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Short communication

Power-law for gravity currents on slopes in the deceleration phase

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ABSTRACT

The power-law for gravity currents on slopes is essentially an asymptotic form of the solution of thermal theory developed in Beghin, Hopfinger, and Britter (*J. Fluid Mech.* 107 (1981) 407–422), when the gravity current is sufficiently far into the deceleration phase. The power-law not only describes the long-term front location versus time relationship but also provides a useful means to estimate the buoyancy contained in the gravity current head. However, the hypothesis that gravity current is sufficiently far into the deceleration phase is hardly satisfied in experiments. In this paper, we re-formulated the power-law, considering the influence of bottom friction, and supplement the formulation by proposing a correct version of the power-law. When the gravity current is not sufficiently far into the deceleration phase, we showed that the power-law still robustly describes the front location versus time relationship, but the amount of heavy fluid in the head can be easily underestimated. The underestimation of heavy fluid in the head also depends on where the gravity current is in the deceleration phase. Therefore, a correction factor is suggested according to the location of gravity current. The amount of heavy fluid in the head, when estimated by the power-law, should be understood as the 'effective' buoyancy in driving the gravitational convection and is deemed as a lower limit for the estimation of buoyancy contained in the head.

1. Introduction

Gravity currents, also known as density currents, are gravitationally driven flows due to a density difference. A number of factors that are likely to cause variations in the density of fluid include

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temperature differentials, dissolved and suspended materials, such as salt and suspended sediments. Lock-exchange flows, in which gravity currents are produced from an instantaneous, finite buoyancy source and propagate on a horizontal boundary, have drawn much attention in the literature (see, for example, Shin et al., 2004; Marino et al., 2005; Cantero et al., 2007). Gravity currents on a slope have been considered less, but are also commonly encountered, such as powder-snow avalanches (Hopfinger, 1983) and spillage of hazardous materials (Fannelop, 1994). On a point of terminology, gravity currents on a slope is more precisely described as a 'thermal cloud', 'gravity cloud', or 'boluse' because the flow is more like a cloud with some tail following. In the literature, the terms are used interchangeably and the general term gravity current is adopted here to be consistent with the recently published work by Maxworthy (2010). For more details about the diversity of gravity currents in geophysical environments and engineering applications, the readers are referred to Allen (1985), Fannelop (1994), and Simpson (1997).

Perhaps the best-known publication on the gravity currents produced from instantaneous buoyancy sources propagating on slopes is due to Beghin et al. (1981). When the buoyancy closed in the lock is instantaneously released on a slope, the produced gravity currents go through an acceleration phase followed by a deceleration phase, according to the front velocity history. Thermal theory, which was developed therein following the famous Morton et al. (1956), has formed the basis for related gravity current studies (see, for example, Dade et al., 1994; Rastello and Hopfinger, 2004). Recently, a series of experiments aiming at gravity currents in the deceleration phase was reported in Maxworthy (2010) and the power-law which describes the front location versus time relationship and gives an estimate for the amount of heavy fluid contained in the head in the deceleration phase was proposed therein. As will be shown in the following section, the power-law for gravity currents in the deceleration phase is essentially an asymptotic form of the solution of thermal theory when the gravity current is sufficiently far into the deceleration phase, while in fact, this hypothesis is hardly satisfied in experiments. Nonetheless, it was reported that the power-law is robust (Maxworthy, 2010) even when this hypothesis is not satisfied, which begs the following questions: how could the power-law possibly be robust when the gravity current is not sufficiently far into the deceleration phase and, more importantly, when applied in this situation, does the power-law provide an accurate estimate for the amount of heavy fluid contained in the gravity current head?

This paper provides the answers to these questions and is organized as follows. The detailed derivation of power-law, including the influence of bottom friction, is presented in Section 2. The expression for an important model constant K_B, which helps determine the amount of heavy fluid contained in the gravity current head, was incorrect in Maxworthy (2010) and is corrected here. The power-law method used to estimate the amount of heavy fluid in the head is introduced in Section 3 and a correction factor is suggested when the gravity current is not sufficiently far into the deceleration phase. Conclusions are drawn in Section 4.

