從信號與系統到控制

單元：數學工具-2

多個複數 的總和

授課老師：連　豐　力

單元學習目標與大綱

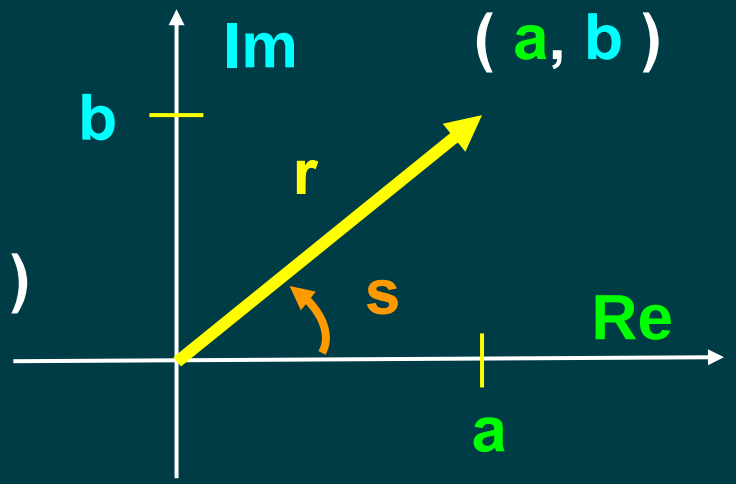
- 討論多個**複數的總和**

複數的表示式

- 對於一個複數，兩種表示式的關係為：

$$r\, e^{j s} = a + j b$$

$$e^{j s} = \cos(s) + j \sin(s)$$



多個複數的總和

- 假設， s = (0) 2π / 12 = 0， r = 1

$$e^{j(0)\frac{2\pi}{12}} = a + jb \qquad e^{js} = \cos(s) + j\sin(s)$$

$$= \cos\left((0)\frac{2\pi}{12}\right) + j\sin\left((0)\frac{2\pi}{12}\right)$$

$$= 1 + j\,0$$

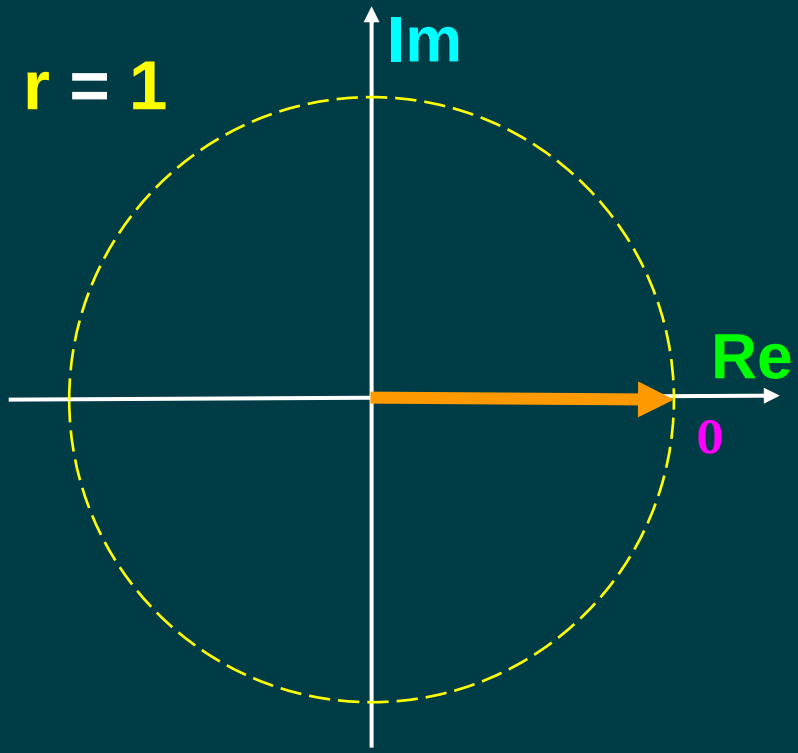
多個複數的總和

