Reexamination of Critical Period for Reservoir Design and Operation

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Abstract: Reservoir operating rules are often based on deterministic models using critical period (CP) analysis, which results in very conservative decisions. CP is defined as the historical hydrologic period that includes the lowest flow or the most severe drought, which represents functionally as the full-empty cycle of reservoir storage under a given firm yield. This paper compares CP with another time scale, the forecast horizon (FH) underlying hedging rule policies for reservoir operation. When the decisions in the initial few periods are not affected by forecast data beyond a certain period, the period is known as FH, and the number of the initial periods is known as the decision horizon. When a reservoir operation policy follows the concept of CP, CP is used as FH and the optimal yield generated within the FH is the firm yield. FH as long as the CP is not realistic because in the real world data for such a long future period is unavailable; and using CP as FH is not effective either, because CP—the longest FH-involving the most severe drought leads to very conservative release decisions, i.e., the “near-zero-risk” decision and the time preference of utility diminishes the influence of hedging for over long periods.


CE Database subject headings: Water supply; Reservoir operation; Optimization; Planning.

Introduction

The concept of critical period (CP), which was interpreted as a historical hydrologic period that included the lowest flow or the most severe drought, was systematically studied by Hall et al. (1969) for reservoir design and operation. They defined CP functionally as the full-empty cycle of reservoir storage under a given firm yield. Following Hall et al. (1969), McMahon and Mein (1986), Oguz and Bayazit (1991), and Montaseri and Adeloye (1999) elaborated the concept of CP and provided different techniques for estimating the value of CP, using either historical records or synthesized data.

CP was primarily determined by the sequent peak algorithm (i.e., Rippl’s method) with consideration of the combination of demand and critical hydrologic conditions (USACE 1997). Once CP is determined, it can be used for several purposes of reservoir design and operations: (1) determining the minimum required reservoir storage capacity using the CP hydrologic inflow series for a target delivery and (2) determining the maximum delivery that can be always produced during the CP, known as the firm yield, given the storage capacity of a reservoir. Following the U.S. Army Corps of Engineers (USACE 1997), besides determining the firm yield, CP has also been used to derive the operation curves that are regularly used by reservoir managers. An operation curve establishes a tentative plan of operation which considers the hydrologic date, flood control, and reservoir sedimentation as well as conservation requirements that can be undertaken by performing detailed sequential routing of the CP and several other periods of low flow. Once the operation curve is developed, “additional sequential routings for the entire period of flow record are then made using the rule curve developed in the CP studies” (USACE 1997).

The operational rules developed with CP are usually extended to deal with drought events. CP is supposed to represent the most severe situation for reservoir operation, but it is based on historical hydrological records and it assumes that the most severe hydrological condition in the future will repeat the observed severe event experienced in the history (Loucks et al. 1981). This assumption forms a limitation for the CP-based method since the future hydrological conditions can differ from the historical record. The CP-based method also leads to very conservative reservoir management only suitable for very risk adverse decision makers (Draper 2001) because the probability of the “most severe” drought is very low (theoretically, it should be zero). In general, since the CP-based method assumes fully known deterministic information, it facilitates the traditional linear decision rules or standard operating policies which ensure an optimal operation under a static situation with a given demand (Shih and ReVelle 1994). Thus, the method is not suitable for deriving decision rules, such as hedging rules, regarding stochastic utility or cost optimization under incomplete information.

With these concerns in mind, this paper examines the rationale of the CP-based rules considering the imperfect forecast of future periods. The question to address is: When the CP-based rules are applied to real world reservoir operations that face future inflow uncertainty, what limitations are imposed? In the following, we first introduce hedging rules and the time scale of hedging for stochastic reservoir operations considering inflow uncertainty. We then show that CP is a special time scale for hedging and discuss
Hedging Rule and Forecast Horizon

Hedging rules for reservoir operations accept a small deficit in current supply to reduce the probability of a severe water shortage in the future (You and Cai 2008a,b). One of the critical questions for hedging research is how long to hedge, given the forecast of an uncertain input. Decision makers always hope to look further to the future, but longer forecasts are less reliable and should have a diminishing influence on decision making. For dynamic reservoir operation optimization models, when it happens that the decisions in the initial few periods are not affected by future data beyond a certain period, the period is known as “forecast horizon (FH)” and the number of the initial periods is known as the “decision horizon (DH).” Most recently, the concept of DH and FH was introduced to stochastic reservoir operation under hedging by You and Cai (2008c). Assuming the objective of a reservoir operation problem is to maximize the utility over all time periods, the expected objective value can be written as

\[
\text{Max}_{r^n} \left\{ B(r, \alpha, N) = EV \left[ \sum_{n=1}^{N} \alpha^n b_n(s_n) \right] \right\}
\]