2. Derivation of the power-law

The configuration of the problem of a gravity current propagating on a slope is sketched in Fig. 1. Here the nomenclature mainly follows Beghin et al. (1981) for the reader's convenience. The density of ambient fluid is taken as ρ_0 and the density of heavy fluid in the lock region is ρ_1, where $\epsilon = \Delta\rho_1/\rho_0 = (\rho_1 - \rho_0)/\rho_0$. The cross-sectional area of the lock, which equivalently represents the amount of heavy fluid in the lock, is $A_0 = h_0 \times l_0$. After an instantaneous removal of the lock gate, the gravity current front develops and the gravity current head approximately takes a semi-elliptical shape with height to length aspect ratio $k = H/L$.

The convection of gravity current is driven by the heavy fluid that is contained within the head. Therefore, the linear momentum with bottom friction term takes the form

$$\frac{d(\rho + k_v\rho_0)S_1HLU}{dt} = B\sin\theta - C_f\rho U^2L, \tag{1}$$

where ρ is the density of mixed fluid in the head, U is the mass-center velocity of the head, t is the time, $k_v = 2k$ is the added mass coefficient (Batchelor, 1967), $S_1 = \pi/4$ is a shape factor (Beghin et al., 1981) by which the cross-sectional area of the semi-elliptical head is defined as S_1HL, C_f is the friction

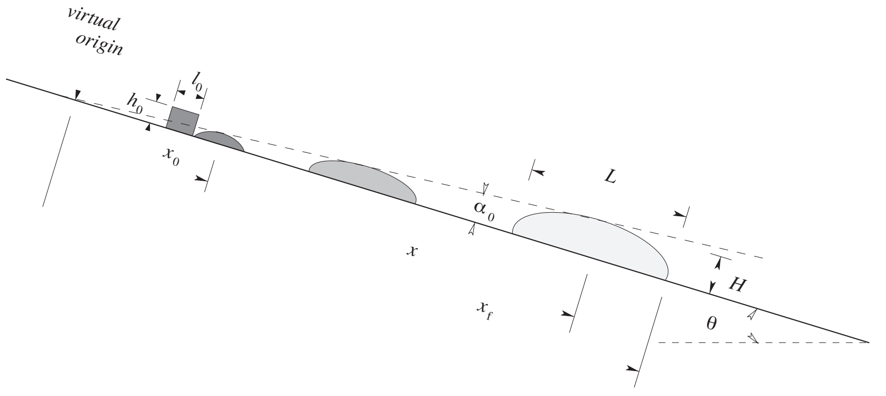

Fig. 1. Gravity current generated from an instantaneous buoyancy source propagating on a slope inclined at an angle of θ. The buoyancy is confined in the shaded region with dimensions of $h_0 \times l_0$. At $t = 0$, the buoyancy is released from quiescent condition and allowed to propagate downslope. H and L are the height and length of the gravity current head. The 'virtual origin', which is defined through extrapolation of H, is located x_0 beyond the initial mass-center location of the heavy fluid. The angle of growth, α_0, is defined by the line joining the gravity current heights, i.e. dH/dx_f. The front location is denoted by x_f and the head mass-center location is denoted by x, where the distances are all measured from the 'virtual origin'.

coefficient on the bottom, which is approximately $C_f \approx 10^{-2}$ for a rough boundary and $C_f \approx 2 \times 10^{-3}$ for a saline cloud (Rastello and Hopfinger, 2004), and $B = g(\rho - \rho_0)S_1HL$ denotes the buoyancy that is related to the amount of heavy fluid contained in the head. Here it is assumed that a fraction χ of heavy fluid in the lock is contained in the head, i.e.

$$B = \chi g \Delta \rho_1 A_0, \tag{2}$$

where $\chi = 1$ was assumed in Beghin et al. (1981) and $\chi < 1$ was reported in Maxworthy (2010). With turbulent entrainment assumptions (Ellison and Turner, 1959), the mass conservation takes the form

$$\frac{d}{dt}(S_1HL) = S_2(HL)^{1/2}\alpha U, \tag{3}$$

where $S_2 = (\pi/2^{3/2})(4k^2+1)^{1/2}/k^{1/2}$ is another shape factor (Beghin et al., 1981) by which the circumference of semi-elliptical head is defined as $S_2(HL)^{1/2}$ and α is the entrainment coefficient.

