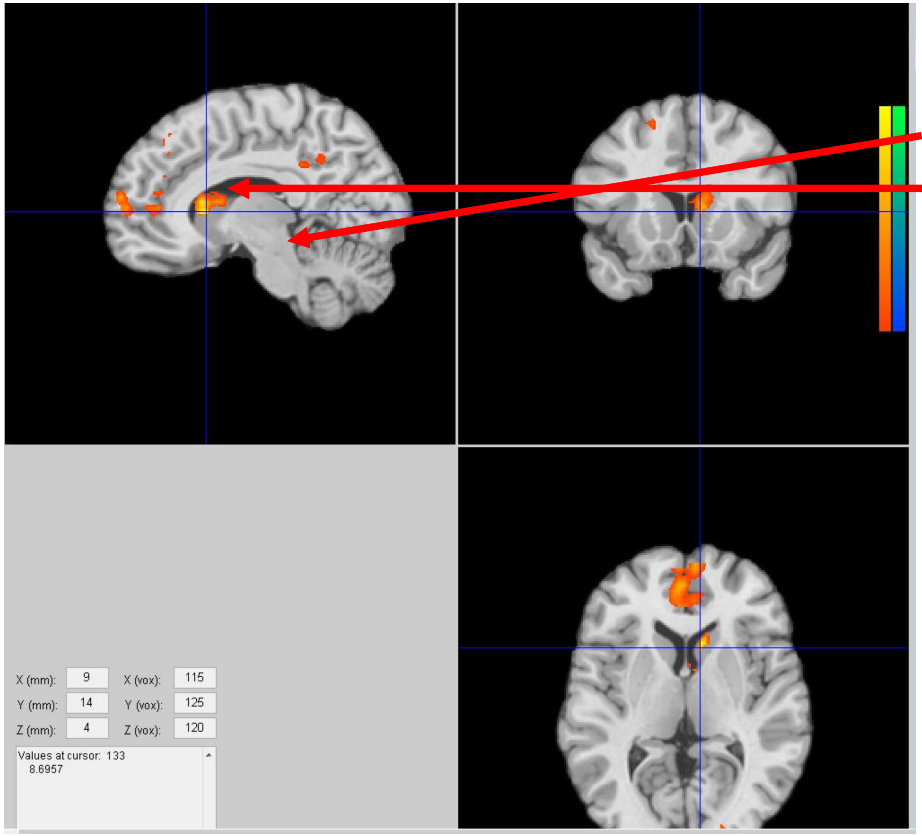
- Bold activities for each voxel across the whole brain
- Data Preprocessing
- BrainVoyager QX

$$
\begin{aligned}
y_1 &= b_0 + b_1 X_{11} + \cdots \cdots \cdots + b_p X_{1p} + e_1 \\
y_2 &= b_0 + b_1 X_{21} + \cdots \cdots \cdots + b_p X_{2p} + e_2 \\
y_3 &= b_0 + b_1 X_{31} + \cdots \cdots \cdots + b_p X_{3p} + e_3 \\
&\vdots \\
y_n &= b_0 + b_1 X_{n1} + \cdots \cdots \cdots + b_p X_{np} + e_n
\end{aligned}
$$


estimate!
β0 ×
β1 ×
β2 ×
Time
=
+
+
+
fMRI signal
= data
design matrix
= model
residuals
= error

