賽局論作業, Ch.7

1. 7.11.24
2. 7.11.30
3. 7.11.32
4. Consider a two person zero-sum game with a payoff matrix M which specifies the payoff to the row player. Let $\underline{m}$ ($\overline{m}$) denote the row (column) player's security level when both players consider to use only pure strategies, and let $\underline{v}$ ($\overline{v}$) denote the row (column) player's security level when both players consider to use mixed strategies.

   (a) Please compare $\underline{m}$ and $\underline{v}$. (Give the mathematical forms of $\underline{m}$ and $\underline{v}$, and analyze their difference rigorously)

   (b) Suppose $\underline{m} = \overline{m}$. Prove that the row player's security level is $\underline{m}$ when both players consider to use mixed strategies. (You could quote the result that $\overline{v} \geq \underline{v}$. Other than that any claim has to be provided with a detailed proof.)

5. Consider a zero-sum game between two players. In a m by n payoff matrix, element $\pi_{i,j}$ stands for the payoff to the row player, when the row player adopts his i-th strategy, and the column player adopts his j-th strategy. Please formulate a constrained optimization problem to find the row player's mixed security strategy. Specify clearly the decision variables, the objective function and the constraints.

解答

1 (a) row player's security strategy is his 3rd one, and column player's security strategy is her 3rd one. $v = 3$.

(b) row player's security strategy is to play the 2nd one with probability 1/7 and the 3rd one with probability 6/7. column player's security strategy is to play the 1st one with probability 4/7 and the 3rd one with probability 3/7. $v = 18/7$.

2 The strategic form is as follows.

	(2,1)	(1,2)
(3,1)	2	0
(2,2)	1	1
(1,3)	0	2

Maximin value = 1 < minimax value = 2. There is no saddle point.

We shall then search for the Nash equilibrium with mixed strategies. Consider Baloney to play (2,1) with probability p, and (1,2) with probability $1 - p$. If Baloney's strategy will make Blotto indifferent among his three pure strategies, then p is chosen such that $2p = 1 = 2(1 - p)$, i.e. p = 1/2. The value of the game is thus 1.

On the other hand, Blotto's strategy will also make Baloney indifferent between his two pure strategies. Moreover, because the value of the game is 1, no matter which pure strategy Baloney chooses, the expected outcome is 1. Let q_1, q_2, q_3 denote the probability that Blotto will use (3,1), (2,2) and (1,3). Then $2q_1 + 1q_2 + 0q_3 = 0, q_1 + 1q_2 + 2q_3 = 1$. As long as $q_1 = q_3$, the above equality holds.

So, in a mixed-strategy Nash equilibrium, Baloney mixes (2,1) and (1,2) with equal probability, and Blotto mixes (3,1) and (1,3) with equal probability. There are more than one Nash equilibrium.

3 Change payoff 4 in figure 7.17 (a) to W to indicate Kirk wins, and all other payoff to L to indicate Kirk loses. Apparently, Spock has some dominant strategies which guarantee her to win. These strategies ask Spock to hit either grid 2 or grid 3 first. If the first bomb is a miss, Spock knows where the ship is. On the other hand, if the first bomb is a hit, the furthest grid is ruled out. And Spock can sink the ship with the remaining two bombs.

4a Let P and Q denote the row player and the column player's space of mixed strategies. Let q and q denote an element of P and Q, respectively.

$$\begin{aligned}
\underline{v} &= \max_{p\in P}\min_{q\in Q} p^t M q \\
&= \max_{p\in P}\min_{j} p^t M_{.j} \\
&\geq \max_{i}\min_{j} M_{ij} \\
&= \underline{m}
\end{aligned}$$

4b From previous problem, $\underline{v} \geq \underline{m} = \overline{m} \geq \overline{v}$. Since $\underline{v} = \overline{v}$, $\underline{v} = \underline{m}$.

5 Let q_i denote the probability for the row player to play his i-th pure strategy, $i = 1, ..., m$. The problem is:

$$\begin{aligned}
\max_{q_i} \quad & s \\
\text{s.t.} \quad & \sum_{i=1}^{m} q_i \pi_{i,j} \geq s, \quad \text{for} \quad j = 1, ..., n
\end{aligned}$$