Mini-symposia

Computational Inverse Scattering Problems and Applications in Imaging (MS I-1, MS I-2)

Organizers:
Xudong Chen
National University of Singapore, Singapore
E-mail: elechenx@nus.edu.sg
Kui Ren
University of Texas at Austin, USA
E-mail: ren@math.utexas.edu

Phase Retrieval with Roughly Known Mask

Albert Fannjiang
Department of Mathematics, University of California, Davis
E-mail: fannjiang@math.ucdavis.edu

Fourier phasing is the problem of retrieving Fourier phase information from Fourier intensity data. The standard Fourier phase retrieval (without a mask) is known to have many solutions which cause the standard phasing algorithms to stagnate and produce wrong or inaccurate solutions. In this talk Fourier phase retrieval is carried out with the introduction of a randomly fabricated mask in measurement and reconstruction. Highly probable uniqueness of solution, up to a global phase, was previously proved with exact knowledge of the mask. Here the uniqueness result is extended to the case where only rough information about the masks phases is assumed. The exponential probability bound for uniqueness is given in terms of the uncertainty-to-diversity ratio of the unknown mask. New phasing algorithms alternating between the object update and the mask update are systematically tested and demonstrated to have the capability of recovering both the object and the mask (within the object support) simultaneously, consistent with the uniqueness result. Phasing with a phase-uncertain mask is shown to be robust with respect to the correlation in the mask as well as the Gaussian and Poisson noises.

Seismic reflection imaging using shape regularization

Sergey Fomel
Bureau of Economic Geology, Jackson School of Geosciences, University of Texas, Austin, USA
E-mail: sergey.fomel@beg.utexas.edu

Seismic reflection imaging can be formulated as an inverse problem for reconstructing reflectivity. This formulation is particularly effective when reconciling data deficiencies, such as incomplete coverage and simultaneous sources. The problem can be regularized by adding constraints on reflectivity, which are different for specular reflections and diffractions. Imaging specular reflections requires contraints on continuity along geological structures. Imaging diffractions (point scatterers) requires constraints on sparsity. I propose a formulation of the problem, where additional constraints are implemented using shaping regularization. Numerical experiments confirm practical validity of the proposed approach.

Reconstruction of scatterers with four different boundary conditions in inverse scattering problems

Xudong Chen[1], *Rencheng Song*[2] *and Xiuzhu Ye*[3]
Department of Electrical and Computer Engineering, National University of Singapore, Singapore[1], Halliburton, Singapore[2] Department of Electrical and Computer Engineering, Beihang University, Beijing, China[3].

This paper deals with a special inverse scattering problem, where four different boundary conditions, i.e., Dirichlet, Neumann, Robin, and transmission boundary, simultaneously exist. We present a general inversion method to simultaneously reconstruct scatterers with four different boundary conditions without prior information on their locations, shapes, or types of boundary condition. In the forward problem, the scattering of mixed scatterers is modeled by the T-matrix method, which provides a unified framework for representing the four different boundary conditions. The objective function considered in the inverse problem is solved by a subspace-based optimization method. The unknowns in the inverse problem are T-matrix coefficients, from which the types of boundary conditions of scatterers are inferred.

Application of Stochastic Optimization Methods to the Inverse Scattering of 2-Dimensional Scatterers

Chien-Ching Chiu
Department of Electrical Engineering, Tamkang University, Tamsui, Taiwan, R.O.C.

The application of stochastic optimization techniques for the reconstruction of 2-dimensional scatterers by the scattered fields is studied in the paper. The method of moment (MOM) used in the frequency domain and finite-difference time-domain (FDTD) technique is employed for electromagnetic analyses for both the forward and inverse scattering problems, while the reconstruction problem is transformed into optimization one during the course of inverse scattering. Then, four techniques including particle swarm optimization (PSO), asynchronous PSO (APSO), dynamic differential evolution (DDE) and self-adaptive DDE (SADDE) are applied to reconstruct the locations and shapes of buried conducting cylinders or the permittivities of buried inhomogeneous dielectric cylinders. The statistical performances of these algorithms are compared. The results show that SADDE outperforms PSO, APSO and DDE in terms of the ability of exploring the optima. However, these results are considered to be indicative and do not generally apply to all optimization problems in electromagnetics.

