

從信號與系統到控制

單元：Z轉換-2

Z轉換範例 – 正時間指數函數

授課老師：連 豐 力

單元學習目標與大綱

- 根據 **Z轉換** 的公式與關係式
- 計算 **正時間指數函數** 的 **Z轉換**
- 介紹 **Z轉換** 後的 **收斂區間** 特性

傅立葉轉換 與 Z轉換



$$r e^{j\omega} = z$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega n} \quad X(z) = \sum_{n=-\infty}^{+\infty} x[n] z^{-n}$$

$$X(e^{j\omega}) = \mathcal{F} \{ x(t) \} \quad X(z) = \mathcal{Z} \{ x[n] \}$$

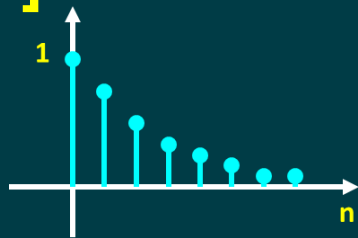
$$x[n] = \mathcal{F}^{-1} \{ X(e^{j\omega}) \} \quad x[n] = \mathcal{Z}^{-1} \{ X(z) \}$$

正時間指數函數的 Z 轉換

$$x[n] = a^n u[n]$$

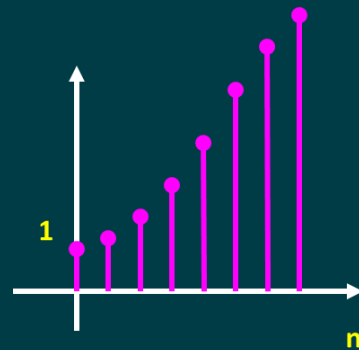
$$|a| < 1$$

$$0 < a < 1$$

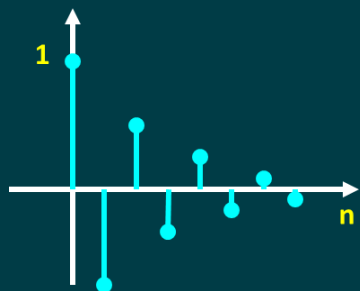


$$|a| > 1$$

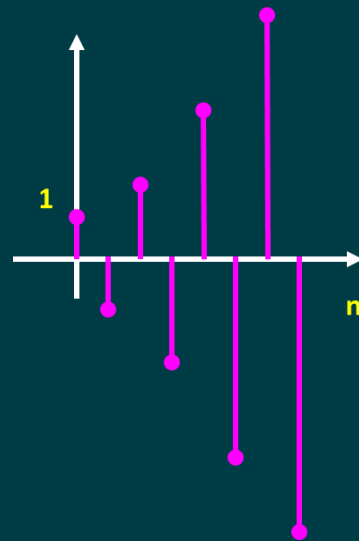
$$a > 1$$



$$-1 < a < 0$$



$$a < -1$$



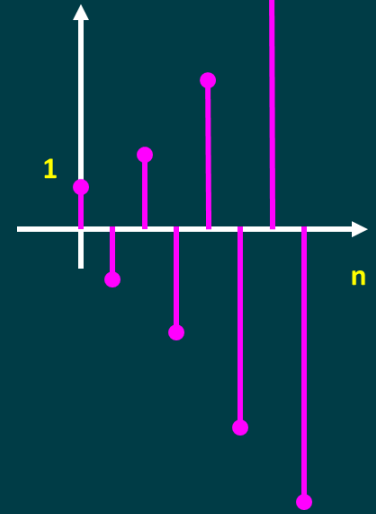
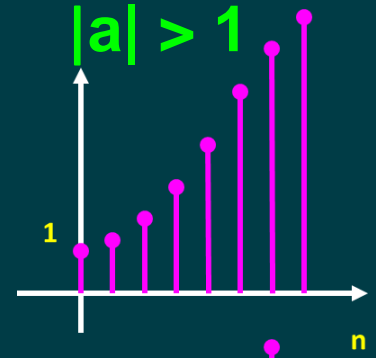
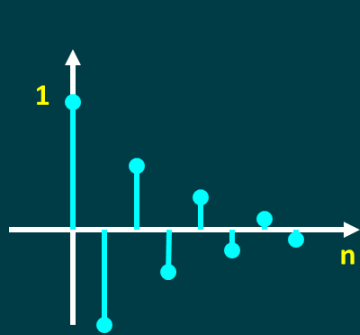
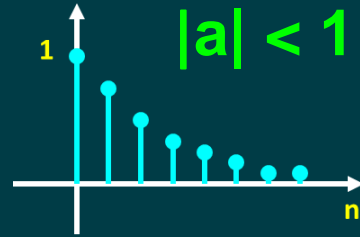
正時間指數函數的 Z 轉換

