On three-dimensional continuous saltating process of sediment particles near the channel bed

Sur le processus de saltation continue tridimensionnelle des particules de sédiment en fond de canal

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ABSTRACT

The purpose of this study is to develop a three-dimensional continuous saltation model which is able to simulate the saltating behaviors of a single particle near the channel bed. A real-time flow visualization system, including two charge-coupled device cameras, an image-grabbing card and an OPTIMAS image-processing software, is developed to measure the three-dimensional saltating trajectories and corresponding velocity components. The averaged dimensionless saltation length, height and width were found to follow Pearson type III distributions and the dimensionless saltation velocity follows a normal distribution. The impacting and rebounding angles increase as the flow transport capacity $T^*$ increases. The rebounding angles, under various impacting conditions, follow a normal distribution. Based on the data collected, an impacting and rebounding mechanism is derived, and thus a three-dimensional saltation model is established to simulate the dynamic characteristics of the saltating particles. Regression equations for the important saltating characteristics, including saltation length, height, width and three-dimensional velocity components were obtained. Based on these equations, bed load equation was derived. This equation tends to slightly overestimate the bed load transport. Nevertheless, the overall accuracy is satisfactory.

RÉSUMÉ

Le but de cette étude est de développer un modèle de saltation continue tridimensionnelle capable de simuler les comportements d’une particule simple près du lit d’un canal. Un système de visualisation d’écoulement en temps réel, comprenant deux caméras à couplage de charge CCD, une carte de capture et un logiciel de traitement d’images OPTIMAS, est mis au point pour mesurer les trajectoires tridimensionnelles de saltation et les composantes de vitesse correspondantes. Les longueur, hauteur et largeur de saltation moyennes adimensionnelles se sont avérées suivre des distributions de Pearson de type III, et la vitesse adimensionnelle de saltation suit une distribution normale. Les angles d’impact et de rebondissement augmentent à mesure que la capacité de transport $T^*$ augmente. Les angles de rebondissement, sous diverses conditions d’impact, suivent une distribution normale. Sur la base des données collectées, un mécanisme d’impact et de rebondissement est obtenu, et ainsi un modèle tridimensionnel de saltation est établi pour simuler les caractéristiques dynamiques des particules en saltation. Des équations de régression ont été obtenues pour les caractéristiques importantes de la saltation, comprenant les longueur, hauteur et largeur de saltation et les composantes tridimensionnelles de la vitesse. A partir de ces dernières, une équation de charge de lit a été obtenue. Cette équation tend à surestimer légèrement le transport solide. Néanmoins, l’exactitude globale est satisfaisante.

Keywords: Saltation, sediment, 3-D.

1 Introduction

Bed load transport is one of the most important sediment transport phenomena. It is the key element that affects the scouring and deposition behaviors near the channel bed. However, due to experimental difficulties, the bed load transport characteristics have seldom been measured very accurately. In the bed load transport mode, the sediment particles move in various forms, such as sliding, rolling and saltating, depending on the particle size and the flow conditions. Previous investigators (Einstein, 1942; Wiberg and Smith, 1987, 1989; Sekine and Kikkawa, 1992) indicated that the majority of bed load transport is in the form of saltation. Efforts by many researchers (Hui and Hu, 1991; Sekine and Kikkawa, 1992; Lee and Hsu, 1994; Nino and Garcia, 1994, 1998) on the mechanism of the saltating process in recent years have constructed the theoretical basis of the saltation phenomena. However, these efforts are confined to two-dimensional single-particle saltating process and theories and experiments...
of the three-dimensional particle saltating mechanism are still lacking.

A series of experiments were conducted in this study to measure the relevant three-dimensional saltation characteristics of the sediment particles near the channel bed. A three-dimensional continuous saltation model was also developed. The model is calibrated and verified with the experimental data with satisfactory results.

2 Experimental setup

The experiments were conducted in a 12-m-long, 0.3-m-wide slope-adjustable recirculating flume. Several combinations of water depth, channel slope, particle size and specific gravity of the particles were tested. The water depth was fixed at 5 cm and the range of the slope was from 0.003 to 0.012. Particles with diameter of 0.6 cm and specific gravities between 2.15 and 1.08 were chosen. Particles of the same size as the saltating particles were glued to form the channel bed. Key hydraulic and sediment transport characteristics are given in Table 1.

Measurements are performed using a newly developed three-dimensional real-time flow visualization system. The system consists of two charged-couple device (CCD) cameras, a Pentium II 266 PC and a Meteor2 Image Grabber Card. Integrating the Image-grabbing program and Image-processing software (OPTI-MAS V5.22), using two of the three color channels (RGB) of each frame, the system can acquire images from both top and side views synchronously, and then transform the images into digitized formats for analysis. The general configuration of the experimental setup is shown in Fig. 1. During the experimental process, the laboratory is kept in complete darkness except for the light sources placed on both sides of each CCD camera. Sediment particles are released about 2 m upstream of the working section. As the saltating particles transit through the viewing windows, images are recorded by the CCD camera at a frame rate of 30 fps (frames per second).

The digitized images are analyzed using the software OPTI-MAS V5.22. Background noise is first eliminated through a logarithmic subtract process in order to highlight the particle images. The area and the centroid of the particle images are then calculated to identify the particle positions. These are then transmitted to a worktable through a dynamic data exchange process. Since the time interval between the successive image frames is constant, the instantaneous saltating velocity vectors are finally obtained.

