Lecture 7: Wireline Channel and Inter-Symbol Interference (ISI)

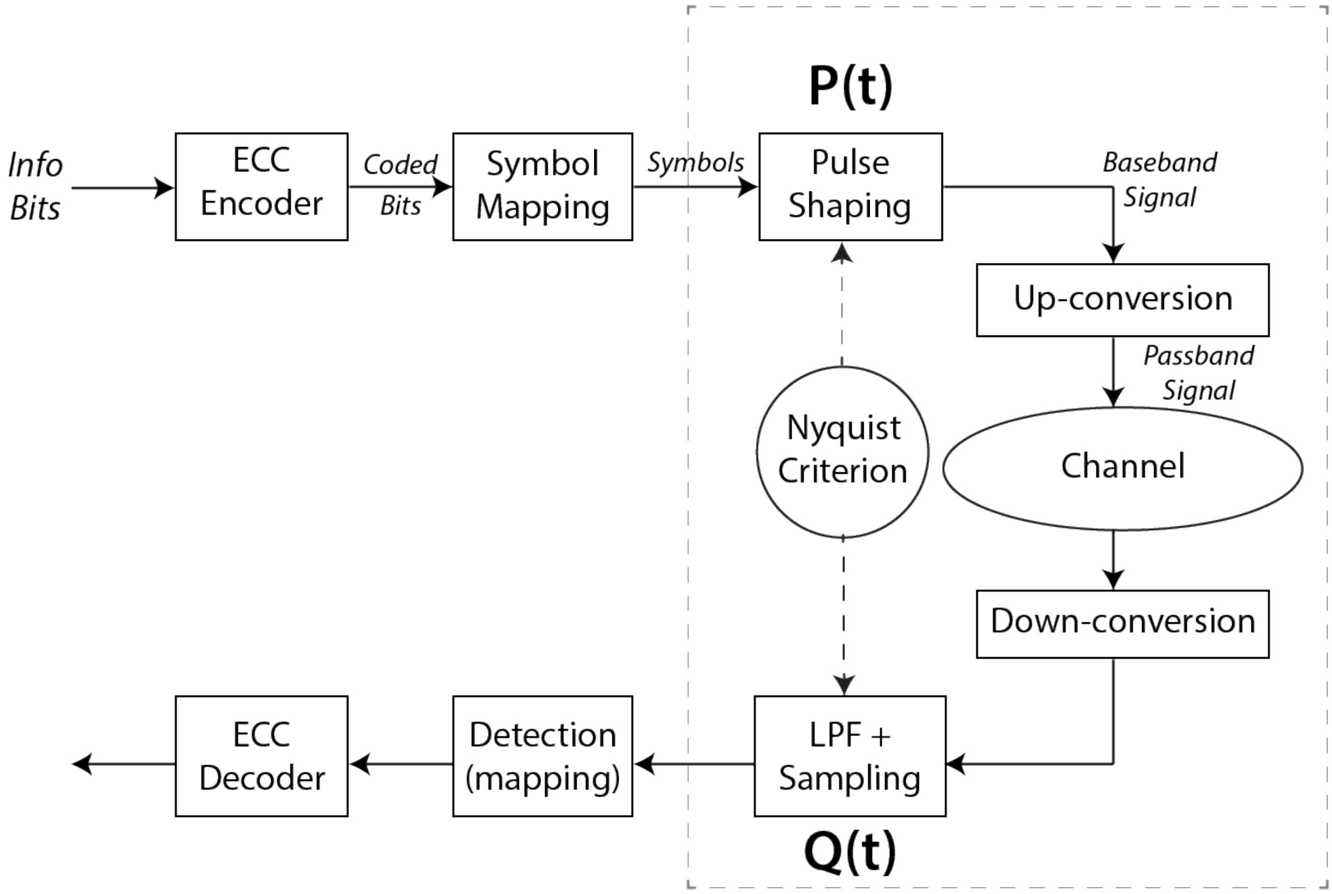
Scribe: 王君璇、朱文瑄 Lecture Date: 4/19, 2017 Lecturer: I-Hsiang Wang

Outline

- Wireline Channel and Inter-Symbol Interference (ISI)
  - Wireline Channel (as a LTI filter)
  - Inter-symbol Interference
  - Mitigation of ISI