- 假設， $s = (0)\ 2\pi / 12 = 0$， $r = 1$

$$e^{j(0)\frac{2\pi}{12}} = a + j\,b$$

$$= \cos\left((0)\frac{2\pi}{12}\right) + j\sin\left((0)\frac{2\pi}{12}\right)$$

$$= 1 + j\ 0$$



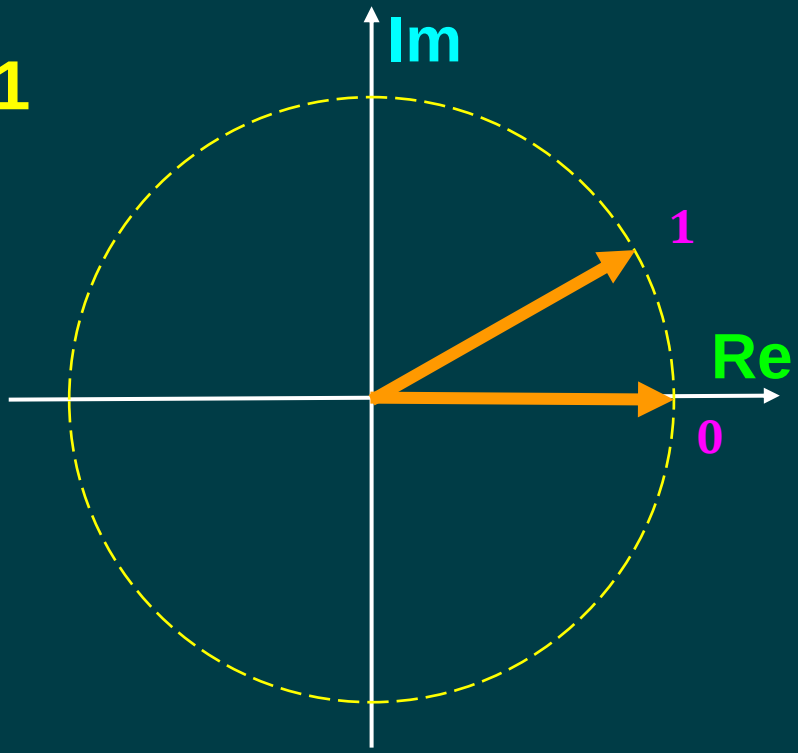
多個複數的總和

- 假設， s = (1) 2π / 12 ，r = 1

$$e^{j(1)\frac{2\pi}{12}} = a + j\,b$$

$$= \cos\left((1)\frac{2\pi}{12}\right) + j\sin\left((1)\frac{2\pi}{12}\right)$$

$$= \sqrt{3}/2 + j\ 1/2$$

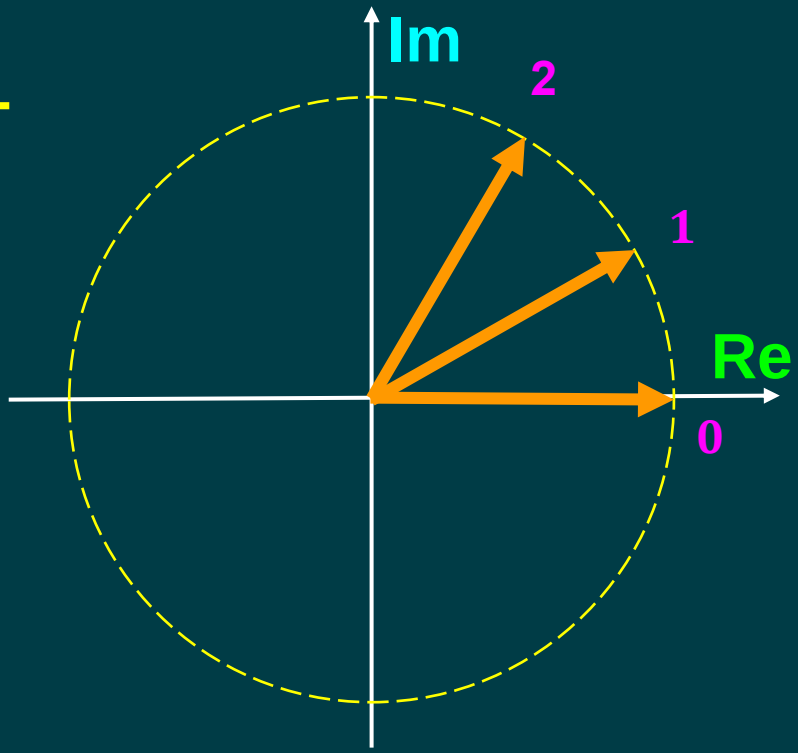


多個複數的總和

- 假設， s = (2) 2π / 12，r = 1

