Homework 1

Due: 10/4, 18:00

1. (**Practice of Different Methods**)
Solve the following initial-value problems (y: dependent variable)

(a) $\frac{dy}{dx} = \frac{1}{x^4 - 1}$, $y(0) = 1$.

(b) $\frac{dy}{dx} = \frac{x^3}{(2y+1)}$, $y(2) = 1$.

(c) $(x^2 - 1)\frac{dy}{dx} = xy + 1$, $y(0) = 1$.

2. (**Discontinuous Coefficients**)
Solve

$$\frac{dy}{dx} + P(x)y = x$$

subject to $y(0) = 0$, where $P(x) = \begin{cases} 1, & x \geq 0 \\ -1 & x < 0 \end{cases}$.

3. (**Nonlinear ODE Made Linear**)
Solve

$$\frac{dy}{dx} = 1 + xe^{-y}$$

subject to $y(0) = 0$.

4. (**Singular Points, Interval of Definition, and Initial Conditions**)
(1) Solve

$$x(x-1)\frac{dy}{dx} = x + y$$

subject to

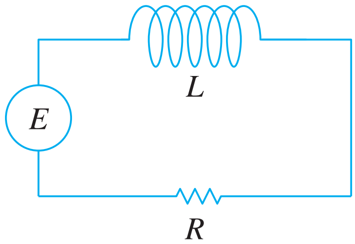
(a) $y(2) = 1$

(b) $y(-1) = 1$

(c) $y(1/2) = 1$

(2) Identify the singular points that cannot be included into the interval of definition.

5. (**LR Circuit with AC Power**)



Consider the above LR circuit, where $E(t) = 10\sin(t)$ volts, $R = 10$ ohms, $L = 0.5$ henry, and initial current $i(0) = 0$.
Find $i(t)$.

6. (**Gompertz Differential Equation**)
English Mathematician B. Gompertz (1779 – 1865) proposed the following equation to model population dynamics:

$$\frac{dP}{dt} = P(a - b\ln P).$$

Suppose the initial population $P(0) = P_0$.

(a) Find $P(t)$.

(b) Find the capacity of population, that is, $P(\infty)$.

(c) Find the threshold of population beyond which its growth rate decreases as population grows.

Do not need to turn in with Homework 1 – turn in with Homework 2.

7. (**Nonexact Differential Equation Made Exact**)
Solve

$$\frac{dy}{dx} = 2 - 2e^{y} + 3e^{2x+y},\ y(0) = 0.$$