Homework 2

1. In principal-agent problems of efforts provision, moral hazard is not a problem if the structure of uncertainty allows the principal to infer precisely whether the agent has failed to perform as desired. Suppose table 6.5 is changed to be:

	$R = 10$	$R = 30$
$e = 1$	$p = 2/3$	$p = 1/3$
$e = 2$	$p = 0$	$p = 1$

   Rewrite the incentive and participation constraints and show that it is possible to design a forcing contract that motivates the worker to work hard, supplying $e = 2$, without placing any risk on him on her. How much is the worked paid if revnues of 10 are realized? How much when revnues are are 30? What are the expected utilities of the two parties? Could the parties do better if effort were observed? Would it be possible to achieve this sort of result if table 6.5 is as follows? Why and why not?

	$R = 10$	$R = 30$
$e = 1$	$p = 1$	$p = 0$
$e = 2$	$p = 1/3$	$p = 2/3$

   (Milgrom and Roberts, Ch6. Quantitative Problems #4.)

2. Reconsider table 6.5 in the handout, but change the agent's utility function to:

$$u(w, e) = w - (e - 1).$$

   Please derive the firm's optimal employment contract and compare the firm's profits when effort is observable and when effort is not observable. (Milgrom and Roberts, Ch6. Quantitative Problems #5.)

3. Reconsider the advantages of relative performance evaluation against an evaluation based solely on the employee's own performance. Here we consider all combinations of the two as well. Suppose manger A's measured performance is:

$$e_A + x_A + x_C,$$

and B's measured performance is:

$$e_B + x_A + x_C,$$

where x_A, x_B and x_C are independent sources of randomness. Suppose it is proposed to base manager A's compensation on his own performance minus δ times B's measured performance. Find the value of δ that minimizes the variance of the performance measure. (Milgrom and Roberts, Ch7. Mathematical Exercises #3.)

4. An entrepreneur can select among investment projects that all cost the same amount but differ in their risk-return characteristics. The set of available projects is described by a curve giving the highest available expected net return (after subtracting the initial cost of the investment) corresponding to any given variance in the return:

$$m = 2v - v^2/2, \quad 0 \leq v \leq 2,$$

where m is the mean return and v is the variance of returns. What project will the entrepreneur choose if he must bear all the returns alone and he has a coefficient of risk aversion of $r > 0$, so that his preferences are given in a certainty equivalent form by $m - rv/2$?

Now suppose that it is possible to share the risk of the investment with an outside investor who has a coefficient of risk aversion fo s. What is the investment choice that maximizes the total certainty equivalent? How should the risk be shared?

(Milgrom and Roberts, Ch7. Mathematical Exercises #4.)