Source: BVQX

Prediction error activated areas

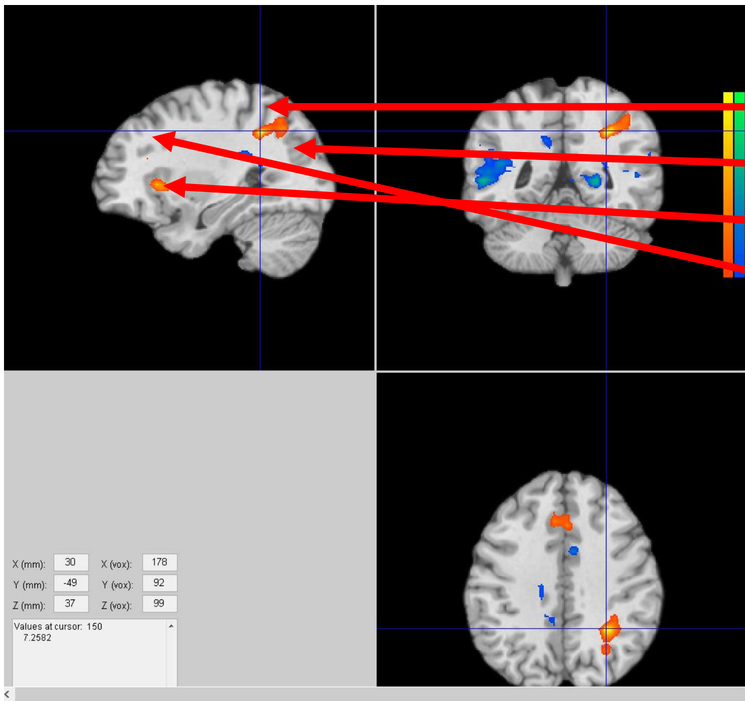


- Caudate (Striatum)
- Superior Frontal

Prediction error activated areas

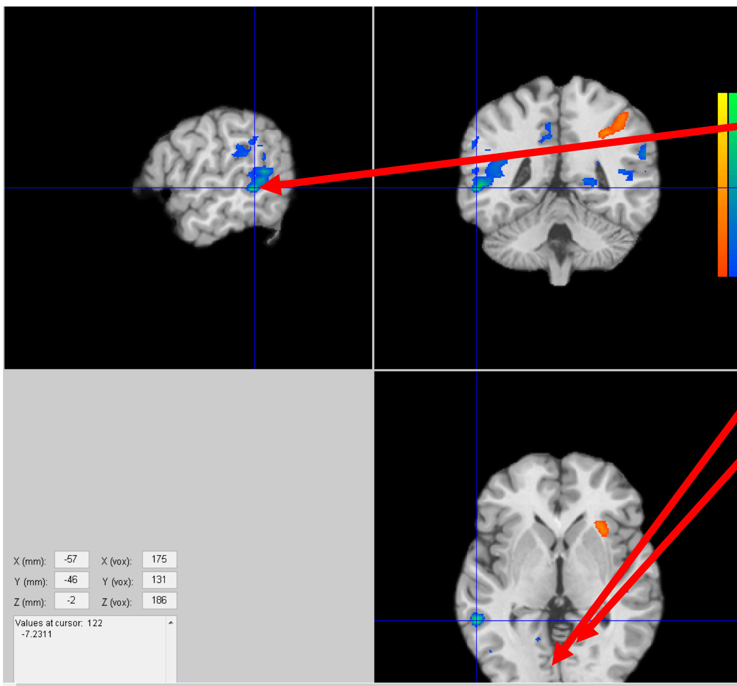
- PreFrontal Cortex (PC)
- Posterior Cingulate Cortex (PCC)

Compound – Simple task



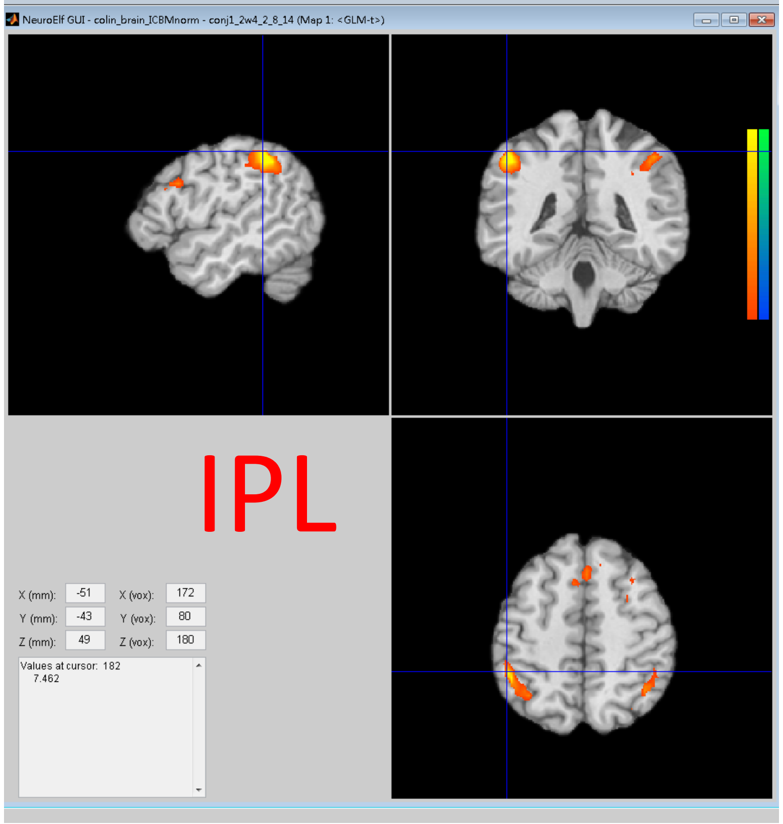
- More active
  - Parietal area (calculation)
  - Precuneus (attention)
  - Insula (risk)
  - Frontal lobe (evaluation)

