Authors:

Vera A. Gonzales, PhD
Kenneth J. Ottenbacher, PhD

Affiliations:

From the School of Allied Health Sciences, University of Texas Medical Branch at Galveston, Galveston, Texas.

Reprints:

All correspondence and requests for reprints should be addressed to Kenneth J. Ottenbacher, PhD, SAHS 4.202, University of Texas Medical Branch at Galveston, 301 University Boulevard, Galveston, TX 77555-1028.

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Research Series Article

Measures of Central Tendency in Rehabilitation Research

What Do They Mean?

Gonzales VA, Ottenbacher KJ: Measures of central tendency in rehabilitation research: what do they mean? *Am J Phys Med Rehabil* 2001;80:141–146.

ABSTRACT

Measures of central tendency including the mean, median, and mode are commonly reported in rehabilitation research. It is believed that the relationship among the mean, median, and mode changes in a specific way when the distribution being analyzed is skewed. A number of widely used textbooks were reviewed to determine how the relationship among the mean, median, and mode is presented in the health sciences and rehabilitation literature. We report a potential misinterpretation of the relationship between measures of central tendency that was identified in several research and statistical textbooks on the subject of rehabilitation. The misinterpretation involves measures of central tendency derived from skewed unimodal sample distributions. The reviewed textbooks state or imply that in asymmetrical distributions, the median is always located between the mode and mean. An example is presented illustrating the fallacy of this assumption. The mean and median will always be to the right of the mode in a positively skewed unimodal distribution and to the left of the mode in a negatively skewed distribution; the order of the mean and median is impossible to predict or generalize. The assumption that the median always falls between the mode and mean in the calculation of coefficients of skewness has implications for the interpretation of exploratory and confirmatory data analysis in rehabilitation research.

Key Words: Skewness, Measurement, Statistics, Averages

A description of the mean, median, and mode is included in every biomedical or health sciences research textbook presenting elementary data analysis procedures. In a review documenting the use of various statistical methods reported in the medical rehabilitation research literature, Schwartz and colleagues[1] stated, "Descriptive statistics were so frequently encountered that, although they were considered a statistical method, their incidence was not separately tabulated." Other investigators[2] have reported that the mean, standard deviation, and related measures of central tendency are the commonest statistics reported in the biomedical literature. Unfortunately, some descriptions of central tendency associated with data that are not normally distributed are misleading and may contribute to statistical misinterpretations. These misinterpretations exist at two different levels of analysis, namely, exploratory or confirmatory.

In describing data analysis procedures, Tukey[3] distinguished between exploratory and confirmatory data analysis. Exploratory data analysis (EDA) uses descriptive procedures and pattern recognition to examine the characteristics of data. In contrast, confirmatory data analysis relies on statistical hypothesis testing to evaluate quantitative data.[4–6] Discussion of data analytic issues in rehabilitation and health sciences research often focuses on statistical inference and confirmatory analysis rather than description and exploratory analysis.[5] Hoaglin, Mosteller, Tukey,[4] and others[7] have argued that one result of this emphasis on confirmatory data analysis is that clinical investigators know P values and F ratios, but often they do not understand or ignore patterns and basic relationships in data.

Tukey[3] contended that EDA should be a component of all research studies involving quantitative data. Exploratory analysis and descriptive examination should be the first step in any rehabilitation research or evaluation study.[7] Royeen and Geiger[7] suggested that EDA be used in clinical research and evaluation as a "preliminary step to determine what the data reveal" (p. 72). They suggested that "subsequent to exploratory data analysis, confirmatory or inferential statistical procedures can be selected based on the findings of the exploratory data analysis."[7] Sinacore, Chang, and Falconer[8] provided an excellent example of EDA in healthcare program evaluation. Their illustration demonstrates how the use of EDA can reveal important information relevant to program effectiveness that might be overlooked if only conventional confirmatory statistical methods are used.

EDA frequently involves examination of various measures of central tendency and the distributional characteristics of symmetry and skewness.[9] Measures of central tendency also have important implications regarding the development of accurate inferences in confirmatory data analysis.[8]

Measures of Central Tendency

The computational characteristics of the mean, median, and mode are familiar to researchers in medical rehabilitation. They will not be described here. The potential error involving measures of central tendency occurs when the mean, median, and mode are derived from skewed unimodal sample distributions. The error involves a misinterpretation regarding relative placement or order of the three measures of central tendency in a skewed distribution. This error is contained in numerous introductory statistics texts. For example, in a text titled *"Basic Statistics for the Health Sciences,"* Kuzma[10] stated that: *In a symmetrical distribution the three measures of central tendency are identical. In an asymmetrical (Fig. 4.2) distribution the mode remains located (by definition) at the peak; the mean is off to the right; and the median is in-between. Left-skewed distributions are the same, but in a mirror-image (p. 38).*