$$e^{j(2)\frac{2\pi}{12}} = a + j\,b$$

$$= \cos\left((2)\frac{2\pi}{12}\right) + j\sin\left((2)\frac{2\pi}{12}\right)$$

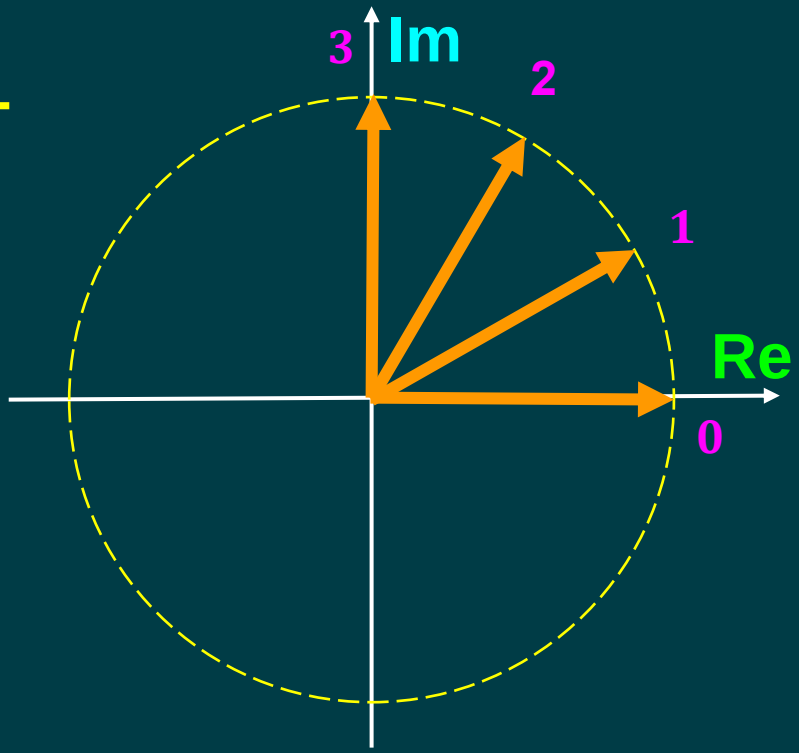


多個複數的總和

- 假設， s = (3) 2π / 12，r = 1

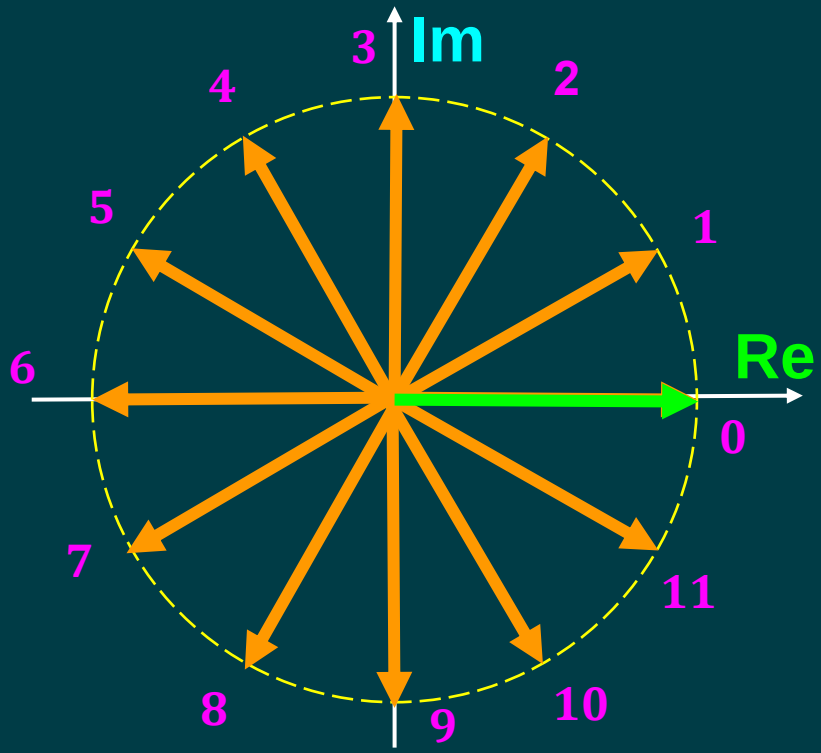
$$e^{j(3)\frac{2\pi}{12}} = a + jb$$

$$= \cos\left((3)\frac{2\pi}{12}\right) + j\sin\left((3)\frac{2\pi}{12}\right)$$



多個複數的總和

- 依此類推，s = (4) 2π / 12
- s = (5) 2π / 12
- …
- s = (11) 2π / 12
- s = (12) 2π / 12 = 2π



