DYNAMICS IN ALGEBRAIC GEOMETRY

1 Description of the course

The aim of this course is to study algebraic varieties from a dynamical point of view. While the general theory of dynamical systems studies the evolution of a topological space under time-dependent transformations, in the context of algebraic geometry we will study how dominant rational self-maps and their iterates act on an algebraic variety. For most of the time we will focus on complex projective manifolds X (or more generally, compact complex manifolds). We will first study the groups of automorphisms $\mathrm{Aut}(X)$ and birational maps $\mathrm{Bir}(X)$ as well as their actions on X. We will also be interested in the actions of $\mathrm{Aut}(X)$ and $\mathrm{Bir}(X)$ on other spaces associated to X such as the cohomology ring and different positive cones. Invariants coming from the theory of dynamical systems (*e.g.* dynamical degree and entropy) will be introduced and discussed. Many concrete examples will be treated in detail including the Cremona groups and the dynamics of complex surfaces. We will also cover some specific topics toward the end of the course such as the Kawamata-Morrison's cone conjecture, the distribution of (pre)periodic subvarieties, or the p-adic approach.

This course will also serve as an opportunity to introduce important theories and concepts (Hodge theory, the minimal model program, positivity, *etc.*) in complex algebraic geometry and we will see how they will be used to formulate and study relevant questions in complex algebraic dynamics.

2 Prerequisites

Students in this course are supposed to be familiar with the following:

- Basic differential geometry and algebraic topology.
- Basic complex geometry (*e.g.* the first three chapters of Huybrechts' "Complex Geometry").

A recommended prerequisite is algebraic geometry at the level of Hartshorne Chap. 1 to 3.

The theory of dynamical systems is not a prerequisite for the course.

3 References

There is no required reference. The following is a list of survey articles related to the course.

- S. Cantat. Dynamics of automorphisms of compact complex surfaces. In *Frontiers in complex dynamics*, volume 51 of *Princeton Math. Ser.*, pages 463–514. Princeton Univ. Press, Princeton, NJ, 2014.
- S. Cantat. Automorphisms and dynamics: a list of open problems. In *Proceedings of the International Congress of Mathematicians—Rio de Janeiro 2018. Vol. I*, pages 615–630, 2018.
- S. Cantat. The Cremona group. In *Algebraic geometry: Salt Lake City 2015*, volume 97 of *Proc. Sympos. Pure Math.*, pages 101–142. Amer. Math. Soc., Providence, RI, 2018.
- C. T. McMullen. Algebra and dynamics. Course note available on the webpage http://www.math.harvard.edu/~ctm/papers/index.html, 2012.
- K. Oguiso. Some aspects of explicit birational geometry inspired by complex dynamics. In *Proceedings of the International Congress of Mathematicians—Seoul 2014. Vol. II*, pages 695–721. Kyung Moon Sa, Seoul, 2014.
- S.-W. Zhang. Distributions in algebraic dynamics. In *Surveys in differential geometry*, Vol. X, pages 381–430. Int. Press, Somerville, MA, 2006.

We will provide further references to students during the semester.