An Iterative Approach to Recover Images of Multiple Targets and Targets with Layered or Continuous Profile

Yu-Hsin Kuo[1] *and* *Jean-Fu Kiang*[2]
Graduate Institute of Communication Engineering, National Taiwan University, Taipei, Taiwan, ROC
E-mail: jfkiang@ntu.edu.tw

Electromagnetic inverse techniques have been widely explored in geophysical survey, target detection, non-destructive testing, medical imaging, and so on. Inverse methods of the scattering type have been widely applied to retrieve the electric properties of possible targets. The linear sampling method (LSM) has been proposed to estimate the target shape, implemented with the singular-value decomposition (SVD) and Tikhonov regularization techniques. With a fairly estimated target shape, the contrast source inversion (CSI) method can be applied to estimate the permittivity and conductivity, in both the target and the background medium.

Integral equation and finite difference methods have been integrated with the CSI method to recover targets immersed in layered background media, or inhomogeneous targets embedded in an inhomogeneous background medium. The LSM and the CSI method have also been applied to inverse multiple targets.

In general, the relative position of multiple targets affects the inverse results. For multiple targets having the same permittivity, the estimated permittivity in different targets sometimes turns out quite different. When the targets are placed close to one another, the permittivity in between the targets is often over-estimated. For a layered target, the permittivity in the external part tends to be under-estimated, and its recovered location may shift inwards.

The permittivity within a target tends to be under-estimated, especially when its electric size is large. With a larger permittivity difference between the target and the background, or between different parts of the target, the error in the recovered permittivity profile becomes larger.

In this work, an iterative approach, based on the linear sampling method and the contrast source inversion method, is proposed to improve the recovered images of multiple targets, layered targets and targets with a continuous profile. The challenges of targets with large electric size or high contrast are partially overcome using this approach in the simulations.

An adaptive phase space method for travel time tomography

Eric Chung
Department of Mathematics, Chinese University of Hong Kong, Hong Kong
E-mail: tschung@math.cuhk.edu.hk

In this work an adaptive strategy for the phase space method for traveltime tomography is developed. The method first uses those geodesics/rays that produce smaller mismatch with the measurements and continues on in the spirit of layer stripping without defining the layers explicitly. The adaptive approach improves stability, efficiency and accuracy. We then extend our method to reflection traveltime tomography by incorporating broken geodesics/rays, for which a jump condition has to be imposed at the broken point for the geodesic flow. In particular we show that our method can distinguish non-broken and broken geodesics in the measurement and utilize them accordingly in reflection traveltime tomography. We demonstrate that our method can recover the convex hull (with respect to the underlying metric) of unknown obstacles as well as the metric outside the convex hull. This is a joint work with Jianliang Qian, Gunther Uhlmann and Hongkai Zhao.

Inverse transport calculations with subspace optimization algorithms
Kui Ren
Department of Mathematics, University of Texas, Austin
E-mail: ren@math.utexas.edu

Inverse boundary value problems for the radiative transport equation play important roles many medical imaging techniques. Despite the rapid progress in the mathematical theory and numerical computation of these inverse problems in recent years, developing robust and efficient reconstruction algorithms remains as a challenging task and an active research topic. We propose here a robust reconstruction method that is based on subspace minimization techniques. The method splits the unknown transport solution (or a functional of it) into low-frequency and high-frequency components, and uses singular value decomposition to analytically recover part of low-frequency information. Minimization is then applied to recover part of the high-frequency components of the unknowns. We present some numerical simulations with synthetic data to demonstrate the performance of the proposed algorithm. This is a joint work with Tian Ding.

A direct imaging method for inverse scattering by unbounded rough surfaces
Bo Zhang
LSEC and Institute of Applied Mathematics, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing, China
E-mail: b.zhang@amt.ac.cn

This talk is concerned with the inverse scattering problem by unbounded rough surfaces. A direct imaging method is proposed to reconstruct the rough surface from the scattered near-field measurement at a constant height above the surface, corresponding to incident plane waves. The reconstruction method is very robust to noises of measured data and works for both penetrable surfaces and impenetrable surfaces with Dirichlet or impedance boundary conditions. The performance of the reconstruction method is analyzed, and numerical experiments are carried out to illustrate that the inversion algorithm is fast, accurate and stable even for the case of multiple scale profiles. This is a joint work with Dr Haiwen Zhang.