3 Dimensional analysis

The parameters that affect the particle saltating motions include: water depth $H$, channel slope $S_0$, mass density of water $\rho_w$, kinematics viscosity $\nu$, saltating particle size $D$, size of the particles that form the channel bed $D_b$, mass density of the saltating particle $\rho_s$, particle surface roughness $\varepsilon$, gravitational acceleration $g$, and shape factor $S$. Using the Buckingham’s $\pi$ method, the following dimensionless equation can be obtained:

$$\hat{F} = f_1 \left( \tau_s, \text{Re}_a, \frac{H D_b}{\rho_s}, \frac{\varepsilon}{D_b}, \frac{\rho_s}{\rho_w}, S \right)$$

where $\hat{F}$ denotes dimensionless saltating characteristics, including height, length and velocity, $\tau_s = \rho_w u^2/(\rho_s - \rho_w)D$ is the dimensionless shear stress, $u_w = (S_0 g H)^{1/2}$ is the shear velocity, and $\text{Re}_a = u_s D/\nu$ is the particle Reynolds number. In our cases, since $H \gg D, D_b \gg \varepsilon$ and $D_b = D$, these three terms can be

<table>
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<th>Particle size, $D$ (cm)</th>
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<th>Depth, $H$ (cm)</th>
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The dynamics of saltating particles are governed by the combined forces. Applying Newton's second law, the equations of motion effects of inertia, added mass, submerged weight, lift and drag are removed from Eq. (1). The saltating particles and bed particles are spheres, so that the shape factor is assumed neglected. Combining \( \tau \) and \( \text{Re} \), and eliminating \( u_\ast \), a dimensionless parameter \( D_\ast = D \left[ (S_G - 1) g/(u_\ast^2) \right]^{1/3} \), where \( S_G = \rho_s/\rho_w \), can be obtained.

Replacing \( u_\ast \) with \( (u_\ast - u_{\ast c}) \), where \( u_{\ast c} \) is the particle critical shear velocity which can be obtained from the Shields diagram (Van Rijn, 1984), \( \tau_{\ast} \) can be transformed into \( T_\ast = (u_\ast^2 - u_{\ast c}^2)/u_{\ast c}^2 = \text{flow transport capacity}. \) Equation (1) can therefore be simplified to:

\[
\vec{F} = f_2(T_\ast, D_\ast)
\] (2)

4 Theoretical model

4.1 Governing equations

The dynamics of saltating particles are governed by the combined effects of inertia, added mass, submerged weight, lift and drag forces. Applying Newton’s second law, the equations of motion of the saltating particles are:

\[
m \ddot{x} = F_L \cdot \left( \frac{\dot{y}}{u_r} \cdot \frac{u - \dot{x}}{\sqrt{(u - \dot{x})^2 + \dot{z}^2}} \right) + F_D \cdot \left( \frac{u - \dot{x}}{u_r} \right)
\]

\[
+ F_G \sin \theta
\] (3)

\[
m \ddot{y} = F_L \cdot \left( \frac{\sqrt{(u - \dot{x})^2 + \dot{z}^2}}{u_r} \right) - F_D \cdot \frac{\dot{y}}{u_r} - F_G \cos \theta
\] (4)

\[
m \ddot{z} = F_L \cdot \left( \frac{\dot{z}}{u_r} \cdot \frac{\dot{z}}{\sqrt{(u - \dot{x})^2 + \dot{z}^2}} \right) + F_D \cdot \left( \frac{\dot{z}}{u_r} \right)
\] (5)

where \( u \) is the flow velocity, \( m = (\rho_s + C_m \rho_w) \pi D^3/6 \) combines the particle mass and added mass of surrounding fluid. The added mass effect means that the relative acceleration between a body and the surrounding fluid will result in an apparent mass (Wiberg and Smith, 1985). \( \dot{x}, \dot{y}, \dot{z}, \ddot{x}, \ddot{y}, \ddot{z} \) represent the components of particle velocity and acceleration oriented parallel, normal and transverse to the sloping bed, respectively.

\[
u_r = \sqrt{(x_u - u)^2 + y^2 + z^2} \] is the velocity of the particle relative to the fluid, \( \theta \) is the channel bed angle, \( F_G = \alpha (\rho_s - \rho_w) g D^3 \) is the submerged body force, \( F_i \) is the lift force, and \( F_D \) is the drag force. The formulations of \( F_L \) and \( F_D \) are given below.

The drag force is caused by a combination of pressure difference and skin friction. It is expressed as:

\[
F_D = C_D A \frac{\rho_w u_r^2}{2}
\] (6)

where \( C_D \) is the drag coefficient and \( A = \beta_1 D^2 \) is the projected area perpendicular to the flow direction, \( \beta_1 = 1/4 \) for sphere particles. The drag coefficient can be calculated using the empirical relation of Swamee and Ojha (1991):

\[
C_D = 0.5 \left[ 16 \left( \frac{24}{\text{Re}} \right)^{1.6} + \left( \frac{130}{\text{Re}} \right)^{0.72} \right]^{2.5}
\]

\[
+ \left[ \frac{40000}{\text{Re}} \right]^{2} - 0.25 \right]^{0.25}
\] (7)

where \( \text{Re} = u_r D/\nu \) is the Reynolds number.