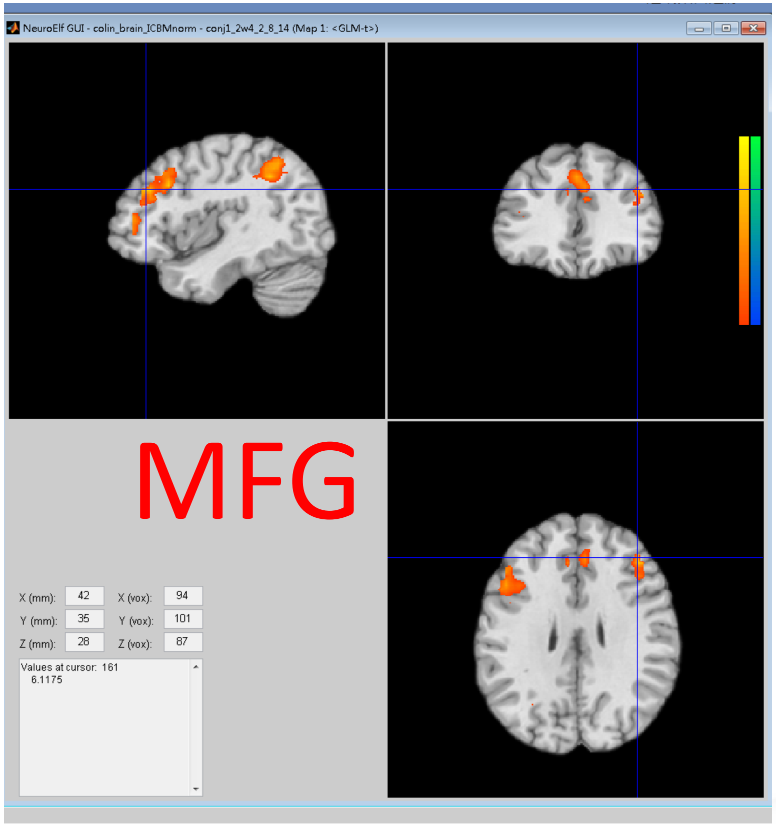
Compound – Simple task

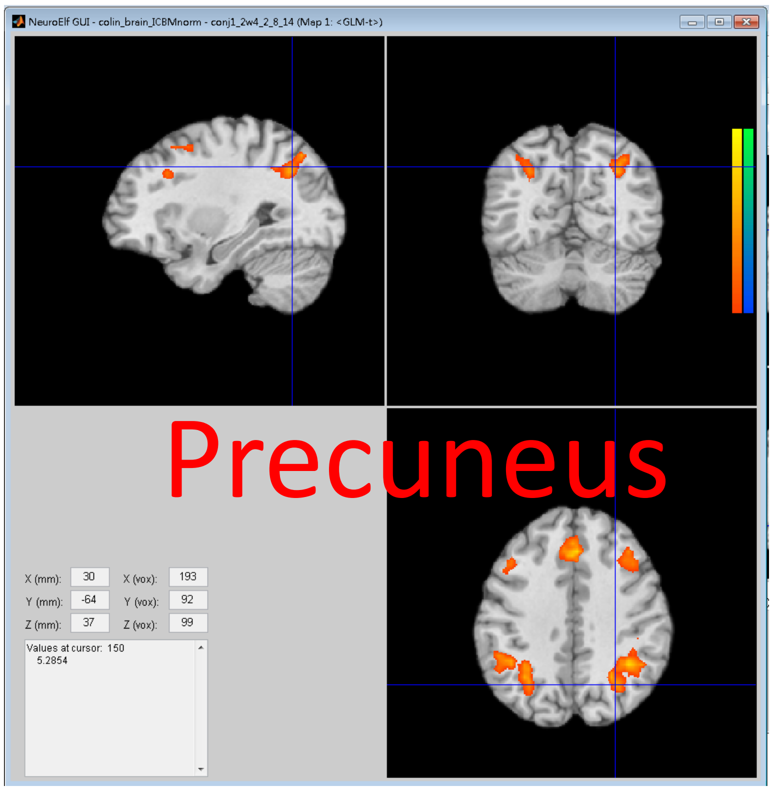


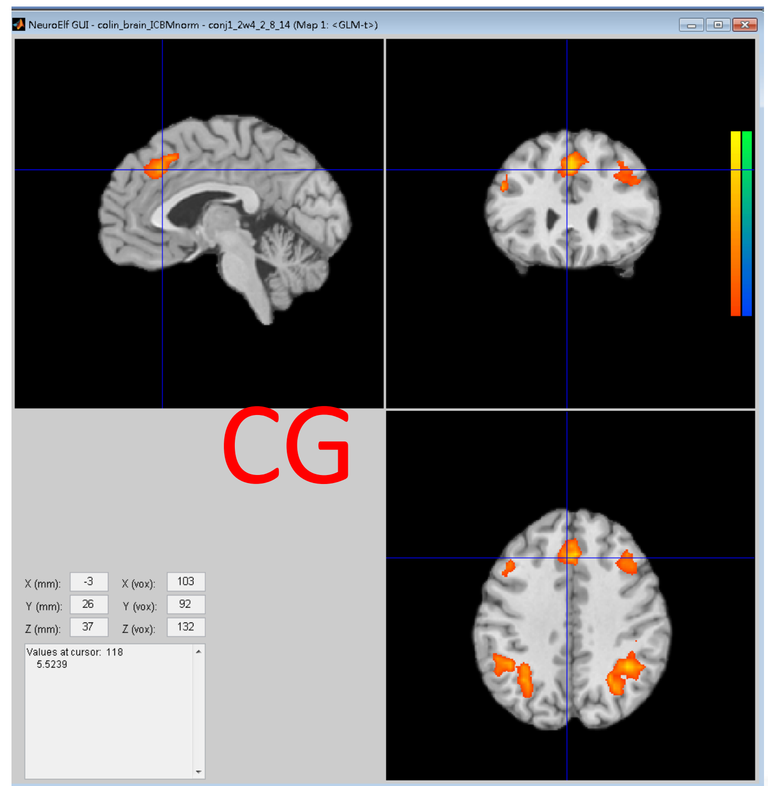
- Less active
  - Temporal gyrus (comprehension)
  - Posterior Cingulate (Learning/Resting/Depression)
  - Parahippocampal Gyrus (visuospatial processing /contextual associations)

(1-2)conj(4-2) (0.001/30)