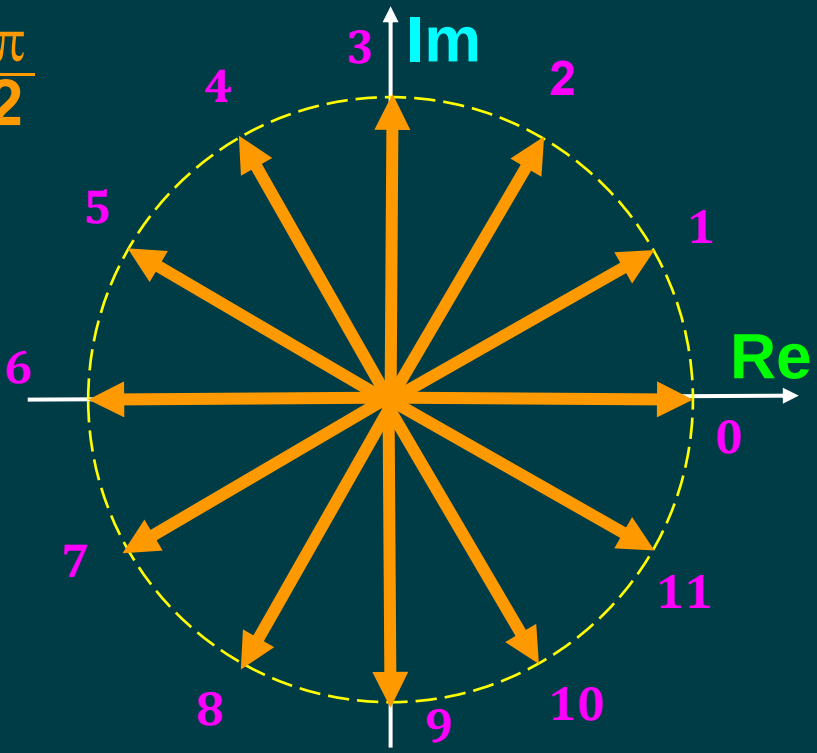
多個複數的總和

$$e^{j(0)\frac{2\pi}{12}} + e^{j(1)\frac{2\pi}{12}} + e^{j(2)\frac{2\pi}{12}}$$

$$+ \ldots + e^{j(11)\frac{2\pi}{12}}$$

$$= 0$$

$$\sum_{k=0}^{11} e^{j\,k\,\frac{2\pi}{12}} = 0$$



多個複數的總和