Numerical Issues of Inverse and Ill-posed Problems (MS II)

Organizer:
Benny Y.C. Hon
Department of Mathematics, City University of Hong Kong, Hong Kong
E-mail: Benny.Hon@cityu.edu.hk

Increasing stability in an inverse problem for the acoustic equation and the Schrödinger equation

Sei Nagayasu
Department of Mathematical Sciences, University of Hyogo, Himeji 671-2201, Japan
E-mail: sei@sci.u-hyogo.ac.jp

We study the phenomenon of increasing stability in the inverse boundary value problems for the acoustic equation and Schrödinger equation. It is known that this inverse problem is ill-posed. Nonetheless, we show that the ill-posedness decreases when we increase the frequency and the stability estimate changes from logarithmic type for low frequencies to a Lipschitz estimate for large frequencies. This work is a joint work with Victor Isakov, Gunther Uhlmann and Jenn-Nan Wang.

Numerical regularization methods of continuation solution of equations of electromagnetic fields

S. Kabanikhin[1] *and* *M. Shishlenin*[2]
Sobolev Institute of Mathematics, Novosibirsk, Russia[1,2]
E-mail: kabanikhin@sscc.ru[1], mshishlenin@ngs.ru[2]

The field continuation problems from the part of the boundary are ill-posed problems. The similar problems can be found, for instance, in geophysics and tomography when the field continuation allows to detect the parameters of the medium outside the investigated domain. We reduce the ill-posed problem to the inverse problem and reformulate it in the operator equation $Aq = f$. For numerical solution of the continuation problem we apply singular value decomposition method and gradient methods. The results of numerical calculations are presented.

Meshless computational methods for solving fractional-order partial differential equations

Dongfang Yun[1], *Yiuchung Hon*[2]
Department of Mathematics, City University of Hong Kong, Hong Kong[1,2]
E-mail: Benny.Hon@cityu.edu.hk[2]

In this talks, we present several meshless computational methods for solving partial differential equations. In particular, we present the recently developed Local Radial Basis Function Collocation Method (LRBFCM) for solving fractional-order partial differential equations (FOPDEs). Numerical examples will be constructed to verify the effectiveness and efficiency of the LRBFCM in solving various types of FOPDEs. For comparison, the numerical result will be compared with several other meshless methods using global radial basis functions (RBFs) from which one of the distinct advantages of LRBFCM in the selection of shape parameter is shown. Since the LRBFCM is not sensitive to the choice of the shape parameter, the well known ill-conditioning problem of RBFs method does not exist and hence can be able to deal with larger scale problems. Numerical examples will also be given to verify the stability of LRBFCM in the case when prescribed perturbations are added.

Keywords: Meshless Computation, Local Radial Basis Function Collocation Method, Fractional-order partial differential equations.

Multiscale support vector approach for solving ill-posed problems

Min Zhong[1], *Yiuchung Hon*[2], *Shuai Lu*[3]
Department of Mathematics, Southeast University, Nanjing 210096, China[1], Department of Mathematics, City University of Hong Kong, Hong Kong[2], School of Mathematical Sciences, Fudan University, Shanghai 200433, China[3]
E-mail: min.zhong@seu.edu.cn[1], Benny.Hon@cityu.edu.hk[2], slu@fudan.edu.cn[3]

Based on the use of compactly supported radial basis functions, we extend in this paper the support vector approach to a multiscale support vector approach (MSVA) scheme for approximating the solution of a moderately ill-posed problem on bounded domain. The Vapniks ϵ-intensive function is adopted to replace the standard ℓ^2 loss function in using the regularization technique to reduce the error induced by noisy data. Convergence proof for the case of noise-free data is then derived under an appropriate choice of the Vapniks cut-off parameter and the regularization parameter. For noisy data case, we demonstrate that a corresponding choice for the Vapniks cut-off parameter gives the same order of error estimate as both the a posteriori strategy based on discrepancy principle and the noise-free a priori strategy. Numerical examples are constructed to verify the efficiency of the proposed MSVA approach and the effectiveness of the parameter choices.

