Homework 2

Due: 10/25, 18:00

1. (**Method of Substitution and Nonexact Differential Equation Made Exact**)
Solve

$$\frac{dy}{dx} = 2 - 2e^{y} + 3e^{2x+y},\ y(0) = 0.$$

Bonus. Solve $\frac{dy}{dx} = 2 - 2e^{y} + 3e^{x+y}$, $y(0) = 0$.

2. (**Method of Substitution**)
Solve

(a)

$$\frac{dy}{dx} = \frac{2}{x} + \left(3 - \frac{1}{x}\right) y + xy^2.$$

(b)

$$\frac{dy}{dx} = 2e^{x^2} + (2x+3)y + e^{-x^2} y^2,\ y(0) = 1.$$

Hint: Choose appropriate $f(x)$ and use the substitution $u = f(x)y$ to convert the equation to the form $u' = P(u)$, where $P(u)$ is a polynomial of u.

3. (**General Solution of Homogenous Linear Differential Equations**)
Find the general solutions of the following:

(a)

$$y^{(4)} - 6y''' + 15y'' - 18y' + 10y = 0.$$

(b)

$$(x-1)^2 y'' + (x-1)y' + 4y = 0.$$