從信號與系統到控制

單元：數學工具-5

多重的連續時間 三角 與 指數 函數 之積分

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單元學習目標與大綱

- 討論連續時間傅立葉級數計算過程中，
- 多重三角函數與指數函數之積分計算過程

連續時間三角函數

$$e^{js} = \cos(s) + j\sin(s)$$

$$a_k = \frac{1}{T}\int_T \left(1+\sin(w_0 t)+2\cos(w_0 t)+\cos(2w_0 t+\frac{\pi}{4}) \right) \boxed{e^{-jkw_0 t}}\, dt$$

$$k = 0$$

$$a_0 = \frac{1}{T}\int_T \left(1+\sin(w_0 t)+2\cos(w_0 t)+\cos(2w_0 t+\frac{\pi}{4}) \right) 1\, dt$$

連續時間三角函數

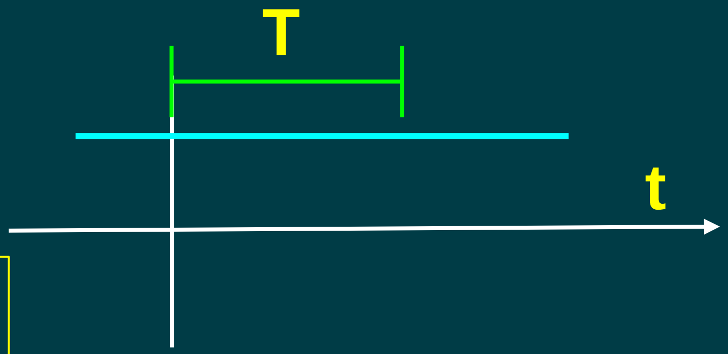
$$a_0 = \frac{1}{T}\int_T \left(\boxed{1} + \boxed{\sin(w_0 t)} + \boxed{2\cos(w_0 t)} + \boxed{\cos(2w_0 t + \frac{\pi}{4})} \right) 1\, dt$$

$$= \frac{1}{T}\int_T 1\, dt + \frac{1}{T}\int_T \sin(w_0 t)\, dt$$

$$+ \frac{1}{T}\int_T 2\cos(w_0 t)\, dt + \frac{1}{T}\int_T \cos(2w_0 t + \frac{\pi}{4})\, dt$$

三角函數一個週期的積分

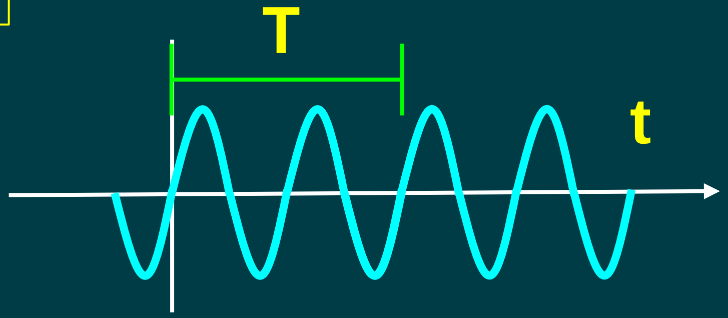
$$\int_T 1 \, dt = T$$



$$\boxed{w_0 = \frac{2\pi}{T}}$$

$$\int_T \sin(k\, w_0\, t)\, dt = 0$$

$$\int_T \cos(k\, w_0 t)\, dt = 0$$



連續時間三角函數

$$a_0 = \frac{1}{T}\int_T 1\,dt + \frac{1}{T}\int_T \underbrace{\sin(w_0 t)\,dt}_{0}$$

$$+ \frac{1}{T}\int_T \underbrace{2\cos(w_0 t)\,dt}_{0} + \frac{1}{T}\int_T \underbrace{\cos(2w_0 t + \frac{\pi}{4})\,dt}_{0}$$

連續時間三角函數

$$a_0 = \frac{1}{T}\int_T 1\,dt$$

$$= \frac{1}{T} \quad T \quad = 1$$

連續時間三角函數

$$e^{js} = \cos(s) + j\sin(s)$$

$k = 1$

$$a_1 = \frac{1}{T}\int_T \left(\boxed{1}+\boxed{\sin(w_0t)}+\boxed{2\cos(w_0t)}+\boxed{\cos(2w_0t+\frac{\pi}{4})}\right)\boxed{e^{-j1w_0t}}dt$$

$$= \frac{1}{T}\int_T e^{-j1w_0t}\,dt + \frac{1}{T}\int_T \sin(w_0t)\,e^{-j1w_0t}\,dt$$

$$+ \frac{1}{T}\int_T 2\cos(w_0t)e^{-j1w_0t}dt + \frac{1}{T}\int_T \cos(2w_0t+\frac{\pi}{4})e^{-j1w_0t}dt$$

三角函數與指數函數一個週期的積分

$$\int_T e^{jk\omega_0 t}\,dt = \boxed{0}$$

$$\int_T \cos(m\omega_0 t)\, e^{jn\omega_0 t}\,dt = \frac{1}{2}T \qquad m = n$$

$$= \boxed{0 \qquad m \neq n}$$

$$\int_T \sin(m\omega_0 t)\, e^{jn\omega_0 t}\,dt = j\frac{1}{2}T \qquad m = n$$

$$= \boxed{0 \qquad m \neq n}$$

連續時間三角函數

$$a_1 = \boxed{\frac{1}{T}\int_T e^{-j1w_0t}\,dt} + \frac{1}{T}\int_T \sin(\boxed{w_0}t)\,e^{-j\boxed{1w_0}t}\,dt$$

$$+ \frac{1}{T}\int_T 2\cos(\boxed{w_0}t)e^{-j\boxed{1w_0}t}dt + \boxed{\frac{1}{T}\int_T \cos(2w_0t+\frac{\pi}{4})e^{-j1w_0t}dt}$$