Keywords: Multiscale support vector approach, Compactly supported radial basis functions, Ill-posed problems, Regularization methods.

Inverse Problems for Elliptic Systems (MS III)

Organizer:
Ching-Lung Lin
Department of Mathematics, National Cheng Kung University, Taiwan
E-mail: cllin2@mail.ncku.edu.tw

Spectral theory of the Neumann-Poincaré operator and applications
Hyeonbae Kang
Department of Mathematics, Inha University, Korea
E-mail: hyeonbaekang@gmail.com

The Neumann-Poincaré (NP) operator is a boundary integral operator arising naturally when solving classical Neumann and Dirichlet boundary value problems using layer potentials. It is not self-adjoint in th usual L^2 space, but can be symmetrized using Plemelj's symmetrization principle. I will talk on the spectral properties of the NP operator and applications. Among applications are uniformity of the regularity estimates and boundary perturbation formula, and plasmonics.

CGO solutions of anisotropic Maxwell's equations
Rulin Kuan
Department of Mathematics, National Taiwan University, Taiwan
E-mail: d97221002@ntu.edu.tw

Complex geometrical optics (CGO) solutions are important tools in inverse problems. A lot of applications of them for isotropic problems are studied. However, it is hard to obtain this kind of solutions for anisotropic materials (such as elastic systems and Maxwell systems) generally. In this talk, we will introduce our recent work on constructing CGO solutions of the time-harmonic anisotropic Maxwell's equations.

Reconstruction of penetrable obstacles in the anisotropic acoustic scattering
Yi-Hsuan, Lin
Department of Mathematics, National Taiwan University, Taiwan
E-mail: d01221001@ntu.edu.tw

We develop an enclosure-type reconstruction scheme to identify penetrable obstacles in acoustic waves with anisotropic medium in $\mathbb{R}^3$. The main difficulty of treating this problem lies in the fact that there are no complex geometrical optics solutions available for the acoustic equation with anisotropic medium in $\mathbb{R}^3$. Instead, we will use another type of special solutions called oscillating-decaying solutions. Even though that oscillating-decaying solutions are defined only on the half space, we are able to acquire needed boundary inputs by the Runge approximation property. Moreover, since we are considering a Helmholtz-type equation, we turn to Meyers' L^p estimate to compare the integrals coming from oscillating-decaying solutions and those from reflected solutions.

Inverse boundary value problem for the Stokes equations in the plane

Ru-Yu Lai
Department of Mathematics, University of Washington, Seattle, USA
E-mail: rylai@uw.edu

We will discuss the global identifiability of the viscosity in an incompressible fluid in two dimensional case. The main focus in this talk will be on deriving that the inverse boundary value problem for the two dimensional Stokes equations and that for a first order system are equivalent. We then show that the uniqueness of the viscosity can be determined by the Cauchy data. This is a joint work with G. Uhlmann and J.-N. Wang.

Inverse Spectral Problems (MS IV)

Organizer:
Chun-Kong Law
Department of Applied Mathematics, National Sun Yat-sen University, Taiwan
E-mail: law@math.nsysu.edu.tw

Direct and inverse discontinuous Sturm-Liouville problems

Guangsheng Wei
College of Mathematics and Information Science, Shaanxi Normal University, China

In this talk, we are concerned with the direct and inverse Sturm-Liouville problems with the discontinuity conditions involved spectral parameter at finite interior points inside a finite interval. We establish the self-adjoint operator-theoretic formulation for this problem, and give the expansion theorem by reference to the self-adjointness of the operator. Furthermore, we provide uniqueness theorems for this inverse spectral and nodal problems.

Uniqueness of the potential function for the vectorial Sturm-Liouville equation on a finite interval

Chung-Tsun Shieh
Deptartment of Mathematics, Tamkang University, Taiwan

In this talk, we consider the Inverse spectral problems for vectorial Sturm-Liouville differential equation on a finite interval. Some uniqueness theorems shall be presented.