(1)

where \( B(r, \alpha, N) \) = N-period overall benefit; \( b_n = \) utility function; \( s_n = \) reservoir storage at the beginning of the \( n \)th period; \( EV[\cdot] \) represents the expected value; \( \alpha = \) discount factor; and \( r^n = \) decision variable vector, \( r^n = (r_1, r_2, \ldots, r_N); \) and \( r^n \in R^N = \) N-dimension Euclidean space, or N-space. According to the FH/DH definition, if the optimal decisions (\( r^n_{opt} \)) in period \([1, t]\) are not affected by the parameters in period \([T+1, N]\), then \( t = \) the DH and \( T = \) the FH. Here, \( N \) represents the study horizon of the optimization problem. The DH/FH problem can be expressed as, “With a given DH, how long of an FH is sufficient for the optimal solution of a dynamic optimization problem?” You and Cai (2008c) derived a local necessary condition for determining FH given DH which is stated as follows: Assume \( B(r, \alpha, N) \) is continuous and piecewise differentiable (i.e., each piece of the function is differentiable, e.g., a piecewise linear utility function) to \( r \) and \( s \), where \( t/\Delta t \) represents the \( \Delta t \)-horizon with a given forecast, \( d \), and also assume for \( \forall n \in \mathbb{Z} \), an upper bound \( (L_n) \) of the utility function in any period \( n \) exists, i.e., \( L_n \geq EV[b_n(\cdot)] \). If there exists a time horizon \( (T) \) which satisfies

\[
- \frac{\partial B(r^n, \alpha, t/\Delta t)}{\partial s_{T+1}} \sup_{T < n \leq N} (\alpha^n L_n) \geq 0
\]

(2)

then \( T = \) forecast horizon for decision horizon \( t \). (The proof is given in the appendix.)

In Eq. (2), \( -\frac{\partial B(r^n, \alpha, t/\Delta t)}{\partial s_{T+1}} \sup_{T < n \leq N} (\alpha^n L_n) \) represents the utility change per one unit of storage change under the current decision policy; \( \frac{\partial B(r^n, \alpha, t/\Delta t)}{\partial s_{T+1}} \) is the residual quantity of one unit additional storage from time horizon \( t \) to \( T \), considering evaporation and seepage loss; \( \sup_{T < n \leq N} (\alpha^n L_n) \) is the maximum potential utility increase with one unit of additional storage in any periods after \( T \). If the change of current policy in DH \( (t) \) causes a larger utility loss than the potential gain in the unplanned future after \( T \), future data should not change the optimality within \( t \). According to the definition of DH and FH, if the operation policy in period \([1, t]\) is not affected by the data in periods \([T+1, N]\), \( T \) is the FH with associated \( t \), the DH.

Examination of Critical Period under the Framework of Forecast Horizon

Both FH and CP represent a time scale for reservoir planning and management decision problems, however, they are different conceptually. CP is defined based on historical records, assumes deterministic hydrologic information, and supports static decision makings. FH is defined as a future time period, adopts both deterministic and stochastic hydrologic information, and supports dynamic decision making. However, CP might have been used as a kind of FH in the real world reservoir operation, in which CP is treated as a time horizon with deterministic and fully known information for the future. This is illustrated in the following using the local necessary condition for FH [Eq. (2)].

The determination of CP can be based on Rippl’s mass curve method, which provides a graphical estimate of reservoir storage required to produce a given water yield (Klemes 1987). Klemes (1979) proved that Rippl’s method was equivalent to an exact variational solution of an optimization reservoir release problem and Morel-Seytoux (1999) illustrated some details of Varlet’s method too. Duranyildiz et al. (1999) showed the use of Varlet’s method to determine the firm yield and the corresponding CP.