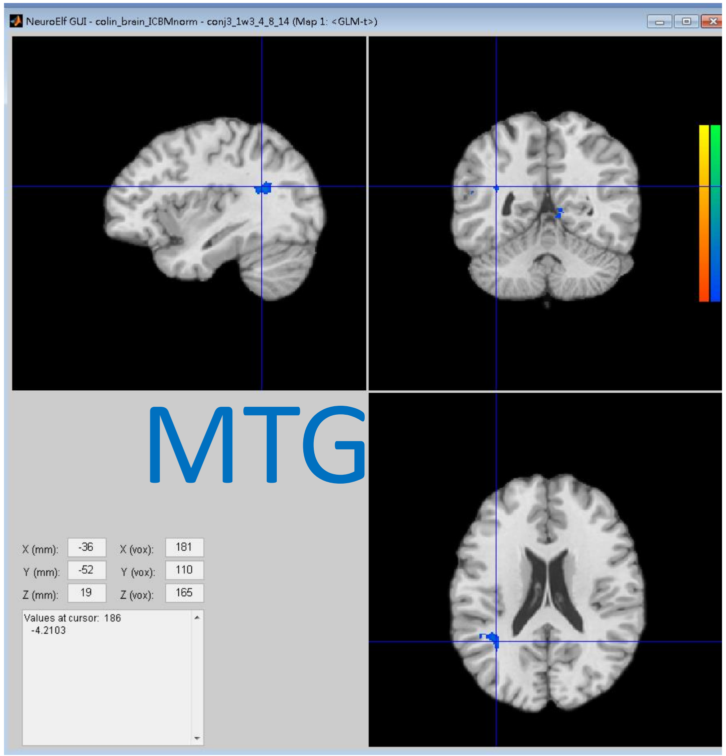


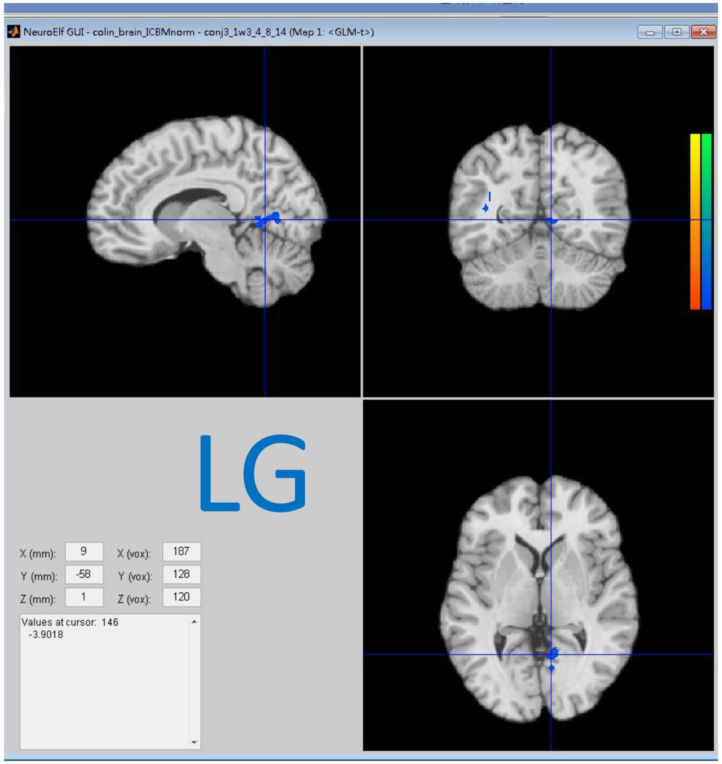



NeuroElf GUI - colin_brain_ICBMnorm - conj1_2w4_2_8_14 (Map 1: <GLM-t>)
Precuneus
X (mm): 30
Y (mm): -64
Z (mm): 37
X (vox): 193
Y (vox): 92
Z (vox): 99
Values at cursor: 150
5.2854