An inverse problem related to some reaction-diffusion models

Wei-Chuan Wang
Center for General Education, National Quemoy University, Taiwan
E-mail: wangwc72@gmail.com

A basic reaction-diffusion model,

$$U_t - U_{xx} = \lambda U - w(x)|U|^p U,$$

is considered first. It is known that steady states of the above problem possess a finite number of nodes. A stationary solution with such oscillation property is referred to as a stripe pattern. In certain situations, a collection of patterns can be found out when there exist steady states with different nodal properties. The inverse problem considered here is to track heterogeneity such as w from the locations of nodes in a collection of stripe patterns. Furthermore, an extension to a FitzHugh-Nagumo type elliptic system,

$$\begin{aligned} -u_{xx} &= \lambda u - w(x)|u|^p u - v \\ -v_{xx} &= u - \gamma v, \end{aligned}$$

is also treated here. This talk consists of results joint with C.N. Chen and C.K. Law.

Polya-Cartwright-Levinson theory and its implications in interior transmission

Lunghui Chen
Department of Mathematics, National Chung Cheng University, Taiwan
E-mail: mr.lunghuichen@gmail.com

In this talk, we will examine the interior transmission eigenvalue problem in inverse scattering theory. The solutions to this transmission problem have a spectral parameter holomorphic in C. Accordingly, the inverse problem with this class of eigenvalues is alternatively considered as an inverse problem retrieving certain spectral invariants in some entire functions by its zero set.

Applications of Carleman Estimates to Inverse Problems (MS V)

Organizer:
Jenn-Nan Wang
Institute of Applied Mathematical Sciences, National Taiwan University, Taiwan
E-mail: jnwang@math.ntu.edu.tw

Inverse Problem for an Hyperbolic-Parabolic System

Patricia Gaitan
Aix-Marseille University, France
Email: patricia.gaitan@univ-amu.fr

Let Ω be a bounded domain of $\mathbb{R}^n$ with C^2 boundary $\partial\Omega$. We denote $\partial\Omega = \Gamma^- \cup \Gamma^+$ such that $\Gamma^\pm = \{x \in \partial\Omega, \quad \pm A(x) \cdot \nu(x) > 0\}$, where $\nu(x)$ is the outer normal to $\partial\Omega$ at $x \in \partial\Omega$. Let $\omega \subset \Omega$ be a non empty subdomain such that $\partial\omega \supset \Gamma^+$.
We consider the following Parabolic-Hyperbolic System

$$\begin{cases} \partial_t u + A(x).\nabla u = F(u,v) & \text{in } \Omega \times (0,T), \\ \partial_t v - \triangle v = G(u,v) & \text{in } \Omega \times (0,T), \\ u(x,t) = h_0(x,t) & x \in \Gamma_-, \\ v(x,t) = h(x,t) & x \in \Gamma, \\ u(x,0) = u_0(x),\ v(x,0) = v_0(x) & x \in \Omega. \end{cases}$$

This system describes the process of tumour-induced angiogenesis. This process allows the tumour to progress from the avascular (lacking blood vessels) to the vascular (possessing a blood supply) state and is initiated and controlled by a diffusive chemical compound, known as tumour-angiogenesis factor (TAF) which is released by the tumour cells into the surrounding tissue.
Here $u(x,t)$ represents the cells density of the blood vessels and $v(x,t)$ is the TAF concentration. The reaction terms F and G are given by

$$F(u,v) = \mu(x)v - \gamma(x)u, \quad G(u,v) = \delta(x)u - k(x)v,$$

where μ, γ, δ, k are time-independant coefficients.
The aim of this work is to reconstruct the two coefficients μ and δ from an interior measurement of only one component and data of two components at a fixed time $\theta \in (0,T)$, that is :

$$v|_{\omega \times (0,T)} \quad \text{and} \quad (u,v)|_{\Omega \times \{\theta\}}.$$