$$x[n] = a^n u[n]$$

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{+\infty} a^n u[n] z^{-n}$$

$$= \sum_{n=0}^{+\infty} a^n z^{-n}$$



正時間指數函數的 Z 轉換

$$x[n] = a^n u[n]$$

$$X(z) = \sum_{n=0}^{+\infty} a^n z^{-n}$$

$$= \sum_{n=0}^{+\infty} (az^{-1})^n \quad |az^{-1}| < 1$$

$$= \frac{(az^{-1})^0 [1 - (az^{-1})^\infty]}{1 - (az^{-1})}$$

正時間指數函數的 Z 轉換

$$x[n] = a^n u[n]$$

$$\begin{aligned} X(z) &= \sum_{n=0}^{\infty} (az^{-1})^n [1 - (az^{-1})^{\infty}] \\ &= \frac{1 - (az^{-1})^{\infty}}{1 - (az^{-1})} \end{aligned}$$

$|az^{-1}| < 1$

$$= \frac{1}{1 - (az^{-1})} = \frac{z}{z - a}$$

正時間指數函數的 Z 轉換

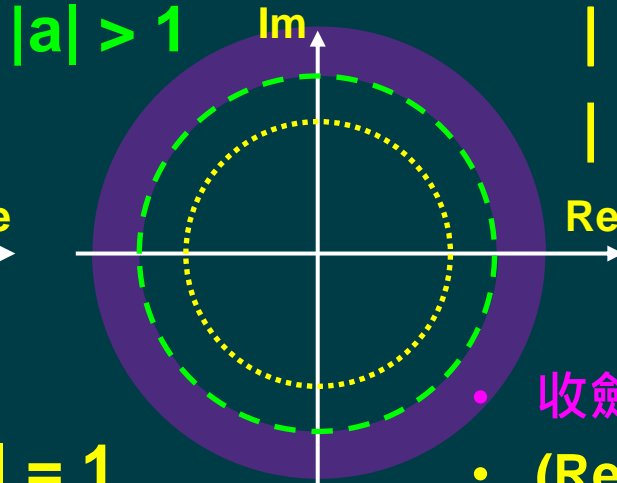
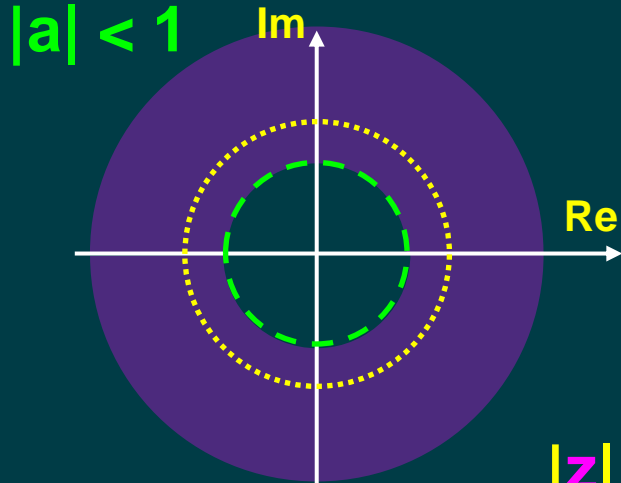
$$x[n] = a^n u[n] \xleftrightarrow{\text{ZT}} X(z) = \frac{z}{z - a}$$

$$|z| > |a|$$

$$|a z^{-1}| < 1$$

$$|a| |z|^{-1} < 1$$

$$|a| < |z|$$



- 收斂區間 (ROC)
- (Region of Convergence)