Several different formulas are available to calculate the lift force. The one proposed by Anderson and Hallet (1986) is used in this study.

\[
F_L = \frac{1}{2} \rho_w A C_L \left( u_{\ast T}^2 - u_{\ast B}^2 \right)
\] (8)

where \( u_{\ast T} \) and \( u_{\ast B} \) are relative velocities at the top and bottom of the particle, \( A = \beta_2 D^2 \) is the cross-sectional area of the particle perpendicular to the flow direction, \( \beta_2 = 1/4 \) for sphere particles, and \( C_L \) is the lift coefficient. The lift coefficient is affected by the particle size, shape, spinning velocity and flow field.

4.2 Impacting and rebounding mechanism

A three-dimensional bed collision model is developed in this study. It is based on the impacting and rebounding concept proposed by Nino and Garcia (1994). Top and side views of the bed impacting process are given in Figs 2 and 3, respectively. The model assumes a channel bed composed of uniformly packed spheres, a saltating particle approaching the bed at an angle \( \theta _{\ast} \), and striking the upstream face of a bed particle at an angle \( \theta _{b} \). The magnitude of the particle salting velocity is reduced due to collision. Coefficients of restitution and friction \( e \) and \( f \) account...
for reductions of the normal and tangential velocity components, respectively. The rebounding angle \( \theta_b \), corresponding velocity components immediately after collision \( u_{p,\text{out}} \) and \( v_{p,\text{out}} \) can be derived as

\[
\tan \theta_i = \frac{e}{f} \tan (\theta_{l,\text{in}} + \theta_b) \tag{9}
\]

\[
u_{p,\text{out}} = f \left( u_{p,\text{in}}^2 + v_{p,\text{in}}^2 \right)^{1/2} \cos(\theta_{l,\text{in}} + \theta_b) \frac{\cos(\theta_i + \theta_b)}{\cos(\theta_i)} \tag{10}
\]

\[
v_{p,\text{out}} = f \left( u_{p,\text{in}}^2 + v_{p,\text{in}}^2 \right)^{1/2} \sin(\theta_{l,\text{in}} + \theta_b) \frac{\sin(\theta_i + \theta_b)}{\cos(\theta_i)} \tag{11}
\]

where \( u_{p,\text{in}} \) is the approaching longitudinal velocity component and \( v_{p,\text{in}} \) is the corresponding vertical velocity component.

In order to describe the three-dimensional phenomena, two lengths, \( l \) and \( r \), are introduced. Length \( l \) is the distance between the impacting point and the center of the bed particle projected on the particle centerline in C–C section, as indicated in Fig. 4. Likewise, length \( r \) pertains to a section parallel to the flow direction, as shown in Fig. 5. The values of \( l \) and \( r \) are assumed to be uniformly distributed. The coefficient \( l \) is in a range between \(-0.5R \) and \( 0.5R \), where \( R \) is the radius of bed particles, and the value of \( r \) is a function of \( l \), the impacting point and the impacting angle.

From geometrical considerations, the range of \( r \) can be expressed as

\[
-\sin \theta_{l,\text{in}} \leq \frac{r}{R\sqrt{1-l^2}} \leq \frac{1}{2\sqrt{1-l^2}} \tag{12}
\]

\[
\text{Figure 3 Side view of the bed impacting process.}
\]

\[
\text{Figure 4 Probability density coefficient } l \text{ in section C–C.}
\]

\[
\text{Figure 5 Probability density coefficient } r \text{ in section A–A.}
\]

\[
\text{Figure 6 Splash function.}
\]

where

\[
\sin \theta_{l,\text{in}} \leq \sin \left( \frac{\pi}{2} - \cos^{-1} \left( \frac{1}{2\sqrt{1-l^2}} \right) \right)
\]

The value of \( r \) is positive when it is to the left of the particle centerline and negative otherwise.

The relation between \( r \) and \( \theta_i \) can be expressed as

\[
\theta_b = \sin^{-1} \left( \frac{r}{R\sqrt{1-l^2}} \right) \tag{13}
\]

\( \theta_b \) is positive facing downstream and negative facing upstream, given \( \theta_{l,\text{in}} \). \( \theta_b \) can thus be determined, and it can be expressed as a conditional probability \( p(\theta_b|\theta_{l,\text{in}}) \). Since \( l \) and \( r \) are uniformly distributed, the probability of every particle impacting position projected onto a horizontal line is the same. That means the locations of the particle impacting points along the channel bed surface are uniformly distributed. According to the geometric relations described above, the probability density functions of \( l \) and \( r \) can be casted in the form of \( p(\theta_{b,\text{in}}|\theta_{l,\text{in}}) \), and thus the splash function and \( \theta_b \) can be derived. The splash function is shown in Fig. 6 and \( \theta_i \) is expressed as

\[
\theta_i = \langle \theta_{\text{out}} \rangle - E \{ \theta_b | \langle \theta_{l,\text{in}} \rangle \} \tag{14}
\]

where \( \langle \rangle \) means average and \( E \) means expectation.