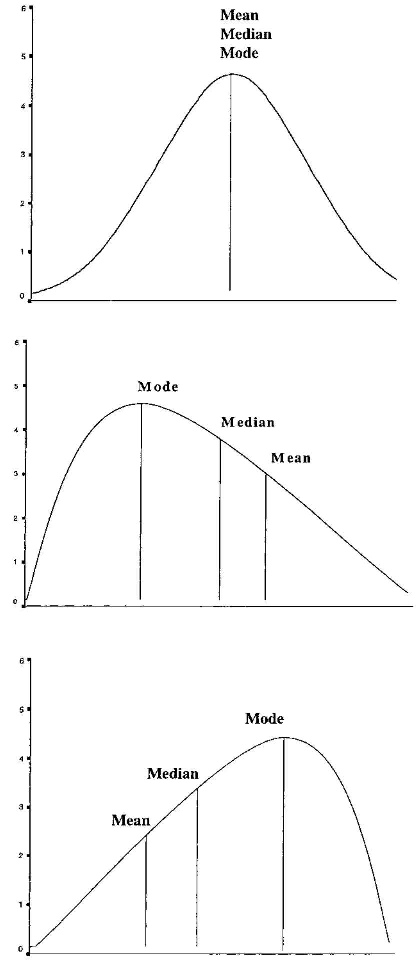

Figure 1. Measures of central tendency in symmetrical and asymmetrical distribution.

Graphs illustrating this relationship are presented in Figure 1.

This presentation is commonly found in rehabilitation and health sciences research textbooks either in a narrative or graphic format. The implication is that in a skewed, unimodal sample distribution, the mean is always pulled furthest toward the tail of the distribution and the median is located between the mode and mean. In such cases, the authors argued that the median is the most appropriate (accurate) statistic to use for descriptive purposes. For example, Vercruyssen and Hendrick[11] stated, "when a distribution is badly skewed the mean is pulled toward the tail and is always higher (or lower) than the median and the mode" (p. 58). They further stated that in cases of skewed data, the median is the preferred measure of central tendency.

In their text, "*Essentials of Biostatistics,*" Elston and Johnson[12] presented graphs similar to those in Figure 1 (see p. 53) and stated that in "symmetric unimodal distributions the mean, the median and the mode are all equal. In a unimodal asymmetric distribution the median always lies between the mean and the mode" (p. 52). Portney and Watkins,[13] in their widely used text "*Foundations of Clinical Research: Applications to Practice,*" also provided graphs similar to those in Figure 1 and stated, "The median will always fall between the mode and the mean in a skewed curve, and the mean is pulled toward the tail" (p. 322).

Although it is true that the median may lie between the mode and mean in many cases when the data are skewed, it is not true for all skewed unimodal distributions. The relative order of the mean and median for unimodal, skewed sample distributions is not predictable in the absolute manner presented in many health and behavioral sciences textbooks. As an illustration of the variation in distributional placement of the median and mean, consider the following numbers 18, 20, 22, 24, 26, 29, 35, 35, and 39. The descriptive data and graphic distribution of the nine numbers are presented in Figure 2. Examination of Figure 2 reveals that the distribution is negatively skewed. The relative order of the three measures of central tendency is not in agreement with the narrative descriptions and graphic distributions commonly included in textbooks presenting basic statistical procedures.[7, 10–16]

The error might be considered trivial if it were not so widespread

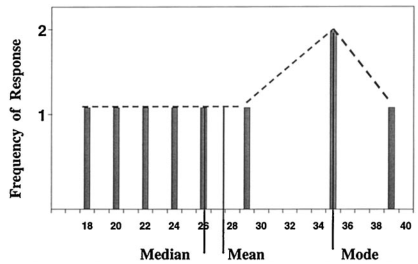

Figure 2. Example of data and relative placement of mode, median, and mean.

and contrary to the spirit of EDA. Correction of the error involves a simple qualification of the interpretation presented above. For skewed, unimodal sample distributions, the mean and median will always be to the right of the mode in a positively skewed distribution, and to the left of the mode in a negatively skewed distribution. The relative position of the mean and median, however, is impossible to predict or generalize. Although it is usually assumed that the median will fall between the mode and mean in skewed distributions, there is no empiric evidence to support the assumption that this is always true, or even true in a majority of cases in the medical rehabilitation research literature.[8]

Implications

The assumption that the mean will always be the extreme value in a skewed, unimodal sample distribution has practical implications in the mathematical calculation of frequently used coefficients of skewness. Coefficients of skewness and other descriptive characteristics of a distribution are often referred to as "moments" of the distribution. Moments are simply the expectations of different powers of the random variable.[17] For example, the first moment about the origin of a random variable *X* is the mean. The second moment about the mean is the variance and the third moment about the mean is the skewness. The third moment will be zero for a symmetric distribution, negative for skewness to the left, and positive for skewness to the right. The statistical characteristics of skewness (third moments) were extensively described by Pearson[18–20] in a series of articles published >70 yrs ago. Several formulas exist to compute third moment (skewness) coefficients. These formulas are based on the mathematical description presented by Pearson. The formula most commonly used in current statistical software packages to compute skewness is shown below:

Skewness (S_k)

$$= \frac{1/n \sum^{n} i=1(x_i - \mu)^3}{\sigma^3} \qquad (1)$$

This formula involves computing the average of the cubed deviations from the mean and then dividing the average by the cube of the standard deviation. By cubing the deviations the signs are preserved providing an indication of whether the distribution is negatively or positively skewed.

Two other formulas used to compute coefficients of skewness (S_k), originally proposed by Pearson,[18–20] are presented below:

$$S_k = \frac{3(\mu - \text{median})}{\sigma} \qquad (2)$$

$$S_k = \frac{\mu - \text{mode}}{\sigma} \qquad (3)$$

In these formulas, S_k equals the coefficient of skewness, μ is population mean (or its estimate), and σ is the population standard deviation (or its estimate).

When the data from the example presented in Figure 2 are analyzed using Equation 1, the result is: 90.04/340.06 = +0.26. This value suggests a positive coefficient of skewness, although the distribution is negatively skewed. When the second equation is used, the result is: [3(27.55 − 26]/6.98 = +0.67. When the data in Figure 2 are analyzed using the third equation, the coefficient of skewness is $(27.55 - 35)/6.98 = -1.07$. This value accurately reflects the negatively skewed characteristic of the sample distribution.

The third equation provides a more accurate coefficient of skewness for the data presented in Figure 2 because it does not depend on the relative order of the mean and median. Rather, the third equation is sensitive to which side of the mode the mean is located on. Unfortunately, the third equation is infrequently used or recommended in basic statistics textbooks or statistical software packages because of the problem encountered when the data are from a bimodal distribution.

It is often recommended that the median be reported as the preferred measure of central tendency if the data are from a non-normal (skewed) distribution. The recommendation is based on the assumption that the median will always be a value between the mode and mean. As Figure 2 illustrates, this is not always the case.

Determining which measure of central tendency is most appropriate for describing a distribution depends on several factors. The scale of measurement of the variable is an important consideration. All three measures of central tendency can be applied to variables on the interval or ratio scales. For data on the nominal scale, only the mode is meaningful. If data are ordinal, both the median and mode can be applied. It is also necessary to consider how the summary measure will be used statistically. The mean is considered the most stable of the three measures of central tendency. If we were to repeatedly draw random samples from a population, the means of those samples would fluctuate less than the mode or median. Each of the measures of central tendency is, in its way, a best guess about any score, but the sense of "best" differs with the way error is regarded. If both the size of the errors and their signs are considered important, and the investigator wants zero error in the long run, then the mean serves as the best "guess." If the investigator wants to be exactly right as often as possible, then the mode is indicated. If, on the other hand, the researcher wants to come as close as possible on the average, irrespective of sign or error, then the median is the best guess. From the point of view of purely descriptive statistics, as apart from inferential work, the median is a most serviceable measure. Its property of representing the typical (most nearly like) score makes it fit the requirements of simple and effective communication better than the mean in many contexts.

Rehabilitation researchers dealing with skewed data should compute all three measures of central tendency to determine which will be the most representative for a given set of numbers. Portney and Watkins[13] noted: "The choice of which index to report with skewed distributions depends on what facet of information is appropriate to the analysis. It is often reasonable to report all three values, to present a complete picture of a distribution's characteristics." The techniques of EDA should also be used to "present a complete picture of a distribution's characteristics" when the data are skewed. Tukey[3] and others[4] pioneered the use of EDA methods that provide detailed information regarding data patterns and characteristics.

Box plots. In the box plot, the upper and lower quartiles of the data are portrayed by the top and bottom of a rectangle and the median portrayed by a horizontal line segment within the rectangle (Fig. 3). The mean can be designated as a plus (+) in the rectangle. The lines from the rectangle, referred to as "whiskers," can extend to represent the minimum and maximum values or the 10th and 90th percentiles (Fig. 3). Outliers may be identified using a defined outlier detection rule and displayed as asterisks or other symbol. Advantages of the box plot include the following: (1) the central rectangle includes 50% of the data, (2) the whiskers show the range of data, and (3) symmetry is indicated by the box and whisker relationship and location of the mean and median. It is easy to compare distributions of more than one sample by constructing side-by-side box plots. Two disadvantages are that box plots do not depict the mode and do not include raw values.