Indeed, roughly speaking, our main result is the following stability inequality

$$||\mu - \widetilde{\mu}||^2_{L^2(\Omega)} + ||\delta - \widetilde{\delta}||^2_{L^2(\Omega)} \leq ||v - \widetilde{v}||^2_{H^3((0,T);\, H^2(\omega))} + ||(v - \widetilde{v})(\cdot,\theta)||^2_{H^2(\Omega)}$$

$$+ ||(u - \widetilde{u})(\cdot,\theta)||^2_{H^1((0,T);L^2(\Omega))} + ||A(x) \cdot \nabla(u - \widetilde{u})(\cdot,\theta)||^2_{L^2(\Omega)}.$$

The key ingredient to obtain such a result is Carleman estimates.

An inverse problem concerning the determination of a discontinuous coefficient in a one-dimensional wave equation

Shumin Li[1] *and Dandan Sheng*[2]
University of Science and Technology of China, China[1,2],
E-mail: shuminli@ustc.edu.cn[1], shdan@mail.ustc.edu.cn[2]

A wave equation with a discontinuous coefficient is discussed. For the solution $\partial_t^2 y - \partial_x(c(x)\partial_x y) = 0$ in $(-T,T) \times \Omega'$ with given $y|_{(-T,T)\times\partial\Omega}$, $y(0,x)$, $\partial_t y(0,x)$, and transmission conditions, an inverse problem concerning the determination of the discontinuous coefficient $c(x)$ is considered by data $\partial_{xt} y(\cdot,0)$, $\partial_{xtt} y(\cdot,0)$ and a Holider stability estimate for the inverse problem is proved by providing that $c(x)$ satisfy a priori condition. Here $T > 0$, $\Omega = (0,1) \subseteq \mathbb{R}$ is a bounded domain, $\Omega' = \Omega \backslash \{a_0, \cdots, a_n\}$ and $0 < a_0 < \cdots < a_{n-1} < 1$, $\{a_0, \cdots, a_{n-1}\}$ is the set of the discontinuity points of $c(x)$. The proof is mainly based on the methodology of applying a weighted estimate called a Carleman estimate which is invented by Bukhgeim and Klibanov in 1981. Thus we first establish a Carleman estimate for a one-dimensional wave equation with a discontinuous coefficient.

Determination of source term in a distributed order time-fractional diffusion equation

Zhiyuan Li
The University of Tokyo
E-mail: zyli@ms.u-tokyo.ac.jp

In this talk, firstly, the forward problem for the initial-boundary-value problem for distributed order time-fractional diffusion equation in bounded multi-dimensional domains is considered. The use of Laplace transform and eigenfunction expansion yield that the equation demonstrates a logarithmic decay of solution at infinity, and the decay rate of the solution for short time is dominated by $(tlogt)^{-1}$ as well as the weak unique continuation of the solution. By the weak unique continuation, coupled with the Duhamel's principle, the uniqueness for inverse problems of determining source term of initial-boundary-value problem of distributed order time-fractional diffusion equation by measuring the Neumann derivative of the solution on partial boundary is established.

The partial unique continuation for the elliptic partial differential systems

Jin Cheng
School of Mathematical Sciences, Fudan University, Shanghai 200433, China
Email: jcheng@fudan.edu.cn

The unique continuation is one of the important properties for the partial differential equations, which can be used to study the inverse problems for partial differential equations. In this talk, we will present an interesting unique continuation result for the elliptic system with constant coefficients. That is, if the one component of the solution

vanishes on a small sub-domain, this component will vanish on the whole connected domain. This partial unique continuation result is very useful for studying inverse problems of the elliptic systems if the measurement can only be made only for some components of the solution. The conditional stability can be proved, which implies that the convergence of the Tikhonov solutions.