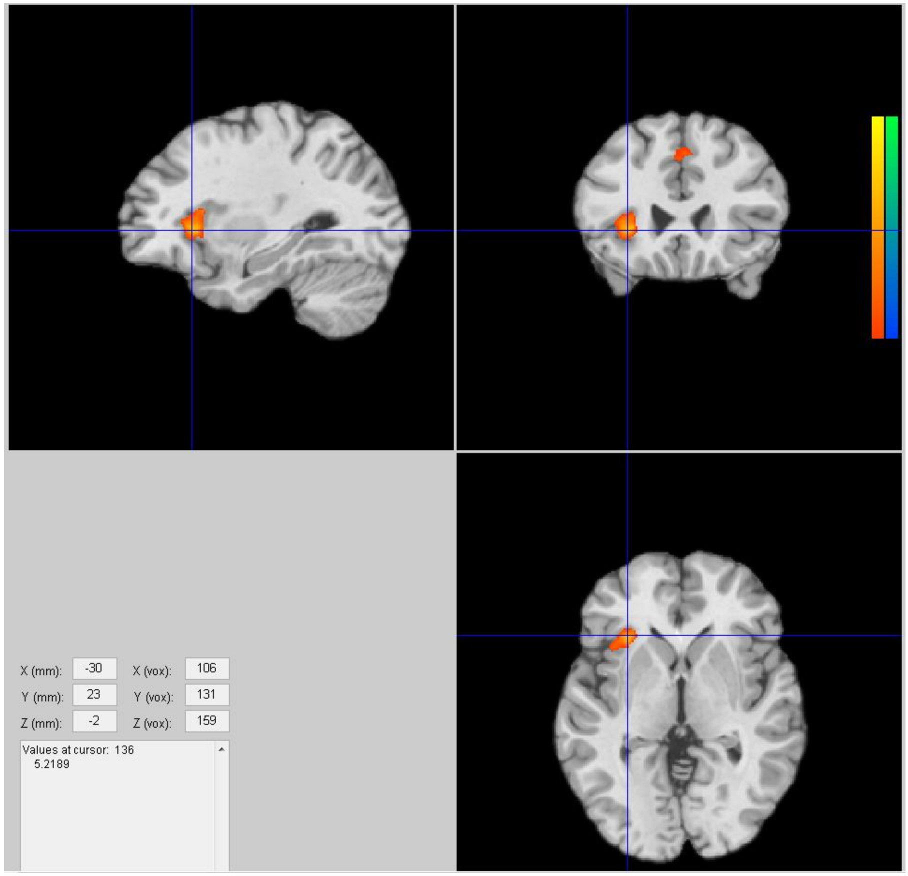

NeuroElf GUI - colin_brain_ICBMnorm - conj1_2w4_2_8_14 (Map 1: <GLM-t>)
CG
X (mm): -3
Y (mm): 26
Z (mm): 37
X (vox): 103
Y (vox): 92
Z (vox): 132
Values at cursor: 118
5.5239

(3-1)conj(3-4) (0.005/20)