From (3)

$$H = \frac{1}{2}\frac{S_2}{S_1}k^{1/2}\alpha x \quad \text{and} \quad L = \frac{1}{2}\frac{S_2}{S_1}k^{-1/2}\alpha x, \tag{4}$$

where x is the distance from the 'virtual origin' to the mass-center of gravity current head. The 'virtual origin' is located x_0 beyond the initial mass-center location of the heavy fluid. In practice, the initial state of the thermal cloud is situated somewhat downstream of the release gate (Beghin et al., 1981). Please note that here the distances are all measured from the 'virtual origin' so an additive constant of x_0 for all the distances is not needed.

Since the front location of the gravity current is a more readily measurable quantity, it is desired to rewrite the solution in terms of the front location, x_f. Using the geometric relation $x_f = x + L/2$ and the identity $\alpha = [2S_1/S_2k^{1/2}]\alpha_0$ for the angle of growth α_0, the following relationship, which translates from the mass-center coordinate system to that using the front location, is derived as

$$x_f = \left(1 + \frac{\alpha_0}{2k}\right)x. \tag{5}$$

Upon substitution of (4) into (1) and using $U = dx/dt$, the momentum equation becomes

$$U\frac{d}{dx}(x^2U) + \epsilon\left(\chi\frac{2}{\pi}\frac{k}{1+2k}\frac{A_0}{\alpha_0^2}\right)\frac{dU^2}{dx} = C - \frac{C_f x U^2}{(1+2k)\alpha_0 S_1}[1 + O(\epsilon)], \tag{6}$$

where

$$C = \frac{4}{\pi}\frac{k}{1+2k}\frac{1}{\alpha_0{}^2}\chi B_0' \sin\theta \quad \text{with} \quad B_0' = \epsilon g A_0, \tag{7}$$

is the driving force term. Here the $O(\epsilon)$ arises because the friction term in (1) is approximated by $C_f\rho_0 U^2 L$ and the difference is of the order of magnitude $O(\epsilon) = O((\rho - \rho_0)/\rho_0)$. With the Boussinesq approximations, that the influence of density variation on the inertia term, $O(\epsilon)$, is neglected but the driving force term, $O(\epsilon g)$, is retained, the following closed-form solution is derived

$$U^2 = U_0^2\left(\frac{x_0}{x}\right)^{4+2C_f/(1+2k)\alpha_0 S_1} + \frac{2}{3+2C_f/(1+2k)\alpha_0 S_1} \times C\frac{1}{x}\left[1-\left(\frac{x_0}{x}\right)^{3+2C_f/(1+2k)\alpha_0 S_1}\right], \tag{8}$$

where U_0 is the initial mass-center velocity. Transforming (8) into the coordinate system using front location, we have

$$\begin{aligned} U_f^2 = {} & U_{f0}^2\left(\frac{x_{f0}}{x_f}\right)^{4+2C_f/(1+2k)\alpha_0 S_1} + \frac{2}{3+2C_f/(1+2k)\alpha_0 S_1} \\ & \times C\left(1+\frac{\alpha_0}{2k}\right)^3 \frac{1}{x_f}\left[1-\left(\frac{x_{f0}}{x_f}\right)^{3+2C_f/(1+2k)\alpha_0 S_1}\right], \end{aligned} \tag{9}$$

where U_{f0} is the initial front velocity and x_{f0}, i.e. $x_{f0} = (1 + \alpha_0/2k)x_0$, is the distance from the 'virtual origin' to the initial front location.

If the gravity current starts from a quiescent initial condition, the solution (9) can be further simplified when the gravity current is sufficiently far into the deceleration phase, i.e. when $x_f/x_{f0} \gg 1$,

$$U_f = \left[\frac{2}{(3+2C_f/(1+2k)\alpha_0 S_1)}\right]^{1/2} C^{1/2}\left(1+\frac{\alpha_0}{2k}\right)^{3/2} x_f{}^{-1/2}. \tag{10}$$

Upon integration, (10) can be rewritten in the following form with an integration constant t_0

$$x_f = K_B\chi^{1/3}B_0'{}^{1/3}(t+t_0)^{2/3} \quad \text{or} \quad x_f{}^{3/2} = K_B{}^{3/2}\chi^{1/2}B_0'{}^{1/2}(t+t_0), \tag{11}$$

where

$$K_B = \left(\frac{36}{6\pi + 4\pi C_f/(1+2k)\alpha_0 S_1}\right)^{1/3}\left(1+\frac{\alpha_0}{2k}\right)\left[\frac{k\sin\theta}{(1+2k)\alpha_0{}^2}\right]^{1/3}. \tag{12}$$

It is worthy to point out that in Maxworthy (2010), $K_M = K_B\chi^{1/3}$. Please note that the power-law (11) is equivalent to the asymptotic form of the front velocity in the deceleration phase, i.e.

$$\frac{U_f x_f{}^{1/2}}{B_0'{}^{1/2}} = \frac{2}{3}K_M{}^{3/2}, \tag{13}$$

which is consistent with that given by Beghin et al. (1981).