$$e^{j(0)\frac{2\pi}{12}} + e^{j(1)\frac{2\pi}{12}} + e^{j(2)\frac{2\pi}{12}}$$

$$+ \ldots + e^{j(11)\frac{2\pi}{12}}$$

$$= 0$$

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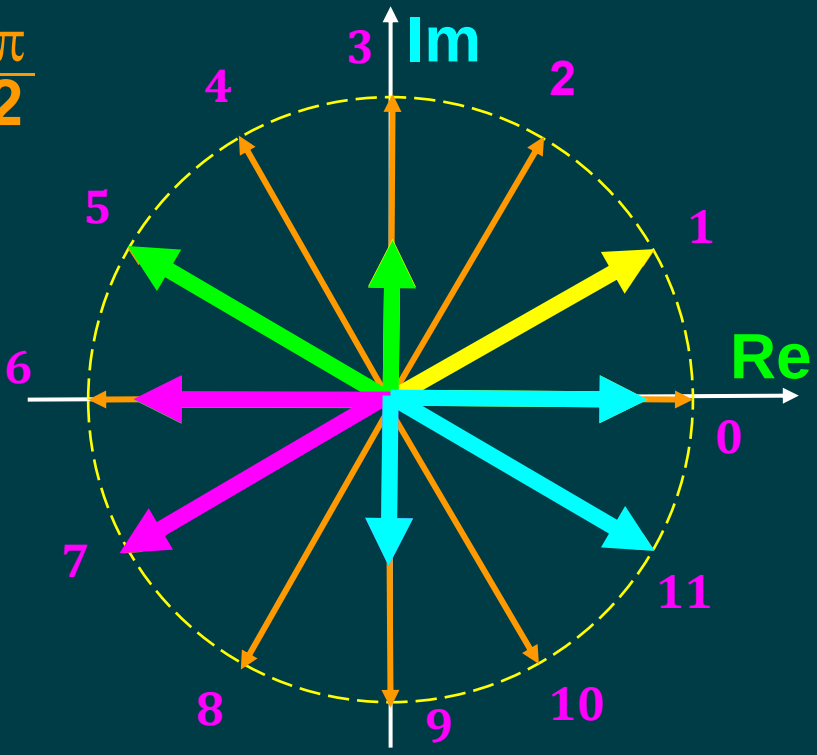
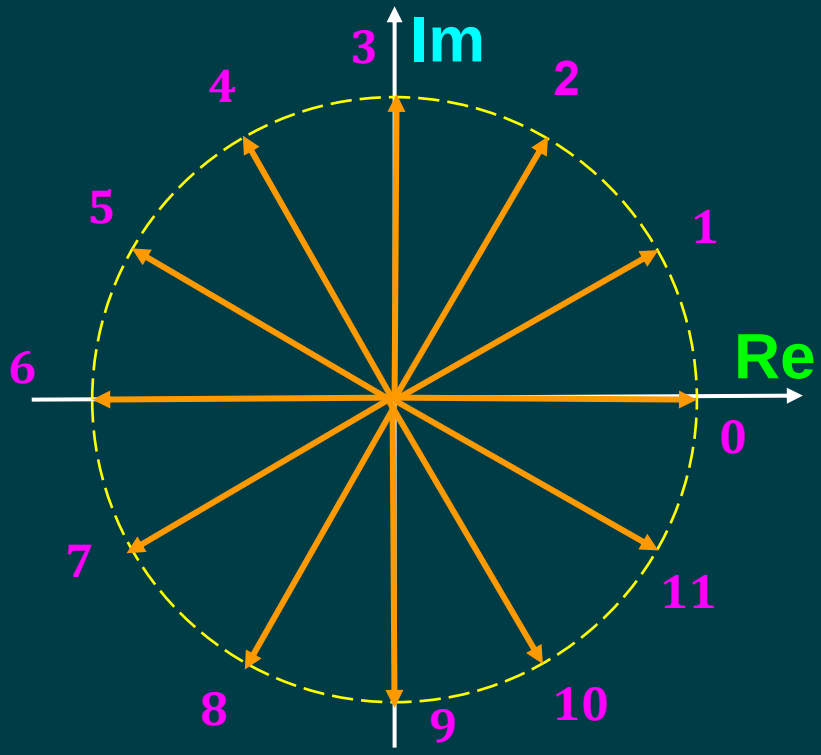