Numerical methods for direct and inverse problems (MS VI)

Organizer:
June-Yub Lee
Department of Mathematics, Ewha Womans University, Korea
E-mail: jyllee@ewha.ac.kr

Variational linear algebra algorithm of extended matrix pseudo-inverse computation

Jay Min LEE[1], *Youngjoo CHUNG*[2] *and YongHoon KWON*[3]
Pohang Accelerator Lab, POSTECH, Pohang 790-784, Korea[1], School of Information and Communications, GIST, Gwangju 500-712, Korea[2], Department of Mathematics, POSTECH, Pohang 790-784, Korea[3]
E-mail: jaymin@postech.ac.kr[1]

A theme of applied mathematics requires numerically reliable matrix inversion computation, even with extreme difficulty since coefficient matrix becomes more ill-conditioned as the matrix size increases. Such a discrete inverse problem, involving conditioned tolerance at under-determined cases of measures, is associated with the information covariance analysis over noises and round-off errors. It is desirable to seek an algebraic invertible matrix representation, that is, a goal of reliable projection implementing matrix topology-invariant feature preservation. We proposed a linear algebra matrix quadrant span algorithm; (i) the -variation (defined in Tikhonov's regularization) algorithm for extended matrix pseudo-inverse computation, (ii) relevant symbolic computation using Mathematica, and (iii) relevant numerical results and interpretation. Our algorithm presents the remarkable enhancement of invertible matrix reliability. Through our numerical simplifications, we showed that the inherent SVD inverse formula is distinguished from other conventional truncated SVD methods. We will discuss the matrix norm feature indicating the smoothness degree of ill-posed matrix conditions. This variational method of pseudo inverse matrix computation is appropriate for the principal component analysis for extracting low-rank invertible matrices. In addition, this robust algorithm will be useful for resolving inverse problems in diverse complex systems.

Numerical approaches for solving nonnegative inverse singular value problems

Min-Hsiung Lin
National Chung Cheng University, Taiwan
E-mail: mhlin@ccu.edu.tw

Inverse singular value problems have been a research focus for decades. Clearly, an inverse singular problem is trivial if the desired matrix is not restricted to a certain structure. This talk will present a numerical procedure, based on the successive projection process, to solve inverse singular value problems for nonnegative matrices subject

to given diagonal entries. Although we focus on a specific type of inverse singular value problems with prescribed diagonal entries, this entire procedure can be straightforwardly applied to other types of structure. Numerical examples are used to demonstrate the capacity and efficiency of our method.

Reconstructing ODFs from human brain by using positive semi-denite tensors

Xu Yi
E-mail: yi.xu1983@gmail.com

In this report, we reconstruct ODFs from human brain by using positive semi-definite tensors. We proposes a novel technique to guarantee nonnegative ODF by minimizing a convex optimization problem, which involves a convex quadratic objective function constrained by the nonnegativity requirement on the smallest Z-eigenvalue of the diffusivity tensor. Based upon convex analysis and optimization theory, we derive its optimality conditions. And then we propose a gradient descent algorithm for solving this problem. We also present formulas for determining the principal directions (maxima) of the ODF. Numerical examples on synthetic data as well as MRI data are displayed to demonstrate our approach. At last, we show the bres image of human brain by using our new method.

A fast direct solver for quasi-periodic scattering problems with material junction points

June-Yub Lee
Department of Mathematics, Ewha Womans University, Korea
E-mail: jyllee@ewha.ac.kr

A number of problems in computational physics require the solution of the Helmholtz equation. The interaction of acoustic or electromagnetic waves with structured, periodic materials is often complicated by the fact that the scattering geometry involves domains where multiple media meet at a single point. We present a robust integral equation method for the calculation of two-dimensional scattering problems in the presence of triple-points, that is problems involving multiple materials meeting at a single point. Our approach involves both the modification of a standard integral representation [V. Rokhlin (1983), Wave Motion, 5, 257-272] and the use of adaptive refinement at geometric singularities [L. Greengard and J.-Y. Lee (2012), J. of Computational Physics, 231, 2389-2395]. The GMRES iterative solver equipped with Fast Multipole Method (FMM) for the second kind integral equation is an optimal algorithm for a single right-hand-side in the sense that it is a linear-timecomplexity algorithm with a reasonably small constant. However, a direct numerical method is more efficient algorithm for multiple right-hand-sides. Our fast direct solver is based on a the interpolative decomposition (ID) that is more useful and produces a near-optimal representation for multilevel compression of the linear system of equations [K. L. Ho and L. Greengard (2012), SIAM J. Sci. Comput., 34, A2507-A2532]. We demonstrate the performance of the scheme with several numerical examples.