連續時間三角函數

$$a_1 = \cdots + \frac{1}{T}\boxed{\int_T \sin(w_0 t)\, e^{-j1w_0 t}\, dt}$$

$$+ \frac{1}{T}\boxed{\int_T 2\cos(w_0 t) e^{-j1w_0 t} dt}$$

$$\int_T \cos(w_0 t)\, e^{j w_0 t}\, dt = \boxed{\frac{1}{2}T}$$

$$\int_T \sin(w_0 t)\, e^{j w_0 t}\, dt = \boxed{j\frac{1}{2}T}$$

$$= \frac{1}{T}\left(2\,\frac{1}{2}T - j\,\frac{1}{2}T\right)$$

$$= 1 - j\frac{1}{2}$$

連續時間三角函數

$$e^{js} = \cos(s) + j\sin(s)$$

$k = 2$

$$a_2 = \frac{1}{T}\int_T \left(\boxed{1} + \boxed{\sin(w_0 t)} + \boxed{2\cos(w_0 t)} + \boxed{\cos(2w_0 t + \frac{\pi}{4})} \right) \boxed{e^{-j2w_0 t}} dt$$

$$= \frac{1}{T}\int_T e^{-j2w_0 t}\, dt \quad + \frac{1}{T}\int_T \sin(w_0 t)\, e^{-j2w_0 t}\, dt$$

$$+ \frac{1}{T}\int_T 2\cos(w_0 t) e^{-j2w_0 t} dt + \frac{1}{T}\int_T \cos(2w_0 t + \frac{\pi}{4}) e^{-j2w_0 t} dt$$

三角函數與指數函數一個週期的積分

$$\int_T e^{jk\omega_0 t}\,dt = \boxed{0}$$

$$\int_T \cos(m\omega_0 t)\, e^{jn\omega_0 t}\,dt = \frac{1}{2}T \qquad m = n$$

$$= \boxed{0 \qquad m \neq n}$$

$$\int_T \sin(m\omega_0 t)\, e^{jn\omega_0 t}\,dt = j\frac{1}{2}T \qquad m = n$$

$$= \boxed{0 \qquad m \neq n}$$

連續時間三角函數

$$a_2 = \frac{1}{T}\boxed{\int_T e^{-j2w_0t}\,dt} + \frac{1}{T}\boxed{\int_T \sin(w_0t)\,e^{-j2w_0t}\,dt}$$

$$+\frac{1}{T}\boxed{\int_T 2\cos(w_0t)e^{-j2w_0t}dt} + \frac{1}{T}\int_T \cos(\boxed{2w_0}t+\frac{\pi}{4})e^{-j\boxed{2w_0}t}dt$$

連續時間三角函數

$a_2 =$

$$+\frac{1}{T}\int_T \cos(2w_0t+\frac{\pi}{4})e^{-j2w_0t}dt$$

連續時間三角函數

$\cos(s) = (e^{js} + e^{-js}) / 2$

$$a_2 = \frac{1}{T}\int_T \boxed{\cos(2w_0t+\frac{\pi}{4})} e^{-j2w_0t}dt$$

$$\boxed{(e^{j(\boxed{2w_0t}+\frac{\pi}{4})}} + \boxed{e^{\boxed{-j(2w_0t}+\frac{\pi}{4})})} / 2 \quad \boxed{(e^{\boxed{-j2w_0t}})}$$

$$= (e^{j\frac{\pi}{4}} + e^{-j(4w_0t+\frac{\pi}{4})}) / 2$$

$$= \frac{1}{T}\int_T \frac{1}{2} e^{j\frac{\pi}{4}} dt + \frac{1}{T}\int_T \frac{1}{2} e^{-j(4w_0t+\frac{\pi}{4})}dt$$

連續時間三角函數

$$a_2 = \frac{1}{T}\boxed{\int_T \frac{1}{2} e^{j\frac{\pi}{4}} dt} + \frac{1}{T}\int_T \frac{1}{2} e^{-j(4w_0 t+\frac{\pi}{4})} dt$$

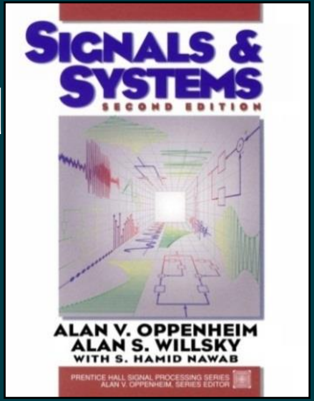
0

$$= \frac{1}{T}\boxed{\frac{1}{2} e^{j\frac{\pi}{4}}}\, T$$

$$= \frac{1}{2}\left[\cos\left(\frac{\pi}{4}\right) + j\sin\left(\frac{\pi}{4}\right)\right]$$

$$= \frac{1}{2}\left[\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right] = \frac{\sqrt{2}}{4} + j\frac{\sqrt{2}}{4}$$

參考文獻



- Alan V. Oppenheim, Alan S. Willsky, S. Hamid
  Signals & Systems,
  Prentice Hall, 2nd Edition, 1997
- SciLab:
  Open source software for numerical computation
  http://www.scilab.org/