Recap:

- Digital Communication

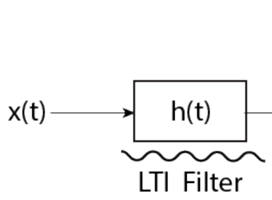


So far, we consider "simple" channels

$$\begin{cases} Waveform & y(t) = x(t) + z(t) \qquad \{z(t)\}\ WGN\ with\ psd\ \frac{No}{2} \\ bit\ level & d[n] = c[n] \oplus w[n] \qquad w[n] \sim Ber(P_b) \end{cases}$$

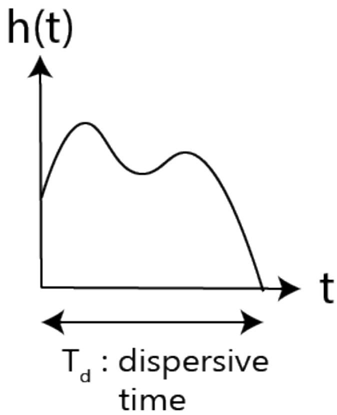
Wireline Channel

- Model (Baseband Waveform level)

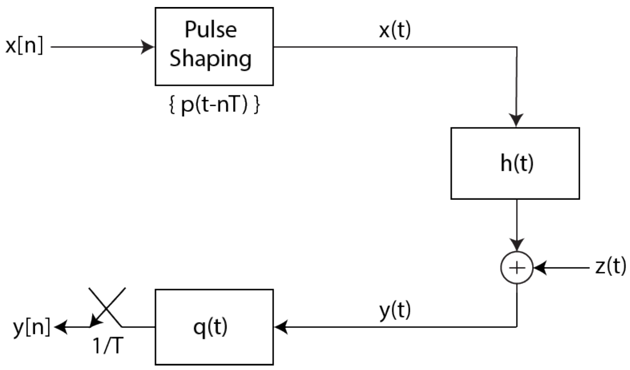


$$y(t) = (x*h)(t) + z(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)\,d\tau + z(t) = \int_{0}^{T_d} x(t-\tau)h(\tau)\,d\tau + z(t)$$

- Features of $h(\tau)$
  1. Causal $h(\tau) = 0$, if $\tau < 0$
  2. Dispersive
     (After the dispersive time $h(\tau)$ is so small that we ignore it.)



Equivalent discrete-time Model (after LTI + sampling)



$$y[n] = (y \ * \ q)(nT) \tag{1}$$

We want to transform the problem as a discrete-time model, such as

$$y[n] \ = \ (x \ * \ h)[n] + z[n] \tag{2}$$

But how can it be transformed? How can we get

$$\{h_l\} \overset{?}{\leftrightarrow} h(\tau)$$

As we know that

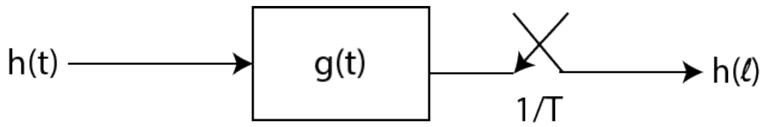
$$\tilde{x}(t) = \sum_{k=1}^{\infty} x[k]g(t-kT), \quad g(t) \triangleq (p*q)(t)$$

so that

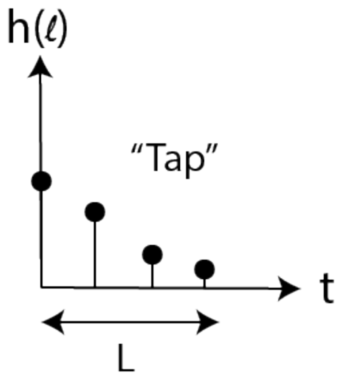
$$\begin{aligned} y[n] &= (y \ * \ q)(nT) \\ &= \int_0^{T_d} \tilde{x}(nT-\tau)h(\tau)\,d\tau + z(t) \\ &= \sum_{k=1}^{\infty} x[k] \int_0^{Td} h_a(\tau)g((n-k)T-\tau)\,d\tau + z[n] \\ &= \sum_{k=1}^{\infty} h_{n-k}\cdot x[k] + z[n] \\ &= (x \ * \ h)[n] + z[n] \end{aligned} \tag{3}$$