Miscellaneous topics on inverse problems (MS VII)

Organizer:
I-Liang Chern
Department of Mathematics, National Central University, Taiwan
E-mail: chern@math.ntu.edu.tw

Inversion formulas for cone transforms arising in application of Compton cameras

Sunghwan Moon
UNIST, Korea.
Email: shmoon@unist.ac.kr

A Compton camera has been suggested for use in single photon emission computed tomography because a conventional gamma camera has low efficiency. It brings about a cone transform, which takes a given function and assigns to it the surface integral of the function over cones determined by the detector position, the central axis, and the opening angle of the Compton camera.

We provide an inversion formula using complete Compton data for 3 and 2-dimensional cases. Numerical simulations were performed to demonstrate the suggested algorithms in dimension 2.

Bayesian Geometric Inverse Problems Arising in Subsurface Flow

Kui Lin
Fudan University, China
E-mail: link10@fudan.edu.cn

In this talk, we present the application of the infinite-dimensional Bayesian framework for the solution to geometric inverse problems that arise in subsurface flow. In particular, we are interested in determining the permeability of the subsurface from pressure measurements, within the framework of an incompressible single-phase Darcy flow. We incorporate prior knowledge in terms of geometric features relevant to the characterization of the geologic properties of the subsurface. These prior models of the permeability lead to the estimation of a finite number of unknown parameters determining the geometry, together with either a finite number of values (piecewise constant permeability) or a finite number of fields (piecewise continuous function permeability). We discuss key aspects of rigorous application of the Bayesian framework, showing existence and well-posedness of the resulting Bayesian posterior. In addition, we introduce novel Markov Chain- Monte Carlo (MCMC) methods in which prior-reversible proposals are defined to exploit the posterior. Finally some numerical examples are presented to illustrate the feasibility of the methodology. This is joint work with Marco Iglesias (University of Nottingham) and Andrew Stuart (University of Warwick).

On Spectral Analysis and a Novel Algorithm for Transmission Eigenvalue Problems

Wei-Qiang Huang[1], *Tiexiang Li*[2]
Department of Applied Mathematics, National Chiao Tung University, Taiwan[1], Department of Mathematics, Southeast University, China[2]
E-mail: txli@seu.edu.cn[2]

The transmission eigenvalue problem, besides its critical role in inverse scattering problems, deserves special interest of its own due to the fact that the corresponding differential operator is neither elliptic nor self-adjoint. In this paper, we provide a spectral analysis and propose a novel iterative algorithm for the computation of a few positive real eigenvalues and the corresponding eigenfunctions of the transmission eigenvalue problem. Based on approximation using continuous finite elements, we first derive an associated symmetric quadratic eigenvalue problem (QEP) for the transmission eigenvalue problem to eliminate the nonphysical zero eigenvalues while preserve all nonzero ones. In addition, the derived QEP enables us to consider more refined discretization to overcome the limitation on the number of degrees of freedom. We then transform the QEP to a parameterized symmetric definite generalized eigenvalue problem (GEP) and develop a secant-type iteration for solving the resulting GEPs. Moreover, we carry out spectral analysis for various existence intervals of desired positive real eigenvalues, since a few lowest positive real transmission eigenvalues are of practical interest in the estimation and the reconstruction of the index of refraction. Numerical experiments show that the proposed method can find those desired smallest positive real transmission eigenvalues accurately, efficiently, and robustly.

Asymptotic analysis for dynamical probe method

Yong-Gwan Ji[1] *and Gen Nakamura*[2]
Department of Mathematics, Inha University, Korea[1,2]
E-mail: jyg0131@naver.com[1]

The dynamical probe method is a method which give a basis for the active thermography to detect unknown cracks, cavities and inclusions in a heat conductor. In this talk, we focus on detecting unknown inclusions. For the numerical realization of this method, there was a problem of overshooting the boundary of inclusions when we probe them from their outside. This problem can be avoided, if we know how close to the unknown boundary of inclusion by looking at the asymptotic behavior of the so-called pre-indicator function defined from measured data. In this talk we will show that this is possible for some cases.