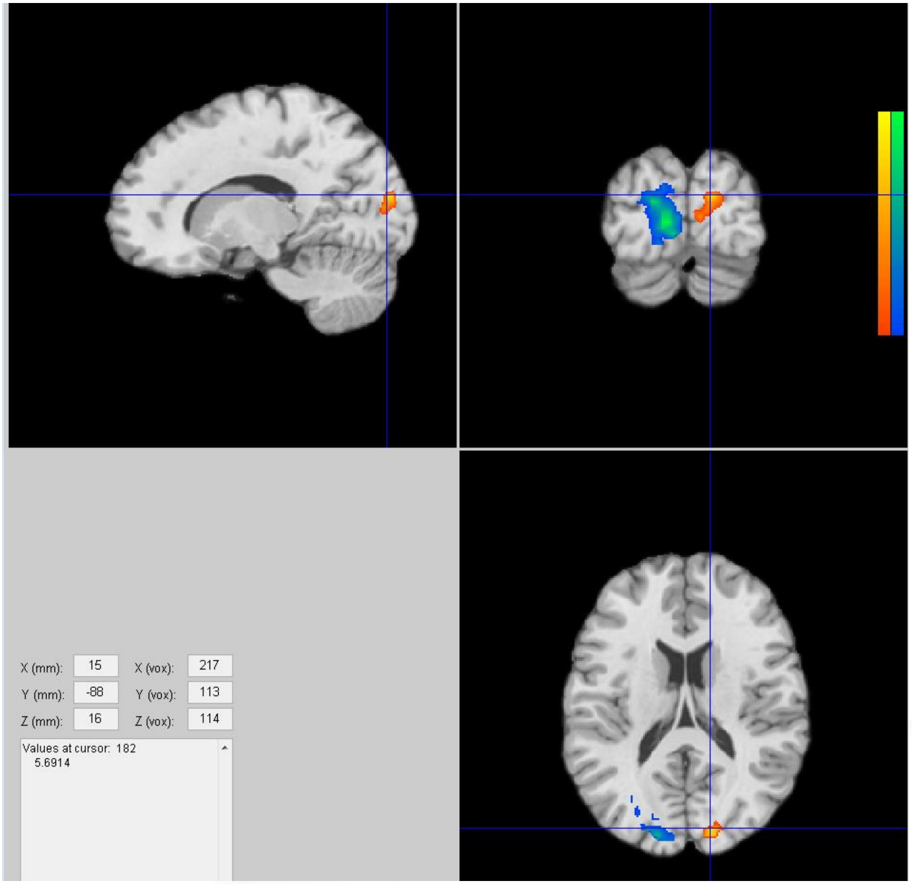




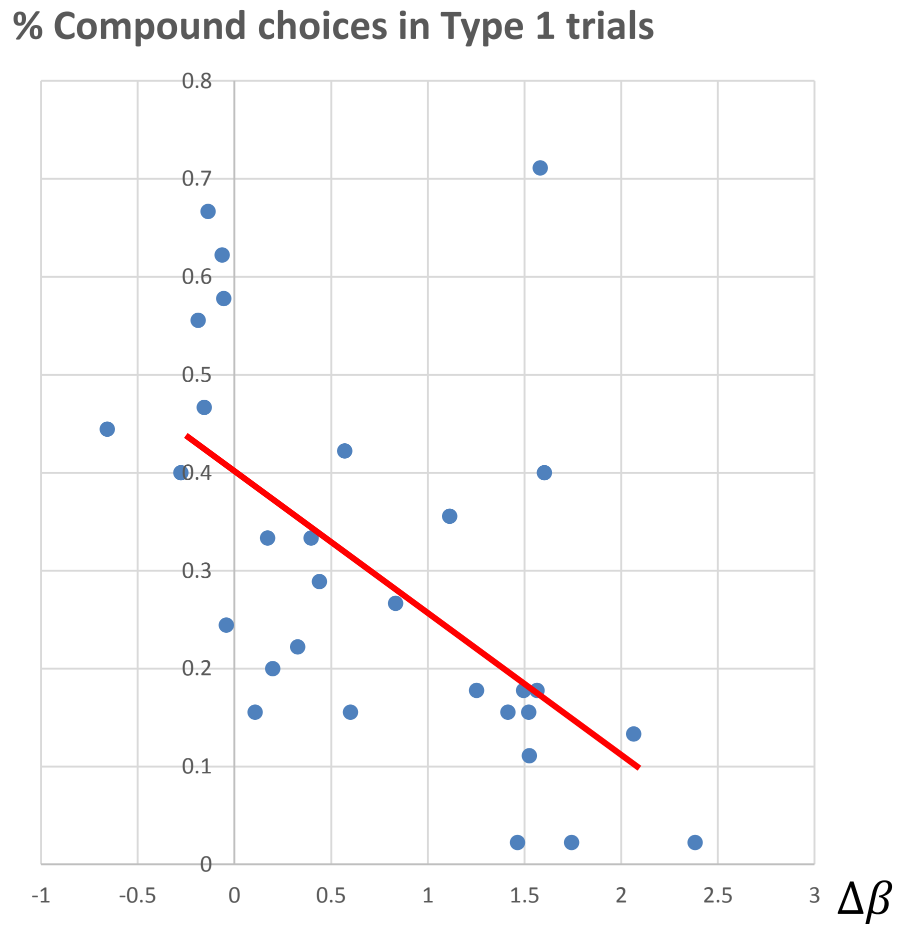
Type 1: Compound - Simple



Type 1: Left (view) - Right



Insula activations predict compound aversion



Regression Statistics	
R square	0.324516
Adjusted R square	0.300392
observations	30

	Coef.	S.E.	t	P-value
const	0.4011	0.0405	9.8993	1.2E-10
$\Delta\beta$	0.1342	0.0366	-3.6677	0.0010

Conclusion

- Aversion from compound lotteries certainly not a random calculation error
- Averting compound lotteries is rooted in probability distortion.
- Compound lotteries involve more evaluation/calculation, thus less preferred (“more risky”?) than simple ones
- When EV/EU difference is insignificant, the form of lottery makes significant impact
- A framing effect