Applying law of energy conservation, the corresponding transverse velocity component can thus be determined. A coefficient \( k_b \) is introduced to represent the energy loss effect. Thus, the transverse velocity component \( w \) can be derived as:

\[
w_{p,\text{out}} = \pm \left[ k_b \left( u_{p,\text{in}}^2 + v_{p,\text{in}}^2 + w_{p,\text{in}}^2 \right) - \left( u_{p,\text{out}}^2 + v_{p,\text{out}}^2 \right) \right]^{1/2} \tag{15}
\]

where the positive or negative sign of \( w \) depends on the particle impacting position.
5 Numerical model

5.1 Initial conditions

The initial conditions include initial position and initial velocity. In our previous single-step saltation model (Lee and Hsu, 1994), the initial position of the particle was assumed to be 0.6D from the average channel bed (Van Rijn, 1984) and the initial longitudinal, vertical and lateral components of the lift-off velocity were assumed to be \( \alpha_1 u_s, \alpha_2 u_s, \) and \( \alpha_3 u_s \), where \( \alpha_1, \alpha_2, \) and \( \alpha_3 \) are coefficients to be calibrated. In this continuous saltating model, the rebounding position and velocity are related to the previous saltation trajectory. According to our experimental observations, the rebounding position is located between 0.6D and 0.75D. The rebounding velocity can thus be calculated using the impacting and rebounding mechanism introduced above.

5.2 Numerical solution scheme

The governing equations are second-order nonlinear ordinary differential equations. They were transferred into a system of first-order ordinary differential equation, shown in Eqs (16)–(21), and then solved by Runge–Kutta and Gill method:

\[
F_1 = \frac{1}{m} \left[ F_l(\dot{x}, \dot{y}, \dot{z}) \cdot \left( \frac{\dot{y}}{u_t} \cdot \frac{u - \dot{x}}{\sqrt{(u - \dot{x})^2 + \dot{z}^2}} \right) \right.
+ F_D(\dot{x}, \dot{y}, \dot{z}) \cdot \left( \frac{u - \dot{x}}{u_t} \right) + F_G \sin \theta \]
\]

\[F_2 = \dot{x}\]

\[
F_3 = \frac{1}{m} \left[ F_l(\dot{x}, \dot{y}, \dot{z}) \cdot \left( \frac{\dot{y}}{u_t} \cdot \sqrt{(u - \dot{x})^2 + \dot{z}^2} \right) \right.
- F_D(\dot{x}, \dot{y}, \dot{z}) \cdot \left( \frac{\dot{y}}{u_t} \right) - F_G \cos \theta \]
\]

\[F_4 = \dot{y}\]

\[
F_5 = \frac{1}{m} \left[ F_l(\dot{x}, \dot{y}, \dot{z}) \cdot \left( \frac{\dot{z}}{u_t} \cdot \frac{\dot{z}}{\sqrt{(u - \dot{x})^2 + \dot{z}^2}} \right) \right.
+ F_D(\dot{x}, \dot{y}, \dot{z}) \cdot \left( \frac{\dot{z}}{u_t} \right) \]
\]

\[F_6 = \dot{z}\]

5.3 Coefficient calibration

The coefficients that need to be calibrated include the lift coefficient \( C_L \), the restitution coefficient \( e \), the friction coefficient \( f \) and energy loss coefficient \( k_b \). The lift coefficients suggested by Wiberg and Smith (1985), Lee (1993), and Lee et al. (2000) are 0.2, 3.1 and 4.6, respectively. According to the experimental investigations conducted in this study, the lift coefficient \( C_L \) is found to increase with the particle Reynolds number (Re_r). The relation is calibrated using the saltation trajectories measured in this study. By minimizing the relative errors of the simulated saltation length, height, lateral displacement and velocity, a regression equation for \( C_L \) is obtained, and shown in Eq. (22) and Fig. 7, respectively.

\[
C_L = 1.45 \times 10^{-5} \times (Re_r)^{1.7325} \quad (R' = 0.73) \quad (22)
\]

The coefficients \( e, f \) and \( k_b \) are calibrated using the bed rebounding data collected. It is suggested that \( e = 0.59 - 0.045T_\ast; \) \( f = 0.96 \). According to the results of the statistical hypothesis test, the distributions of \( k_b \) are found to be independent of flow conditions and can be represented by a normal distribution with average and standard deviation equal to 0.75 and 0.1563, respectively. To reflect this range, \( k_b \) is set to fall between 0.5 and 1.

5.4 Verification

The model is verified using the experimental data of Nino and Garcia (1998), Lee et al. (2000), Lee and Hsu (1994), and Lee (1993). The results compare favorably, as shown in Figs 8–10.