where

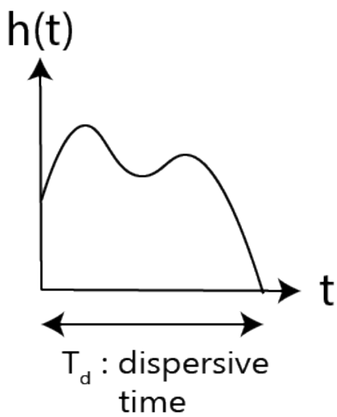
$$h_l \triangleq \int_0^{T_d} h_a(\tau)g(lT-\tau)\,d\tau = (h_a * g)(lT) \tag{4}$$

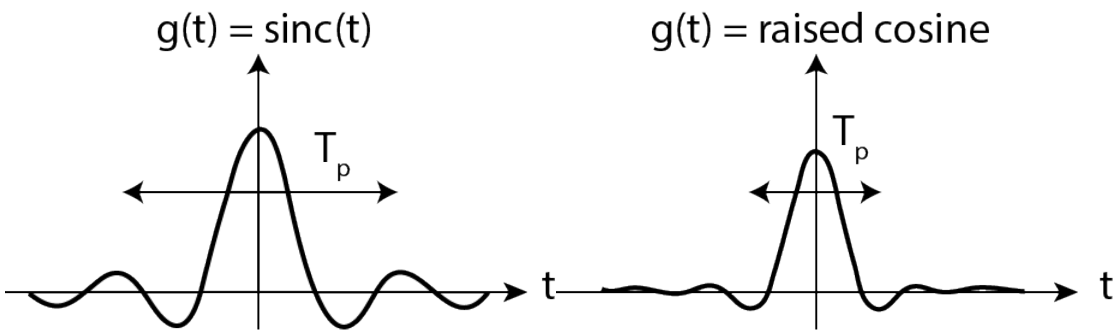


Hence, the equivalent digital filter $\{h_l\}$ is just the convolution of $h(\tau)$ and pulse $g(\tau)$ sampled at rate $\frac{1}{T}$
The following figure is an L-"Tap" Channel, where L is the range of index l where h_l is nonzero.



- The relation among L (number of non-zero taps in h_l)
  - T_d (dispersion of $h_a(\tau)$)
  - T_p (dispersion of $g(\tau)$)
  - T (symbol time)





Approximation Assumption:$g(t) << 1$ for $|\tau| > T_p$
The T_p of raised cosine is much less than sinc function.

As we know,

$$y[n] = \sum_{k=1}^{\infty} h_{n-k} \cdot x[k] + z[n] \tag{5}$$

where

$$h_l = \int_0^{T_d} h_a(\tau) g(lT - \tau)\, d\tau$$

How can we find out the number of taps? Let's look at the upper and lower bound of l.
For lower bound of l:

$$lT - \tau \geq -\frac{T_p}{2} \;\Rightarrow\; l \geq \frac{\tau - \frac{T_p}{2}}{T}$$

$$l_{min} = \frac{0 - \frac{T_p}{2}}{T} = -\frac{T_p}{2T} \tag{6}$$

For upper bound of l:

$$lT - \tau \geq \frac{T_p}{2} \;\Rightarrow\; l \geq \frac{\tau + \frac{T_p}{2}}{T}$$

$$l_{max} = \frac{T_d + \frac{T_p}{2}}{T} = -\frac{2T_d + T_p}{2T} \tag{7}$$

We can easily find out

$$y[n] = \sum_{k=1}^{\infty} h_{n-k} \cdot x[k] + z[n] = \sum_{l=\infty}^{\infty} h_l \cdot x[n-l] + z[n] \approx \sum_{l=-\frac{-T_p}{2T}}^{\frac{T_d + \frac{T}{2}}{2T}} h_l \cdot x[n-l] + z[n] = \sum_{l=0}^{L} h_l \cdot x[n-l] + z[n]$$

In summary,

$$y[n] \approx \sum_{l=0}^{L-1} h_l \cdot x[n-l] + z[n] \tag{8}$$

$$L \approx \frac{T_d + T_p}{T}$$

h_l depends on

1. Symbol Time T ($T = \frac{1}{2W}$, $W \leq B_b$, Sampling rate $= \frac{1}{T}$)
2. Pulse shaping $g(t)$
3. Channel impulse response $h_a(t)$

In practice, $\{h_l\}_{l=0}^{L-1}$ are measured directly by sending a known sequence $\{p[n]\}$ called "pilot". This procedure is called "training", enable R_x to learn the channel taps.