多個複數的總和

$$\sum_{k=0}^{N-1} e^{j(k)\frac{2\pi}{N}} = 0$$

$$\sum_{k=0}^{N-1} e^{j(k)\frac{2\pi}{N}\boxed{n}} = 0$$

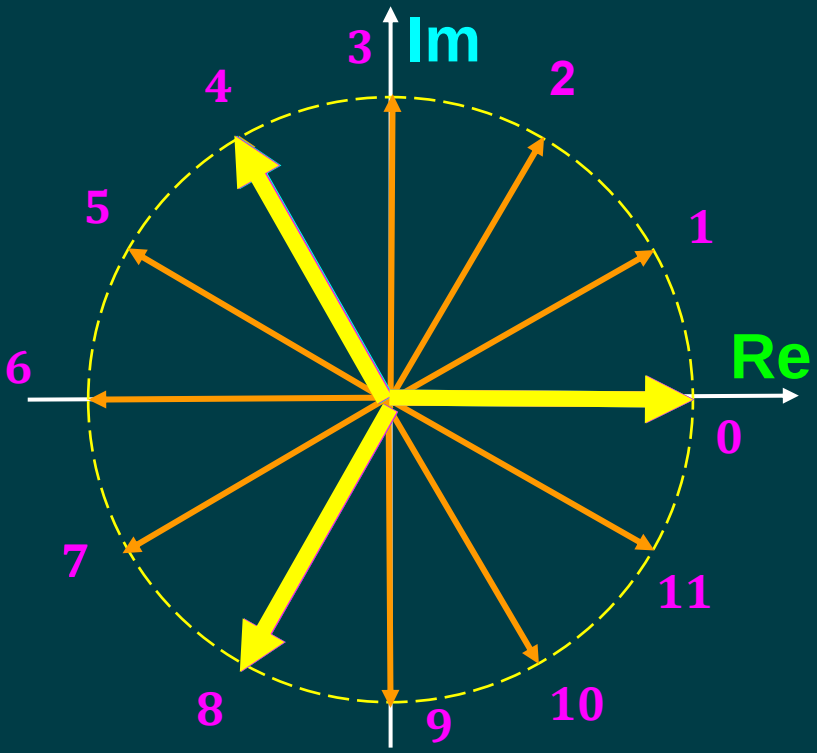


多個複數的總和

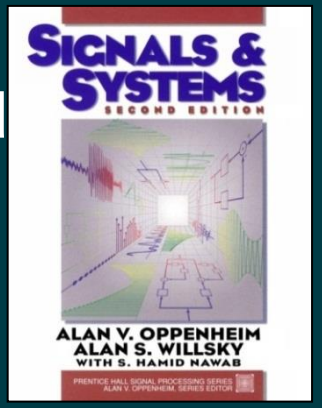
$$\sum_{k=0}^{N-1} e^{j(k)\frac{2\pi}{N}} = 0$$

n = 4

$$\sum_{k=0}^{N-1} e^{j(k)\frac{2\pi}{N}\boxed{4}} = 0$$



參考文獻



- Alan V. Oppenheim, Alan S. Willsky, S. Hamid
  Signals & Systems,
  Prentice Hall, 2nd Edition, 1997

- **SciLab**:
  Open source software for numerical computation
  http://www.scilab.org/