When the front location and time are plotted as $x_f{}^{3/2}$ against t, $K_M{}^{3/2}B_0'{}^{1/2}$ represents the slope of the linear regression line and can be deduced directly from the experimental data. Meanwhile, the slope of the linear regression line would be $K_B{}^{3/2}B_0'{}^{1/2}$ if the gravity current head were to contain the full charge of buoyancy in the lock. Here the solution of K_B, (12), does not converge to that presented as (3.4) in Maxworthy (2010) even when $C_f = 0$. Where the difference between (12) and (3.4) in Maxworthy (2010) really comes from is not immediately apparent from viewing the two expressions. However, the power-law certainly was derived from the thermal theory, therefore the expression for K_B has to be corrected here.

As reported in Beghin et al. (1981), the aspect ratio of 'thermal cloud' can increase from $k \approx 0.17$ to 0.53 and the entrainment coefficient can increase from $\alpha \approx 0.081$ to 0.59 as the bottom slope increases from 0° to 90°. It would not be possible to show directly the dependence of K_B on θ without explicit relationships of $k(\theta)$ and $\alpha_0(\theta)$. Here both k and α_0, are measured quantities that depend on the slope

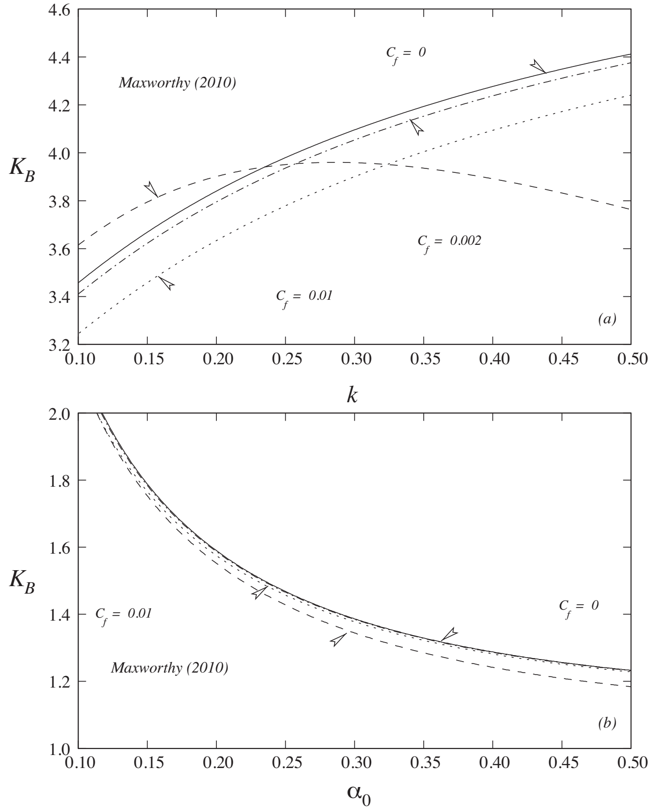

Fig. 2. Functional dependence of K_B on k and α_0 at different friction coefficients C_f. Panel (a): K_B versus k while $\alpha_0 = 0.0336$; panel (b), K_B versus α_0 while $k = 0.25$. Solid line represents K_B according to (12) with $C_f = 0$; dash-dot line (-·-) represents K_B with $C_f = 2 \times 10^{-3}$; dotted line (···) represents K_B with $C_f = 10^{-2}$. Dashed line represents K_B from (3.4) in Maxworthy (2010). The bottom slope is set at $\theta = 10.6^{\circ}$ for illustrative purposes.

angle. To illustrate the functional dependence of K_B on k and α_0, Fig. 2(a) shows K_B for $0.1 \leq k \leq 0.5$ while α_0 is fixed; Fig. 2(b) shows K_B for $0.1 \leq \alpha_0 \leq 0.5$ while k is fixed. For illustrative purposes, friction coefficients $C_f = 0$, 2×10^{-3}, 10^{-2} are chosen, the bottom slope is set at $\theta = 10.6^{\circ}$, the angle of growth is set at $\alpha_0 = 0.0336$ in Fig. 2(a), and the aspect ratio is set at $k = 0.25$ in Fig. 2(b). Since both k and α_0 increase with increasing bottom slope, K_B should not be interpreted as either monotonically increasing or decreasing with slope angle.

