Homework 3

Due: 12/6, 18:00

1. (**Solving a System of Linear Differential Equations**) [10]
Solve $x(t), y(t)$ in the following system of linear differential equations.

$$\begin{cases} t^2(x'' + y'') + t(x' + y') + 4x = t \\ t(x' + y') + y = \dfrac{1}{t^2} \end{cases}$$

2. (**Solving a Nonlinear Differential Equation**) [10]
Solve $y(t)$ in the following initial value problem:

$$y'' = -\frac{gR^2}{y^2},\ y(0) = R,\ y'(0) = 2\sqrt{gR}.$$

Hint. $2\sinh^2 x = \cosh 2x - 1$.

3. (**Power Series Solution about an Ordinary Point**) [10]
Solve the DE below using power series centered at $x = 0$.

$$(x^2 + x - 2)y'' - 2(2x + 1)y' + 6y = 0.$$

4. (**Method of Frobenius**) [10]
Use the method of Frobenius, find two linearly independent solutions of the following DE about the singular point $x = 0$.

$$xy'' + y = 0.$$

Hint. Recall that if $r_1 > r_2$ are the two roots of the indicial equation and $r_1 - r_2 \in \mathbb{Z}$,

$$y_1(x) = \sum_{n=0}^{\infty} c_n x^{n+r_1},\ c_0 \neq 0, \quad y_2(x) = \underbrace{C}_{\text{can be 0}} y_1(x) \ln x + \sum_{n=0}^{\infty} d_n x^{n+r_2},\ d_0 \neq 0.$$

5. (**Laplace Transform and its Inverse Transform**) [20]
Evaluate the following:

(a) $\mathscr{L}\left\{(t + \cos t)\sinh 2t\right\}$ [5]

(b) $\mathscr{L}\left\{(t^3 - 3t^2 + 3t - 1)\mathcal{U}(t - 2)\right\}$ [5]

(c) $\mathscr{L}^{-1}\left\{\dfrac{3e^{-s}}{(s^3-1)}\right\}$ [5]

(d) $\mathscr{L}^{-1}\left\{\dfrac{6s^2-14}{(s-3)^2(s^2+2s+5)}\right\}$ [5]

6. (**Solving IVP with Laplace Transform**) [10]
Solve the following initial value problem: $y(\pi)=1,\ y'(\pi)=-1,$

$$y''+4y'+4y=g(t)=\begin{cases}\cos 2t & t<2\pi\\ e^{-(t-2\pi)}\cos 2t & t\ge 2\pi\end{cases}$$

7. (**What Laplace Transform does not Take Care of**) [10]
Solve the following initial value problem: $y(0)=1,\ y'(0)=3,$

$$y''-y=g(t)=\begin{cases}0 & t<0\\ t^3 & t\ge 0\end{cases}$$

Hint. Be careful at the solution you found: it has to satisfy $y''-y=0$ for $t<0$.