INTRODUCTION TO THE FINITE ELEMENT METHOD

Department of Mechanical Engineering

National Taiwan University

Fall 2019

HOMEWORK #3

Due October 24, 2019

1. (Reddy) Problems 2.1

In Problem 2.1–2.5, construct the weak form and, whenever possible, quadratic functionals.

Problem 2.1: A nonlinear equation:

$$-\frac{d}{dx}\left(u\frac{du}{dx}\right)+f=0 \quad \text{for} \quad 0<x<L$$

$$\left(u\frac{du}{dx}\right)\Big|_{x=0}=0 \quad u(1)=\sqrt{2}$$

2. (Reddy) Problems 2.6

Problem 2.6: Compute the coefficient matrix and the right-hand side of the N-parameter Ritz approximation of the equation

$$-\frac{d}{dx}\left[(1+x)\frac{du}{dx}\right]=0 \quad \text{for} \quad 0<x<1$$

$$u(0)=0, \quad u(1)=1$$

Use algebraic polynomials for the approximation functions. Specialize your result for $N=2$ and compute the Ritz coefficients.

3. (Reddy) Problems 2.16

Problem 2.16: Find a one-parameter approximate solution of the nonlinear equation

$$-2u\frac{d^2u}{dx^2}+\left(\frac{du}{dx}\right)^2=4 \quad \text{for} \quad 0<x<1$$

subject to the boundary conditions $u(0)=1$ and $u(1)=0$, and compare it with the exact solution $u_0=1-x^2$. Use (a) the Galerkin method, (b) the least-squares method, and (c) the Petrov–Galerkin method with weight function $w=1$.

4. Term project outline: One copy for each team. You should include the following:

(a) Title of Term Project

(b) Name of Team Members

(c) Motivation and Objective

(d) Method of Approaches

(e) Expected Difficulties

(f) Expected Results

(g) Time Schedule