3. Estimation for the buoyancy in gravity current head

The power-law (11) provides a useful means for the estimation of the buoyancy contained in gravity current head. When $x_f^{3/2}$ is plotted against t, the numerical value of coefficient K_M can be derived using linear regression of the front location data in the deceleration phase. The fraction of heavy fluid that is contained in the gravity current head, χ, is estimated via

$$\chi = \left[\frac{K_M}{K_B}\right]^3. \tag{14}$$

In other words, $K_M = K_B$ would indicate the gravity current head carrying the full amount of heavy fluid in the lock into the deceleration phase. Note here that the estimate for K_M has influence not only

Table 1
K_A derived from the solution of (15), as shown in Fig. 3(b), in different ranges of x_f/x_{f0}. The power-law is of the form $(x_f/x_{f0})^{3/2} = K_A{}^{3/2}(\tilde{t}+\tilde{t}_0)$, where $\tilde{t}_0$ is the $\tilde{t}$-intercept. Here the model equation is designed such that $K_B = 1$. $(1 - R^2)$-value of linear regression is also listed for reference.

Range of x_f/x_{f0}	K_A	$\tilde{t}_0$	$[K_A/K_B]^3$	$1 - R^2$
2.0–2.5	0.969	0.300	0.911	1.792×10^{-5}
2.5–3.0	0.984	0.213	0.952	3.248×10^{-6}
2.0–3.0	0.977	0.258	0.934	3.176×10^{-5}
3.0–3.5	0.990	0.161	0.971	8.132×10^{-7}
3.5–4.0	0.994	0.126	0.981	2.521×10^{-7}
3.0–4.0	0.992	0.144	0.976	1.840×10^{-6}
4.0–4.5	0.996	0.102	0.987	9.114×10^{-8}
4.5–5.0	0.997	0.084	0.991	3.698×10^{-8}
4.0–5.0	0.996	0.093	0.989	2.341×10^{-7}

on the estimation for the buoyancy in the head but also on the front velocity in the deceleration phase, cf. (13).

However, mindful that in addition to the correction made on K_B, the derivation of power-law (11) is not possible without an important hypothesis, i.e. $x_f/x_{f0} \gg 1$, which is in fact not really attained in experiments. As an example, the experiment 18/9/07 reported in Fig. 16 of Maxworthy (2010) shows that the front location is in the limited range of $1.0 \leq x_f/x_{f0} \lesssim 3.0$ and the power-law is approached for the front location approximately in the range of $2.0 \lesssim x_f/x_{f0} \lesssim 3.0$. This reported observation begs the following questions: how could the power-law be robust when the gravity current is not sufficiently far into the deceleration phase? More importantly, how accurate is the estimation of buoyancy contained in the head provided by the power-law when applied in this situation?

To answer the proposed questions, we consider a model equation, cf. (9),

$$\tilde{U}_f^2 = \left(\frac{2}{3}\right)^2 \left(\frac{x_{f0}}{x_f}\right) \left[1 - \left(\frac{x_{f0}}{x_f}\right)^3\right], \tag{15}$$

where the $\tilde{()}$ denotes dimensionless variables and the gravity current is assumed to start from a quiescent initial condition. Here the length and time scales are chosen as $A_0{}^{1/2}$ and $A_0{}^{1/4}/(\epsilon g)^{1/2}$, respectively. The influence of bottom friction is minor and is neglected in (15) for illustrative purposes. Without loss of generality, here we assume that $\chi = 1$. The coefficient $(2/3)^2$ in the model equation (15) is chosen such that $K_B = 1$. While in Maxworthy (2010) K_M is used to designate the experimental or measured value, here we use K_A in place of K_M in the power-law to ease the confusion on whether the power-law is applied to experimental or theoretically derived results.