Typical number of taps for different medium:

1. Voiceband/Dial-up modem $L \approx 1$ to 20
2. DSL modem $L \approx 100$ to 200
3. Optical fiber $L \approx 200$ to 300

Inter-Symbol Interference(ISI)

$$\begin{aligned} y[n] &= h_0 x[n] + h_1 x[n-1] + h_2 x[n-2] + \cdots + h_{L-1} x[n-L+1] + z[n] \\ &= h_0 x[n] + I[n] + z[n] \\ I[n] &\triangleq h_1 x[n-1] + h_2 x[n-2] + \cdots + h_{L-1} x[n-L+1] \end{aligned} \tag{9}$$

where $I[n]$ is Inter-Symbol Interference
Let's take a look at a pretty easy example. Suppose that L = 3:

$$\begin{aligned} \underline{h} &= [h_0\ h_1\ h_2] \\ y[0] &= h_0 \cdot x[1] + z[1] \\ y[1] &= h_0 \cdot x[2] + h_1 \cdot x[1] + z[2] \\ y[2] &= h_0 \cdot x[3] + h_1 \cdot x[2] + h_2 \cdot x[1] + z[3] \\ y[3] &= h_0 \cdot x[4] + h_1 \cdot x[3] + h_2 \cdot x[2] + z[4] \\ \vdots &= \vdots \\ y[n] &= h_0 \cdot x[n] + I[n] + z[n] \quad \text{where } I[n] = \sum_{l=1}^{L-1} h_l \cdot x[n-l] \end{aligned}$$

- Naive Idea
  - Sequential Detection
    * First, detect $\hat{x}[1]$ from $y[1]$
    * Second, subtract $h_1\hat{x}[1]$ from $y[2]$ and get $\hat{y}[2] = h_1 x[2] + z[2] + h_1(x[1] - \hat{x}[1])$
  - The issue is ”error propagation”.
  - Therefore, ISI is indeed a new challenge we need to deal with designing wide-band system.

- Methods
  1. Rx methods
     (a) Linear Equalization + SSC(successive interference cancellation)

$$\tilde{y}[k] = \underline{v_k}^T \cdot \underline{y}$$

$$\underline{\tilde{y}} = V \cdot \underline{y} = V \cdot H \cdot \underline{x} + V \cdot \underline{z}$$

       i. Zero-Forcing
       ii. Matched filter
       iii. Minimum Mean Square Error (MMSE)
       - Equivalent matrix vector form

$$\begin{aligned} \underline{y} &= [y[1]\ y[2]\ \cdots\ y[n]]^T \\ \underline{x} &= [x[1]\ x[2]\ \cdots\ x[n]]^T \\ \underline{z} &= [z[1]\ z[2]\ \cdots\ z[n]]^T \\ \underline{y} &= H \cdot \underline{x} + \underline{z} \end{aligned} \tag{10}$$

       if we take the previous example for L = 3,

$$\text{then we can represent } H = \begin{bmatrix} h_0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ h_1 & h_0 & 0 & \cdots & 0 & 0 & 0 \\ h_2 & h_1 & h_0 & \cdots & 0 & 0 & 0 \\ 0 & h_2 & h_1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & h_0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & h_1 & h_0 & 0 \\ 0 & 0 & 0 & \cdots & h_2 & h_1 & h_0 \end{bmatrix}$$

     (b) Maximum Likelihood Sequence Detection (MLSD)
       - Viterbi Algorithm can ”optimally” solve the problem

    * Computation Complexity $\approx 2^L$, complicated when L is large
    * intensible for wideband system

2. Tx method
   (a) Tomlinson-Harashima Precoding
   (b) Dirty-Paper Coding (DPC)
      – A technique for efficient transmission of digital data through a channel subjected to some interference known to the transmitter
      – Consisting of precoding the data in order to cancel the effect caused by the interference
3. Tx + Rx Method
   – Orthogonal Frequency Division Multiplexing (OFDM)