6 Results and discussions

6.1 Experimental results

The important saltation characteristics measured in this study include saltation length (SL), height (SH), width (SW), velocity (SV), and impacting and rebounding angles. The saltation length, height and width are defined to be the longitudinal, vertical and lateral distances between the takeoff and landing points of the saltation particle. The corresponding data are given in Tables 2 and 3. A typical three-dimensional trajectory is shown in Fig. 11. The relations between the dimensionless saltation length, height, width, velocity and the flow transport capacity \( T_\ast \) are given in Figs 12–15. The saltation lengths are found to be in the range of 10 to 30 times the particle diameter, the saltation widths in the range of 1 to 2 times the particle diameter, and the saltation heights in the range of 2 to 3 times the particle diameter. The saltation velocity is about 8 to 10 times the shear velocity. The dimensionless saltation length increases significantly as \( T_\ast \) increases, and other parameters are more significantly related to the variations of \( D_\ast \). Comparing the experimental data with same \( D_\ast \), the above-mentioned dimensionless parameters increase as \( T_\ast \) increases.
The values of the longitudinal and lateral impacting and rebounding angles are shown in Figs 16–19, respectively. The averaged longitudinal rebounding angle is between 18° and 32°, and the corresponding averaged impacting angle is between 7° and 15°. The averaged lateral rebounding angle is between 5° and 12°, and the corresponding averaged impacting angle is between 3° and 5° degrees. These values decrease as $T_*$ increases. Chi-square test was applied to determine the distributions of the dimensionless salting length, height, width, and velocity. The dimensionless values introduced here are different from the dimensionless values mentioned above. They were obtained by dividing the relevant values by the corresponding mean values, namely, the averaged salting length $(SL_{\text{mean}})$, height $(SH_{\text{mean}})$, and width $(SW_{\text{mean}})$. These dimensionless values were found to follow Pearson type III distributions, and the dimensionless salting velocity obtained by dividing the salting velocity by the averaged salting velocity $(SV_{\text{mean}})$ follows a normal distribution. The distribution functions are given in Eqs (23)–(26), with the corresponding data histograms provided in Figs 20–23, respectively.

$$f\left(x = \frac{SL}{SL_{\text{mean}}}\right) = \frac{\lambda \beta_1}{\Gamma(\beta_1)} (x - \varepsilon_1)^{\beta_1 - 1} e^{-\lambda_1(x-\varepsilon_1)}$$  \hspace{1cm} (23)$$

$$f\left(x = \frac{SH}{SH_{\text{mean}}}\right) = \frac{\lambda \beta_2}{\Gamma(\beta_2)} (x - \varepsilon_2)^{\beta_2 - 1} e^{-\lambda_2(x-\varepsilon_2)}$$  \hspace{1cm} (24)$$
where \( \sigma_x \) is the standard deviation of the random variable \( x \), \( \bar{x} \) is its mean, \( \beta = \left(2/C_x\right)^2 \), \( C_x \) is the skew coefficient of the random variable \( x \), \( \lambda = \sqrt{\beta/\sigma_x} \), \( \varepsilon = \bar{x} - \sigma_x \sqrt{\beta} \) and \( \Gamma \) is gamma function.

These distributions are similar to the distributions suggested in our previous work (Lee et al., 2000).

In order to investigate the bed impacting mechanism, a number of impacting and rebounding angles in both lateral and longitudinal directions were measured. The variance of the rebounding angles is about two times that of the impacting angles. The rebounding angles are not significantly related to impacting angles, as indicated in Fig. 24. This shows the inherent randomness of the rebounding mechanism. The variance of the lateral rebounding angles is also about two times that of the corresponding impacting angles. Under various impacting conditions, the rebounding angles follow a normal distribution, as shown in Fig. 25.

The coefficients \( e \), \( f \) and \( k_b \) were calibrated with the experimental data. It is found that \( f \) is a constant about 0.96, \( e \) falls between 0.3 and 0.7 and decreases as the flow intensity increases. A regression equation is obtained as:

\[
e = 0.59 - 0.045T_*,
\]  

(27)

The coefficient \( k_b \) is normally distributed with average and standard deviation equal to 0.75 and 0.1563, respectively. The
goodness of fit is shown in Fig. 26. According to this equation, we can conclude that a higher flow intensity will suppress the rebounding velocity and yield smaller values of $e$.

6.2 Numerical simulation results

The calibrated numerical model was used to simulate a series of continuous saltation trajectories under 100 different combinations, including 10 particle sizes and 10 flow intensities. Sizes of the saltating particles simulated were between 0.00625 and 0.2 mm and the flow transport intensity $T_s$ was between 1.58 and 100. One hundred and ten continuous saltation steps were simulated in each simulation and the trajectories of the last 100 steps were used to analyze the saltation characteristics. A typical trajectory is shown in Fig. 27.

The simulated dimensionless saltating length $SL/D$ increases as $D_s$ and $T_s$ increase. However, when $T_s$ is small, $SL/D$ is sensitive to the variations of the particle size, and it varies inversely with $D_s$. The simulated $SH/D$ increases as $T_s$ and $D_s$ increase. Under the same $T_s$ condition, $SH/D$ increases as $D_s$ increases. The variations of $SH/D$ is significant when $D_s$ is large. The simulated $SW/D$ increases as $T_s$ and $D_s$ increase.

The simulated dimensionless longitudinal saltation velocity component $SV_x/U_s$ increases as $T_s$ and $D_s$ increase. Under same $T_s$ condition, a larger $D_s$ leads to a larger $SV_x/U_s$. However, the simulated lateral saltation velocity component $SV_z/U_s$ varies inversely with $T_s$. This is because stronger flow intensity will suppress the lateral motion. The relevant simulation results are provided in Fig. 28.

