<u>INTRODUCTION TO THE FINITE ELEMENT METHOD</u>

Department of Mechanical Engineering

National Taiwan University

Fall 2019

HOMEWORK #6

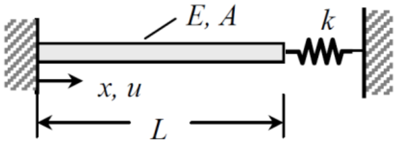
Due December 5, 2019

1. (Reddy) Problem 6.2

Problem 6.2: Determine the first two longitudinal frequencies of a rod $(E,\ A,\ L)$ fixed at one end and spring-supported at the other:

$$-EA\frac{\partial^2 u}{\partial x^2} + \rho A\frac{\partial^2 u}{\partial t^2} = 0 \quad \text{for } 0 < x < L$$

$$u(0) = 0, \quad \left(EA\frac{du}{dx} + ku\right)\bigg|_{x=L} = 0$$



Use (a) two linear finite elements and (b) one quadratic element.

2. (Reddy) Problem 6.16

Problem 6.16: Consider the partial differential equation arising in connection with unsteady heat transfer in an insulated rod:

$$\frac{\partial u}{\partial t} - \frac{\partial}{\partial x}\left(a\frac{\partial u}{\partial x}\right) = f \quad \text{for } 0 < x < L$$

$$u(0,t) = 0, \quad u(x,0) = u_0, \quad \left[a\frac{\partial u}{\partial x} + \beta(u - u_\infty) + \hat{q}\right]\bigg|_{x=L} = 0$$

Following the procedure outlined in Section 6.2, derive the semidiscrete variational form, the semidiscrete finite element model, and the fully discretized finite element equations for a typical element.