Z轉換

$$x[n] = a^n u[n]$$

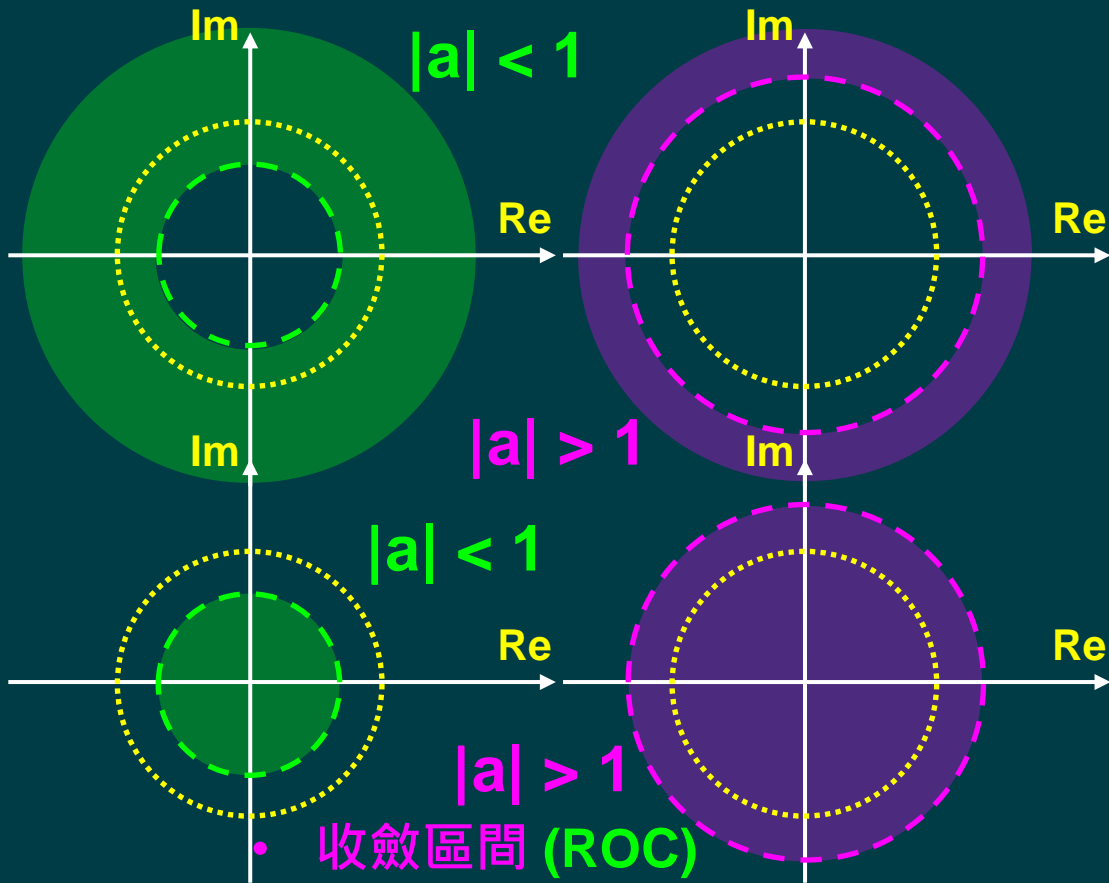
$$X(z) = \frac{z}{z - a}$$

→ $|z| > |a|$

$$x[n] = -a^n u[-n-1]$$

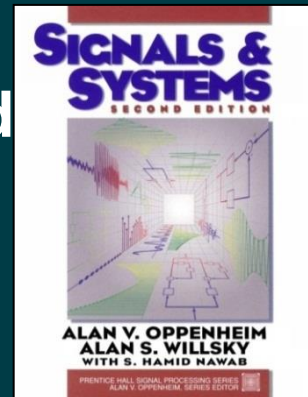
$$X(z) = \frac{z}{z - a}$$

→ $|z| < |a|$



參考文獻

- Alan V. Oppenheim, Alan S. Willsky, S. Hamid
Signals & Systems,
Prentice Hall, 2nd Edition, 1997



- **SciLab:**
Open source software for numerical computation
<http://www.scilab.org/>

指數函數的 Z 轉換

$$x[n] = a^n u[n] \quad |a| < 1$$

$$X(z) = \frac{z}{z - a} \quad |z| > |a|$$

$$x[n] = -a^n u[-n-1] \quad |a| < 1$$

$$X(z) = \frac{z}{z - a} \quad |z| < |a|$$