The relations between the simulated longitudinal and horizontal impacting and rebounding angles under various $T_s$ and $D_s$ are
Table 2 Important saltation characteristics measured in the experiments

<table>
<thead>
<tr>
<th>Group</th>
<th>Runs</th>
<th>SL (cm)</th>
<th>SH (cm)</th>
<th>SW (cm)</th>
<th>SV (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-01</td>
<td>48</td>
<td>7.37 (27.8)</td>
<td>1.29 (27.0)</td>
<td>0.67 (64.9)</td>
<td>44.25 (17.6)</td>
</tr>
<tr>
<td>S-02</td>
<td>43</td>
<td>8.75 (33.1)</td>
<td>1.41 (30.6)</td>
<td>0.73 (58.8)</td>
<td>58.68 (17.3)</td>
</tr>
<tr>
<td>S-03</td>
<td>43</td>
<td>8.09 (33.6)</td>
<td>1.24 (29.6)</td>
<td>0.69 (63.6)</td>
<td>50.32 (17.7)</td>
</tr>
<tr>
<td>S-04</td>
<td>46</td>
<td>9.41 (34.8)</td>
<td>1.42 (37.3)</td>
<td>0.76 (70.6)</td>
<td>64.19 (17.1)</td>
</tr>
<tr>
<td>S-05</td>
<td>44</td>
<td>10.38 (28.4)</td>
<td>1.53 (23.8)</td>
<td>0.91 (67.1)</td>
<td>57.23 (18.6)</td>
</tr>
<tr>
<td>S-06</td>
<td>47</td>
<td>10.80 (34.4)</td>
<td>1.68 (35.1)</td>
<td>0.83 (66.2)</td>
<td>65.14 (19.6)</td>
</tr>
<tr>
<td>S-07</td>
<td>70</td>
<td>9.53 (31.9)</td>
<td>1.48 (28.1)</td>
<td>0.75 (64.3)</td>
<td>39.46 (16.7)</td>
</tr>
<tr>
<td>S-08</td>
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<td>10.80 (36.1)</td>
<td>1.50 (33.6)</td>
<td>0.83 (67.6)</td>
<td>59.48 (16.4)</td>
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<td>12.27 (31.2)</td>
<td>1.70 (31.2)</td>
<td>0.97 (56.5)</td>
<td>62.50 (15.7)</td>
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<td>11.54 (32.2)</td>
<td>1.51 (30.8)</td>
<td>0.76 (56.8)</td>
<td>47.61 (15.3)</td>
</tr>
<tr>
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<td>12.26 (37.9)</td>
<td>1.58 (37.3)</td>
<td>0.86 (71.0)</td>
<td>67.66 (20.2)</td>
</tr>
<tr>
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<td>13.92 (30.7)</td>
<td>1.75 (32.0)</td>
<td>1.01 (62.0)</td>
<td>71.67 (15.3)</td>
</tr>
<tr>
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<td>49.80 (16.0)</td>
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<tr>
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<td>54.44 (18.3)</td>
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<td>1.00 (53.6)</td>
<td>62.36 (11.6)</td>
</tr>
<tr>
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<td>16.53 (34.2)</td>
<td>1.65 (32.5)</td>
<td>0.93 (71.3)</td>
<td>66.07 (20.7)</td>
</tr>
<tr>
<td>S-17</td>
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<td>12.24 (32.6)</td>
<td>1.30 (30.0)</td>
<td>0.75 (73.8)</td>
<td>35.15 (15.6)</td>
</tr>
<tr>
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<td>16.21 (32.5)</td>
<td>1.44 (27.2)</td>
<td>0.99 (55.0)</td>
<td>40.79 (17.6)</td>
</tr>
<tr>
<td>S-19</td>
<td>51</td>
<td>18.45 (31.2)</td>
<td>1.53 (26.7)</td>
<td>0.98 (65.9)</td>
<td>49.11 (16.6)</td>
</tr>
</tbody>
</table>

Variation in percentage is shown in parenthesis.

Table 3 Averaged impacting and rebounding angles measured

<table>
<thead>
<tr>
<th>Group</th>
<th>Runs</th>
<th>x–y plane Rebounding angles (°)</th>
<th>Impacting angles (°)</th>
<th>x–z plane Rebounding angles (°)</th>
<th>Impacting angles (°)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>29.03 (36.5) 15.44 (30.0) 12.02 (36.5) 4.23 (69.2)</td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>23.13 (48.2) 14.05 (32.1) 12.78 (35.8) 4.23 (75.8)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>S-03</td>
<td>43</td>
<td>23.04 (38.7) 14.80 (26.0) 12.73 (37.8) 4.23 (57.9)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>S-04</td>
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<td>21.74 (49.5) 13.81 (31.9) 12.72 (38.9) 4.23 (83.7)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>S-05</td>
<td>44</td>
<td>28.02 (38.7) 15.17 (23.2) 13.72 (39.8) 4.23 (76.8)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-06</td>
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<td>24.62 (49.7) 14.41 (31.1) 13.71 (37.6) 4.23 (121.5)</td>
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<td></td>
</tr>
<tr>
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<td>32.12 (37.1) 15.13 (37.0) 14.72 (39.8) 4.23 (60.2)</td>
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<td></td>
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<tr>
<td>S-08</td>
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<tr>
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<td>25.32 (45.1) 13.72 (29.8) 14.70 (37.6) 4.23 (83.7)</td>
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</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
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<tr>
<td>S-15</td>
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<td>25.69 (39.9) 11.58 (28.3) 14.65 (37.6) 4.23 (79.8)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>S-16</td>
<td>48</td>
<td>19.53 (50.3) 10.83 (26.1) 14.64 (37.6) 4.23 (63.7)</td>
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<td></td>
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</tr>
<tr>
<td>S-17</td>
<td>71</td>
<td>19.19 (51.3) 10.12 (42.5) 14.63 (37.6) 4.23 (76.5)</td>
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<tr>
<td>S-18</td>
<td>102</td>
<td>19.40 (43.2) 8.31 (68.0) 14.62 (37.6) 4.23 (71.5)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-19</td>
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<td>18.03 (43.9) 7.47 (34.8) 14.61 (37.6) 4.23 (80.3)</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Variation in percentage is shown in parenthesis.

