Homework 1

Due: 10/4, 17:00

1. (**Practice of Different Methods**)
Solve the following initial-value problems (y: dependent variable)

(a) $\dfrac{dy}{dx} = \dfrac{1}{x^4 - 1}$, $y(0) = 1$.

(b) $\dfrac{dy}{dx} = \dfrac{x^3}{(2y + 1)}$, $y(2) = 1$.

(c) $(x^2 - 1)\dfrac{dy}{dx} = xy + 1$, $y(0) = 1$.

2. (**Discontinuous Coefficients**)
Solve

$$\frac{dy}{dx} + P(x)y = x$$

subject to $y(0) = 0$, where $P(x) = \begin{cases} 1, & x \geq 0 \\ -1 & x < 0 \end{cases}$.

3. (**Nonlinear ODE Made Linear**)
Solve

$$\frac{dy}{dx} = 1 + xe^{-y}$$

subject to $y(0) = 0$.

4. (**Singular Points, Interval of Definition, and Initial Conditions**)
(1) Solve

$$x(x - 1)\frac{dy}{dx} = x + y$$

subject to

(a) $y(2) = 1$

(b) $y(-1) = 1$

(c) $y(1/2) = 1$

(2) Identify the singular points that cannot be included into the interval of definition.