Fig. 3 shows the front velocity versus front location and $(x_f/x_{f0})^{3/2}$ versus $\tilde{t}$ that are solved numerically from the model equation (15). It is evident from Fig. 3(a) that the asymptotic relationship for the front velocity, i.e. $\tilde{U}_f = (2/3)(x_f/x_{f0})^{-1/2}$, is not strictly observed until $x_f/x_{f0} \gtrsim 5.0$ and the front location and time relationship follows the power-law (11) with $K_A = 1$ only when $\tilde{t} \gtrsim 8.0$, as illustrated by the dashed line in Fig. 3(b). For a limited range of x_f/x_{f0}, that is shorter than $x_f/x_{f0} \approx 5.0$, the power-law still seems to work but the numerical value of coefficient K_A becomes less than unity.

The numerical solution to (15) is then used to validate the power-law. When $(x_f/x_{f0})^{3/2}$ is plotted against $\tilde{t}$, K_A and $\tilde{t}_0$ can be deduced from the slope of the linear regression line and the $\tilde{t}$-intercept, respectively. Table 1 lists the numerical value of K_A and $\left[K_A/K_B\right]^3$, that are derived from the power-law when applied to the solution of model equation (15) in a limited range of x_f/x_{f0} as listed. When applied in a limited range of front location, the power-law appears to be very robust even when $x_f/x_{f0} < 5.0$, as elucidated by the $(1 - R^2)$-value. Note that K_A approaches to unity as x_f/x_{f0} increases and when $x_f/x_{f0} \gtrsim 5.0$, the estimated fraction of heavy fluid in the head, $\left[K_A/K_B\right]^3$, is expected to be less than unity by within 1%. However, as opposed to our assumption of $\chi = 1$, the estimated fraction of heavy fluid in the gravity current head is less than unity by as much as 7% when the power-law is applied to the front location in the range of $2.0 \leq x_f/x_{f0} \leq 3.0$, as an example. Of course, $[K_A/K_B]^3$ should approach to unity if the hypothesis $x_f/x_{f0} \gg 1$ is satisfied, as shown by the dashed line in Fig. 3(b). However,

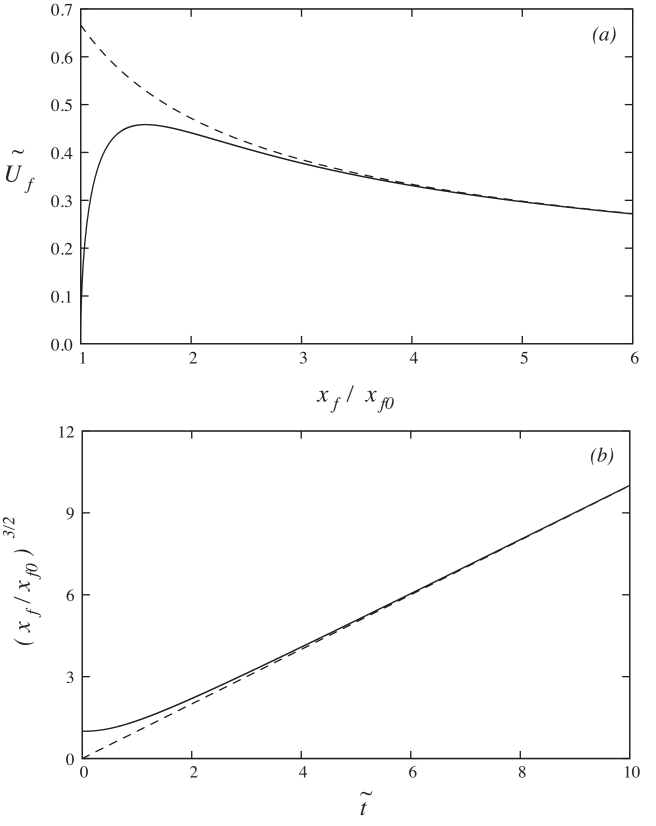

Fig. 3. Solution for the model equation (15). Panel (a): the solid line shows the front velocity ($\tilde{U}_f$) versus front location (x_f/x_{f0}) and the dashed line represents the asymptotic relation $\tilde{U}_f = (2/3)(x_f/x_{f0})^{-1/2}$; panel (b): the solid lines shows $(x_f/x_{f0})^{3/2}$ versus $\tilde{t}$ and the dashed line represents the power-law relation $(x_f/x_{f0})^{3/2} = K_A{}^{3/2}(\tilde{t} + \tilde{t}_0)$, where $K_A = 1$ and $\tilde{t}_0 = 0$.