Here, we show that under the assumptions associated with its definition, CP is the FH when the CP-based rule is used for reservoir operation. We start from Rippl’s mass curve. For an unconstrained reservoir (i.e., infinite storage), Klemes (1979) defined a problem to minimize the following cost function

\[
F(Y) = \int_0^T L(Y') dt
\]

(3)

where \( Y' = \) reservoir yield; \( Y = \) total water availability during time period \([0, T]\); and \( L(Y') \) denotes the convex loss function. Klemes used the calculus of variation to solve this problem. Following the Euler-Lagrange differential equation

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial Y'} \right) - \frac{\partial L}{\partial Y} = 0
\]

(4)

Because the loss is function of only the yield \( Y' \)

\[
\frac{\partial L}{\partial Y} = 0
\]

(5)

\[
\frac{\partial L}{\partial Y'} = L'(Y')
\]

(6)

where \( L'(Y') = \) derivative of \( L(Y') \). Following these conditions, Klemes proved that the “firm yield” corresponding to the minimum loss was

\[
Y' = (X_T + S_0 - S_T)/T
\]

(7)

where \( X_T = \) total inflow during time period \([0, T]\); \( S_0 = \) initial storage; and \( S_T = \) terminal reservoir storage. Taking the integral of Eq. (7) with respect to \( t \) from 0 to \( T \), we have

\[
Y = \int_0^T Y' dt = \int_0^T (X_T + S_0 - S_T)/T dt = X_T + S_0 - S_T
\]

(8)

Since \( X_T \) and \( S_0 \) are given constants for the problem, we have

\[
- \frac{\partial S_T}{\partial t} = \int_0^T Y' dt
\]

(9)
Since Klemes’ problem is to minimize a loss function, if we treat loss as negative utility, i.e., \(-B(r^T, \alpha, t|d) = F(Y)\), then we can apply the local necessary condition [Eq. (2)] to Klemes’ problem described above. Klemes’ problem assumes an unconstrained reservoir without considering evaporation and seepage losses; then in Eq. (2), \(\delta s_T/\delta s_t = 1\) (i.e., the residual quantity of one unit additional storage from time horizon \(t\) to \(T\) remains unchanged because of no evaporation and seepage loss). Additionally, \(L'(Y')\) equals \(\partial L(Y')/\partial Y'\) according to Eq. (6). Thus, the left-hand side (LHS) of the local necessary condition [Eq. (2)] can be rewritten as

\[
- \frac{\partial B(r^T, \alpha, t|d)}{\partial s_t} \frac{\partial s_T}{\partial s_t} = \frac{\partial F(Y)}{\partial s_T} = \frac{\partial L(Y')}{\partial Y'} = -L'(Y') \tag{10}
\]

Therefore, Eq. (2) applied to the problem defined above can be written as

\[
-L'(Y')|_{0 \leq t \leq T} > \sup -L'(Y')|_{t > T} \tag{11}
\]

While for a constrained reservoir with a given capacity, when the reservoir is not full, \(\delta s_T/\delta s_t = 1\) ignoring evaporation and seepage loss over time, and Eq. (10) and Eq. (11) still hold; when the reservoir is full, spill occurs with additional inflow and no additional water can be saved for future use, thus \(\delta s_T/\delta s_t = 0\), the LHS of Eq. (2) is infinite, and Eq. (2) holds.

For a convex loss function \(L, L'\) monotonically increases with respect to \(Y'\); in other words, \(-L'\) decreases with respect to \(Y'\). So, we have

\[
Y'|_{0 \leq t \leq T} < Y'|_{t > T} \tag{12}
\]

which means the yield during the period \([0, T]\) is the smallest over an infinite time period, which is consistent with the concept of CP—the yield any time in the future will not be lower than the yield generated during the CP given that the future hydrological events reproduce the historic hydrologic cycle. Therefore, according to the local necessary condition \(T\) which is equivalent to CP here, is the FH.

Conclusions

When a reservoir operation policy is made following the concept of the CP, CP is actually used as FH and the optimal yield generated within the FH is the firm yield. From the point view of hedging, if reservoir managers use the forecast information in FH as long as in duration as CP, they are supposed to consider the most severe situation in the future provided that the future repeats the history. Thus, CP is the longest FH for reservoir operation. This is not realistic because in the real world, the forecast for a future period as long as CP is very difficult. Furthermore, this method may be ineffective for two reasons. First, the longest FH involving the most severe drought in the far future, which may be insignificant to current operations, leads to very conservatism release decisions, i.e., “zero-risk” decision. Second, FH based on a duration as long as CP diminishes the influence of hedging due to the time preference (i.e., economic discount) as well as growing uncertainty of reservoir inflow. Therefore, CP may not be used a realistic forecast horizon for stochastic reservoir operation analysis due to the assumptions involved in the concept, which probably lead to too conservative policies for reservoir operation. However, we do not mean that CP is a not totally inappropriate for reservoir design and planning. Public decision making usually tends to be a risk aversion as what a framework based on CP does.