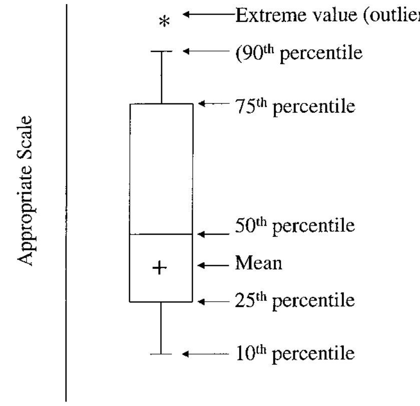

Figure 3. Sample box plot showing distribution and descriptive statistics.

Stem-and-leaf plots. The stem-and-leaf plot is a hybrid between a table and a graph because it shows numerical values, but also presents a profile of the data distribution. Figure 4 depicts a stem-and-leaf plot of effect size values (d-indexes) computed from a meta-analysis examining the effectiveness of medical rehabilitation in persons with stroke.[21] The numbers in the vertical column (*on the left*) represent the "stem" in Figure 4 and the numbers in the row to the right represent the "leaf" values. Each stem-and-leaf combination indicates a number in the data set. For example, for the stem 0.9 (*left column*), the following "leaf" values are found in the row to the right, 0119. These stem-and-leaf values represent the following four numbers, 90, 91, 91, and 99. Stem-and-leaf plots can include the raw data values and re-

Stem	Leaf	
1.4	4	
1.3	3 3	
1.2		
1.1		
1.0	0	
0.9	0 1 1 9	
0.8	1 1 1 1 1 3 8 8 8 8 8 9	
0.7	1 1 2 3 3 6 7 7 7 7 7 9	
0.6	1 1 1 2 2 4 5 5 6 6 6 6 6 6 7 9 9	
0.5	1 2 5 7 8 8 9	
0.4	0 0 1 1 1 1 1 1 3 3 4 4 4 4 4 5 7 8 9	(Mean = 0.40)
0.3	1 1 1 1 1 2 2 3 3 3 3 3 3 3 3 4 4 4 5 7 9	(Median = 0.33)
0.2	1 1 1 1 1 2 2 2 2 2 2 2 2 4 4 7 7 7 7 8 9	
0.1	0 1 2 2 2 2 2 2 2 2 2 3 3 3 3 4 4 5 6 6 7 7 8 8 9 9 9	(Mode = 0.12)
0.0	0 0 0 0 0 0 0 0 1 2 4 4 4 5 5 7 7 7 7 8 8 8 8 9 9 9	

Mean =	0.40	Minimum =	0.00
Median =	0.33	Maximum =	2.22
SD =	0.33	Q_1 =	0.14
N =	173	Q_3 =	0.65

Figure 4. Stem-and-leaf plot of effect size values (d-indexes) from stroke rehabilitation trials.[21]

lated descriptive statistics, that is, mean, median mode (Fig. 4). In addition, the shape of the distribution is "graphically" displayed in a stem-and-leaf plot. Depending on the size of the data set, it may be necessary to change the measurement units by multiplying by some power of 10 or to truncate the data (that is, to ignore some digits on the right) to get values suitable for the stem-and-leaf display. There are also methods to transform the data, for example, by taking logarithms. The rules for making the diagram can be modified if one finds that some other variation does a better job for a particular data set. For example, each leaf can be two digits rather than one, with a comma separating the leaves on a single row so that 4 24,38,45 would represent 424, 438, and 445. Tukey[3] and others[4] provided detailed information and instructions regarding how to create stem-and-leaf plots.

CONCLUSIONS

Accurate EDA relies on the ability to compute and interpret basic descriptive statistics. Any error or misinterpretation at this fundamental level of data analysis may lead to contradictory results and contribute to a form of statistical confusion that Cook and Campbell[22] referred to as statistical conclusion invalidity. In a discussion of data analysis procedures in medical rehabilitation research, Findley[23] advised clinical researchers that "you cannot bypass the data description phase of analysis" (p. 92). Findley[23] noted that "you need to be sure that your choice of central value and dispersion truly represent your data. If you decide to move to more analyses, such as testing differences between groups, then it is even more important that the values for each group that you subject to statistical tests truly represent that group" (p. 92). If basic statistical values such as measures of central tendency do not reflect the intended characteristics, the potential for statistical misinterpretation is increased.

Frequently, errors associated with statistical conclusion validity in medical rehabilitation research are impossible to eliminate. For example, problems associated with low statistical power and type 2 errors are often difficult to resolve in clinical or field based studies with relatively small sample sizes. Obstacles associated with the interpretation of basic descriptive statistics, such as measures of central tendency, can be resolved more directly if authors and rehabilitation researchers follow the logic of sound quantitative reasoning and treat descriptive and exploratory statistics with the attention and respect they deserve.

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CME Self-Assessment Exam

Answers

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CME Article Number 2:
M. Inouye, et al.

1. B
2. D
3. D
4. D
5. D