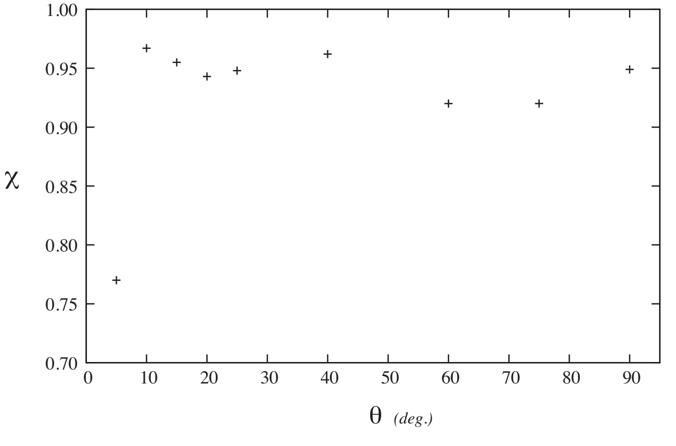

Fig. 4. The fraction of heavy fluid contained in the head, χ, versus slope angle, θ. χ is estimated via (14) using the data provided in Beghin et al. (1981) where K_M is back calculated via (13) and K_B is computed using the measured quantities provided therein.

due to the application of power-law to a limited range of x_f/x_{f0}, the estimated fraction of heavy fluid in the head falls below unity. The power-law (11) provides a robust means for the estimation of heavy fluid in the gravity current head, nevertheless, the amount of heavy fluid in the head could be easily underestimated when the gravity current is not sufficiently far into the deceleration phase. To remedy the underestimation, we could multiply the estimated fraction by a correction factor, i.e. $[K_B/K_A]^3$, when the power-law is approached for the front location in the ranges as listed.

It would also be of interest to analyse how the value of χ varies with bottom slope. Using the data provided for $U_f {x_f}^{1/2}/{B_0'}^{1/2}$ and other measured quantities in Beghin et al. (1981), it is possible to back calculate K_M and K_B and estimate χ. Fig. 4 shows the dependence of χ on θ. For the whole range of slope angles considered therein, i.e. $5^\circ \lesssim \theta \leq 90^\circ$, it is evident that the fraction of heavy fluid contained in the head exceeds 75%. In the subrange $5^\circ \lesssim \theta \lesssim 10^\circ$, it is likely that the fraction increases with increasing slope angle, while in the subrange $10^\circ \lesssim \theta \leq 90^\circ$, the fraction falls within $0.9 < \chi < 1.0$ and appears to slightly decrease with increasing slope angle. Considering the original experiments and measurements might be subject to uncertainties, more precise estimates for χ await further studies.

4. Conclusions

The power-law for gravity currents on slopes is essentially an asymptotic form of the solution of thermal theory, when the gravity current is sufficiently far into the deceleration phase. The power-law not only describes the front location versus time relationship but also provides a robust means for the estimation of heavy fluid contained in the gravity current head. The expression for an important model constant K_B in the formulation is now corrected.

The hypothesis that gravity current is sufficiently far into the deceleration phase is usually not satisfied in experiments. Even though the power-law still works in this situation, the derived constant K_M is easily underestimated and consequently so is the amount of heavy fluid contained in the head. When the gravity current is not sufficiently far into the deceleration phase, the shortcoming of under-estimation of K_M can be remedied by multiplying a correction factor according to where the gravity current is in the deceleration phase.

It is worthy to point out that the power-law method is based on the momentum equation. The amount of heavy fluid in the head estimated with the power-law should be understood as the 'effective' buoyancy that plays the role as the driving mechanism for the 'thermal cloud' convection. In other words, the amount of heavy fluid in the head so estimated is deemed as a lower limit for the estimation. As shown in previous work (e.g. Dai et al., 2012), the gravity current head may contain more heavy fluid in the head region as part of it is so diluted or dissipated from the moving head and is considered 'ineffective' in driving the gravitational convection of 'thermal cloud'.

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