Thus, we argue that for deriving appropriate hedging policies, a new time horizon of reservoir operation is needed. The horizon will be based on reservoir functions, storage characteristics, and hydrologic inflow uncertainty. Moreover, the economic loss function will also likely be important for hedging (Draper and Lund 2004; You and Cai 2008a,b). Considering these factors, the local necessary condition can be used as a principle to determine a more appropriate forecast horizon, which was illustrated through theoretical analysis and numerical modeling by You and Cai (2008c).

Appendix. Proof of the Local Necessary Condition [Eq. (2)]

Theorem

Assume \(B(r, \alpha, N)\) is continuous and piecewise differentiable to \(r\) and \(s\); also assume for all \(n \in Z\), an upper bound (\(L_n\)) of the utility function in any period \(n\) exists, i.e., \(L_n > EV[b^0(s)|d]\). If there exists a time horizon (\(T\)) which satisfies

\[
- \frac{\partial B(r^T, \alpha, t|d)}{\partial s_{t+1}} \frac{\partial s_{T+1}}{\partial s_{t+1}} \sup \geq \sup (\alpha L_n), \quad \forall T < n \leq N \tag{13}
\]

Then \(T=\) forecast horizon for decision horizon \(t\).

Proof

We use proof by contradiction. Define \(P\) as a condition and \(Q\) as a conclusion; we want to prove \(P > Q\). To apply the method of proof of contradiction, we suppose that \(P\) and \(not-Q\) are true and look for a contradiction with known facts, i.e., “\(P\) true” = > “\(not-Q\) is false.” In our proof, we use a variation of the proof by contradiction, which is called the contrapositive method, by which if we can show \(not-Q\) = > \(not-P\), then we can conclude that \(P = > Q\). For our problem, we assume \(T\) is not FH and \(B(r^T, \alpha, T|d)\) is not the projection of \(B(r^N, \alpha, N)\) (the optimal solution of the \(N\)-period problem) in period \([1, T]\); with this assumption, if Eq. (7) is false, then we prove the theorem.

If \(T\) is not the FH and \(B(r^T, \alpha, T|d)\) is not the projection of \(B(r^N, \alpha, N)\) in period \([1, T]\), then there exists a decision \(r^T\), \(r^T\neq r^T\), satisfying

\[
B(r^T, \alpha, T|d) + EV\left[ \sum_{n=t+1}^{N} \alpha^n b_n(r_n, s_n) \right] \geq \sum_{n=t+1}^{N} \alpha^n b_n(r_n, s_n)|_{r_{n+1}=r^n_{T+1}} > B(r^T, \alpha, T|d)
\]

\[
+ EV\left[ \sum_{n=t+1}^{N} \alpha^n b_n(r_n, s_n)|_{r_{n+1}=r^n_{T+1}} \right] \tag{13}
\]

As defined before, \(r^T\) is the decision variable vector, \(r^T = [r_1, r_2, r_3, \ldots, r_T]\), \(B(r^T, \alpha, T|d)\) and \(B(r^T, \alpha, T|d)\) are the total utility in \(T\) periods under the two release decisions, \(r^T\) and \(r^T\), respectively; and we assume \(B(r^T, \alpha, T|d)\) is close to \(B(r^T, \alpha, T|d)\). Eq. (13) can be rearranged to

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Following the principle of differentials, if \( s_T(r^T) \) is close to \( s_T(\hat{r}^T) \) and \( B(\hat{r}^T, \alpha, T | d) \) is close to \( B(r^T, \alpha, T | d) \), then the LHS of Eq. (15) can be expressed by a differential form

\[
\begin{align*}
\text{LHS} & \geq \frac{B(r^T, \alpha, T | d) - B(\hat{r}^T, \alpha, T | d)}{s_T(r^T) - s_T(\hat{r}^T)} \\
& = -\frac{\partial B(r^T, \alpha, T | d)}{\partial s_T} \frac{ds_T}{ds_T} \\
& = -\frac{\partial B(r^T, \alpha, T | d)}{\partial s_T} \frac{ds_T}{ds_T}.
\end{align*}
\]