provided in Fig. 29. The averaged longitudinal impacting angle is below 3°. This indicates that the trajectories are close to a sliding motion. The average impacting and rebounding angles increase as the flow intensity $T_s$ increases. This is because a saltating particle contains higher energy when $T_s$ and $D_s$ are large, generating a large momentum transfer as it hits the channel bed thus increases the longitudinal impacting and rebounding angles. The lateral impacting and rebounding angles are not sensitive to the variations of $T_s$.

The simulated frequency histograms of the dimensionless saltation length, height, width, and velocity $SL/SL_{mean}$, $SH/SH_{mean}$, $SW/SW_{mean}$ and $SV/SV_{mean}$ are shown in Fig. 30, where $SL_{mean}$, $SH_{mean}$, $SW_{mean}$ and $SV_{mean}$ represent mean saltating length, height, width and velocity, respectively. The dimensionless saltating length, height and width are found to follow Pearson type III distributions, similar to the experimental observations. The simulated saltation velocity follows a uniform distribution. Based on averaged values of the 110 simulation runs,
Three-dimensional continuous saltation model

(a) longitudinal trajectory (x-y plane)

(b) lateral trajectory (x-z plane)

(3D) 26 May 2000

(c) trajectory plotted in a 3-D diagram

Figure 11 Typical experimental trajectories.

Five regression equations were obtained:

\[
\frac{SL}{D} = 2.2583D_{*}^{0.1543}T_{*}^{0.9132} \quad (R^2 = 0.8217) \quad (28)
\]

\[
\frac{SH}{D} = 0.3010D_{*}^{0.4522}T_{*}^{0.3345} \quad (R^2 = 0.7718) \quad (29)
\]

\[
\frac{SW}{D} = 0.3521D_{*}^{0.0683}T_{*}^{0.8606} \quad (R^2 = 0.8472) \quad (30)
\]

\[
\frac{SV}{U_{*}} = 3.099D_{*}^{0.1537}T_{*}^{0.1381} \quad (R^2 = 0.9662) \quad (31)
\]

\[
\frac{SV}{U_{*}} = 0.4522D_{*}^{0.0906}T_{*}^{0.080} \quad (R^2 = 0.7741) \quad (32)
\]

where \( R^2 \) is the correlation coefficient.

Figure 12 Relations between the dimensionless saltation length \( SL/D \) and \( T^* \).

Figure 13 Relations between the dimensionless saltation height \( SH/D \) and \( T^* \).

Figure 14 Relations between the dimensionless saltation width \( SW/D \) and \( T^* \).

Figure 15 Relations between the dimensionless saltation velocity \( SV/U^* \) and \( T^* \).
Figure 16  Variations of the longitudinal rebounding angles.

Figure 17  Variations of the longitudinal impacting angles.

Figure 18  Variations of the lateral rebounding angles.

Figure 19  Variations of the lateral impacting angles.

Figure 20  Distributions of the dimensionless saltation length measured.

Figure 21  Distributions of the dimensionless saltation height measured.

7 Conclusions

A flow visualization technique was developed to measure the three-dimensional particle saltation motions near the channel bed. Based on the geometric configuration of the sediment particles forming the channel bed and a stochastic description of the impact positions, a model able to simulate the three-dimensional
Figure 22 Distributions of the dimensionless saltation width measured.

Figure 23 Distributions of the dimensionless saltation velocity measured.

Figure 24 Distributions of the longitudinal rebounding angles.

Figure 25 Distributions of the lateral rebounding angles.

Figure 26(a) Relationship between $e$ and $T^*$.  

Figure 26(b) Relationship between $f$ and $T^*$.  

Figure 26(c) Distributions of $k_b$.  

Three-dimensional continuous saltation model 385
Figure 27  Typical simulated three-dimensional trajectory.

Figure 28(a) Relations between the simulated dimensionless saltation length SL/D and $T_\ast$.

Figure 28(b) Relations between the simulated dimensionless saltation height SH/D and $T_\ast$.

Figure 28(c) Relations between the simulated dimensionless saltation width SW/D and $T_\ast$.

Figure 28(d) Relations between the simulated dimensionless longitudinal saltation velocity $SV_x/U_\ast$ and $T_\ast$.

Figure 28(e) Relations between the simulated dimensionless lateral saltation velocity $SV_z/U_\ast$ and $T_\ast$. 
impacting and rebounding mechanism is derived, and then incorporated into a three-dimensional continuous saltation model. The model was calibrated and verified with the experimental data, and checked to yield satisfactory results.

According to the experimental observations, the averaged longitudinal rebounding angle is between $18^\circ$ and $32^\circ$ and the corresponding averaged impacting angle is between $7^\circ$ and $15^\circ$. These values decrease as the flow intensity increases. The averaged dimensionless saltation length, height and width were observed to follow Person type III distributions and the averaged dimensionless saltation velocity follows a normal distribution. Regression equation for the lift coefficient $C_L$ is obtained. The lift coefficient increases as the particle Reynolds number increases.

