賽局論作業

1. 9.8.12, (a), (d) and (e). in the texbook
   For (e), please exclude the asymptotic attractors considered in (a) and (b) and change the condition "$a < c$ or $d < b$" to "$a < c$ and $d < b$".

2. Reconsider the fitness game exemplified by the story of dodos that we discussed in class. Suppose that instead of two types of genes, Dove and Hawk, there are n types of genes. Let $p = (p_1, ..., p_n)^T$ denote the fractions of the population hosting each replicator. Let A denote the $n * n$ payoff matrix. The fitness $f(p) = (f_1(p), ..., f_n(p))^T$ is hence given by $f(p) = Ap$, and the average fitness $\bar{f}(p) = p^T f(p)$. In this case, the replicator equation is:

$$p_i' = p_i(f_i(p) - \bar{f}(p)), \quad i = 1, ...n. \tag{1}$$

   (a) Suppose (p^*, p^*) is a Nash equilibrium of the game with a normal form A. Please prove that p^* is a rest point of the replicator dynamics (1).

   (b) Suppose that $(1, 0, ..., 0)$ is an asymptotic attractor of the replicator dynamics described in (1). Please prove that $((1, 0, ..., 0), (1, 0, ..., 0))$ is a Nash equilibrium.

3. Consider the following game:

	s_1	s_2	s_3
s_1	0,0	6,-3	-4,-1
s_2	-3,6	0,0	5,3
s_3	-1,-4	3,5	0,0

   In a payoff vector, the 1st element denotes the row player's payoff and the 2nd element denotes the column player's payoff. s_i denotes a strategy, $i = 1, 2, 3$.

   (a) Please find all the Nash equilibrium.

   (b) Please find all the evolutionarily stable strategies of this game.

4. Consider the following replicator equations of (p_1, p_2, p_3):

$$\begin{aligned} p_1' &= p_1(p_2 - p_3), \\ p_2' &= p_2(p_3 - p_1), \\ p_3' &= p_3(p_1 - p_2), \end{aligned}$$

where $p_i' = dp_i/dt$, $i = 1, 2, 3$.

(a) Please find all the rest point(s) for these equations.

(b) Please prove that $d(p_1p_2p_3)/dt = 0$.

(c) Please find all the asymptotic attractor(s) for these equations. (Feel free to use the claim in (b), even if you fail to provide a proof to it.)