賽局論作業

Please read sections 15.1–15.3 and section 21.4.

1. Consider a duopoly with Cournot (quantity) competition. The market demand is:
$$q = 100 - p,$$
where q and p are the quantity and the price, respectively. The unit cost of each firm is a constant: \$10. So, given the opponent's output q_j, firm i considers to:
$$\max_{q_i}(90 - q_i - q_j)q_i.$$
Please solve for the Cournot equilibrium (or Nash equilibrium).

2. Reconsider the previous problem. It is common knowledge that the unit cost of firm 1 is \$10. But now, the constant unit cost of firm 2 could be either \$10 or \$20. Firm 2 knows its own cost. Firm 1 considers the unit cost of firm 2 to be \$10 or \$20 with equal probabilities. Please calculate firm 1's output in a Bayes-Nash equilibrium.

3. Consider the first-price sealed bidding for an object. Two bidders have independent private value. A bidder knows his own value, but other people consider the value to be either v_1 or v_2 with probability p_1 and p_2, respectively where $v_2 > v_1$. If two bidders submit the same bid, a fair random draw will decide who gets the object and the winner has to pay for the bid. It is known that in a symmetric Bayes-Nash equilibrium, a bidder with value v_1 will submit a bid $b = v_1$. On the other hand, a bidder with value v_2 will play a mixed strategy over a continuous range $[x, y]$.
   (a) What is x precisely?
   (b) In equilibrium, what is the expected gain of a bidder with v_2? (It's a function of v_1, v_2, p_1, p_2.)
   (c) What is y precisely?

4. An all-pay sealed bidding works very much like a first-price sealed bidding except that every bidder has to pay his/her submitted bid

even when he/she loses. Bob and Carol are the only bidders interested in A's house. It is common knowledge that their valuations of this house are independently and uniformly distributed between 0 and 1, and each one knows his own private value. In a symmetric Bayes-Nash equilibrium, each bidder adopts a bidding function $B(v)$ where v is a bidder's private value and $B' > 0$. Please solve for $B(v)$ step by step. (We'll ignore the case of a tie since its probability is zero.)

(a) With a private value v, how should Bob decide his optimal bid β to maximize his expected wealth? Write out Bob's objective function.

(b) Let C be the inverse function of B. Please rephrase Bob's objective function in terms of C.

(c) Solve for $B(v)$ from the FOC.

解答

1 $q_1 = q_2 = 30$.

2 Let q_2^h (q_2^l) be the output of firm 2 when its unit cost are \$20 (\$10). In a BNE,

$$q_2^h = \frac{80 - q_1}{2}, \quad q_2^l = \frac{90 - q_1}{2}.$$

Firm 1 hence considers to:

$$\max_{q_1} 0.5q_1(90 - q_1 - q_2^h) + 0.5q_1(90 - q_1 - q_2^l).$$

$q_1 = 95/3$.

3a The bidder with v_2 plays against an opponent who bids v_1 with probability p_1, and some random bid in the range $[x, y]$ with probability p_2. Let $E\pi(b)$ denote the expected profit of the bidder with v_2 when he bids b. Then $E\pi(b)$ stays constant in the range $[x, y]$ (or at least in an area which probability measure is 1).

Suppose that $x < y < v_1$. For a bidder with value v_2, since he is willing to randomize, it must be that $E\pi(b)$ stays the same in the range of $[x, y]$. But note that $E\pi(x) = 0$ and $E\pi(y) > 0$. So he shouldn't randomize in this range.

Suppose $x < v_1 < y$.

$$E\pi(b) = \begin{cases} (v_2 - v_1)(p_1/2 + p_2 F(v_1)) & \text{if } b = v_1, \\ (v_2 - b)p_2 F(v_1 - \epsilon) & \text{if } b = v_1 - \epsilon \text{ where} \epsilon > 0. \end{cases}$$

In some area $[v_1 - \epsilon, v_1)$ with a strictly positive measure, we have $E\pi(v_1) > E\pi(v_1 - \epsilon)$, but this is the area that the bidder should bid with zero probability. In sum, $x \geq v_1$.

On the other hand, if $y > x > v_1$, For a bidder with value v_2, $E\pi(b)$ stays the same in the range of $[x, y)$. Changing to bid $v_1 + \epsilon$ will not hurt the winning probability when compared with the bid of x, but will lower the payment when winning. Hence $x = v_1$.

3b

$$\forall b \in [v_1, y], (v_2 - b)(p_1 + p_2 F(b)) = c,$$

where c is some constant. Because $F(v_1) = 0$, $c = (v_2 - v_1)p_1$.

3c From above, $(v_2 - y) = (v_2 - v_1)p_1$, $y = v_1 p_1 + v_2 p_2$

4a

$$\max_\beta (v - \beta)P(\beta > B(w)) - \beta[1 - P(\beta > B(w))],$$

where w is Carol's private value.

4b

$$\max_\beta (v - \beta)C(\beta) - \beta[1 - C(\beta)],$$

4c The FOC gives:

$$vC'(\beta) = 1.$$

Because $\beta = B(v)$, It means:

$$vC'(B(v)) = 1, \quad \text{or} \quad v\frac{dv}{db} = 1,$$

where $b = B(v)$. Since $v = db/dv$, $B(v) = v^2/2 + k$. It's clear that $B(0) = 0$, so $k = 0$ and $B(v) = v^2/2$.