The averaged lateral rebounding angle is between $5^\circ$ and $12^\circ$, and the corresponding averaged impacting angle is between $3^\circ$ and $5^\circ$. The rebounding angles under various impacting conditions follow a normal distribution.

The simulated dimensionless saltating length, height, width and velocity, increase as $D_*$ and $T_*$ increase. The simulated dimensionless saltating length, height and width follow Pearson type III distributions, while the simulated dimensionless saltation velocity follows an uniform distribution.

**Acknowledgments**

This study was supported by the National Science Council of Taiwan, the Republic of China. The writers would like to thank the staff of the Hydraulic Research Laboratory of National Taiwan University for their support in conducting the experiments.

**Notation**

\[ A = \text{Cross-sectional area of the particle} \]
\[ \text{perpendicular to} \]
\[ \text{the flow direction} \]
\[ C_b = \text{Bed load concentration} \]
\[ C_D = \text{Drag coefficient} \]
\[ C_L = \text{Lift coefficient} \]
\[ C_m = \text{Virtual mass coefficient} \]
Figure 30 Distributions of the simulated dimensionless saltation length, height, width and velocity.

\begin{align*}
C_x & = \text{Skew coefficient of random variable } x \\
D & = \text{Saltating particle size} \\
D_b & = \text{Size of the particles that form the channel bed} \\
D_* & = \text{Particle dimensionless parameter} \\
d_{50} & = \text{Mean particle size} \\
e & = \text{Restitution coefficient} \\
F_D & = \text{Drag force} \\
F_G & = \text{Submerged body force} \\
F_L & = \text{Lift force} \\
f & = \text{Friction coefficient} \\
g & = \text{Gravitational acceleration} \\
H & = \text{Water depth} \\
k_b & = \text{Energy loss coefficient} \\
l & = \text{A random variable which is the distance between the impacting point and the center of the bed particle projected on the particle centerline in C–C section, as shown in Fig. 4.} \\
m & = \text{Particle total mass} \\
q_b & = \text{Bed load transport rate} \\
R & = \text{Radius of the particles that form the channel bed} \\
R' & = \text{Correlation coefficient} \\
Re & = \text{Fluid Reynolds number} \\
Re_* & = \text{Particle Reynolds number} \\
r & = \text{A random variable which is the distance between the impacting point and the center of the bed particle parallel to the flow direction, as shown in Fig. 5.} \\
S & = \text{Shape factor} \\
S_0 & = \text{Channel slope} \\
S_G & = \text{Specific gravity} \\
SL, SH, SW, SV & = \text{Saltation length, height, width and velocity, respectively} \\
SH_{\text{mean}} & = \text{Mean saltation height} \\
SL_{\text{mean}} & = \text{Mean saltation length} \\
SV_{\text{mean}} & = \text{Mean saltation velocity} \\
SV_x & = \text{Longitudinal saltation velocity} \\
SV_z & = \text{Lateral saltation velocity} \\
SW_{\text{mean}} & = \text{Mean saltation width} \\
T_* & = \text{Parameter of flow transport capacity} \\
u & = \text{Flow velocity} \\
u_T & = \text{Relative velocity} \\
u_{TB} & = \text{Relative velocity at the bottom of the particle} \\
u_{TT} & = \text{Relative velocity at the top of the particle} \\
u_{p,\text{in}} & = \text{Horizontal particle velocity component before collision} \\
u_{p,\text{out}} & = \text{Horizontal particle velocity component after collision} \\
u_s & = \text{Shear velocity} \\
u_{4c} & = \text{Critical shear velocity} \\
v_{p,\text{in}} & = \text{Vertical particle velocity component before collision} \\
v_{p,\text{out}} & = \text{Vertical particle velocity component after collision} \\
x, y, z & = \text{Longitudinal, vertical, and transverse distance}
\end{align*}
\( \dot{x}, \dot{y}, \dot{z} = \) Longitudinal, vertical, and transverse components of the particle velocity

\( \ddot{x}, \ddot{y}, \ddot{z} = \) Longitudinal, vertical, and transverse components of the particle acceleration

\( \bar{x} = \) Mean value of random variable \( x \)

\( \alpha_1 = \) Coefficient of the initial longitudinal component of the lift-off velocity

\( \alpha_2 = \) Coefficient of the initial vertical component of the lift-off velocity

\( \alpha_3 = \) Coefficient of the initial transverse component of the lift-off velocity

\( \beta^2_D = \) Projected area perpendicular to the flow direction

\( \varepsilon = \) Particle surface roughness

\( \theta = \) Channel bed angle

\( \theta_h = \) As shown in Fig. 3

\( \theta_{in} = \) Impacting angle (incident angle)

\( \theta_{out} = \) Takeoff angle

\( \theta_r = \) Rebounding angle

\( v = \) Kinematic viscosity

\( \rho_s = \) Specific gravity of the saltating particle

\( \rho_w = \) Specific gravity of water

\( \sigma_x = \) Standard deviation of random variable \( x \)

\( \tau_* = \) Dimensionless shear stress

\( \Phi = \) Dimensionless sediment transport rate

References


