

從信號與系統到控制

單元：離散F轉換-9

傅立葉轉換範例 - 週期脈衝函數

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單元學習目標與大綱

- 根據 傅立葉轉換 有關 週期信號 的關係式
- 計算 週期脈衝函數 的 傅立葉轉換

週期信號的 傅立葉轉換 表示式

- 一個 週期信號 的 傅立葉轉換 的關係式：

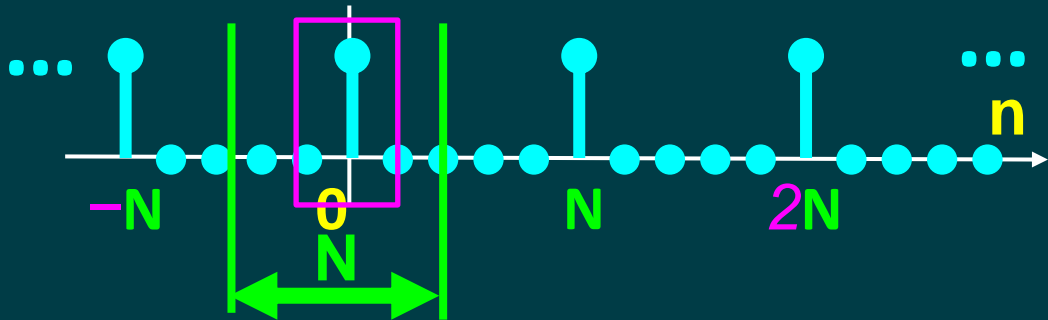
$$\begin{aligned}
 & x[n] \xleftrightarrow{\text{FT}} X(e^{j\omega}) \\
 & = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} = \sum_{k=\langle N \rangle} \sum_{r=-\infty}^{+\infty} 2\pi a_k \delta(\omega - k\omega_0 - r2\pi)
 \end{aligned}$$

The diagram illustrates the Fourier Transform of a periodic signal. The top part shows the relationship between the discrete-time signal $x[n]$ and its discrete-time Fourier Transform $X(e^{j\omega})$. The middle part shows the signal as a sum of complex exponentials and the transform as a sum of impulses. The bottom part is a plot of the frequency spectrum with impulses at discrete frequencies.

The frequency spectrum plot shows a horizontal axis labeled ω . The impulses are located at $0, \omega_0, 2\omega_0, 3\omega_0, \dots, (N-1)\omega_0, N\omega_0 = 2\pi, 2N\omega_0 = 4\pi, \dots$. The impulses at 0 and $N\omega_0 = 2\pi$ are highlighted in yellow. The impulses at 0 and $N\omega_0 = 2\pi$ are labeled 0 and $N\omega_0 = 2\pi$ respectively. The impulses at 0 and $N\omega_0 = 2\pi$ are labeled 0 and $N\omega_0 = 2\pi$ respectively. The impulses at 0 and $N\omega_0 = 2\pi$ are labeled 0 and $N\omega_0 = 2\pi$ respectively.

週期脈衝函數的傅立葉轉換

$$x[n] = \sum_{k=-\infty}^{+\infty} \delta[n - kN]$$



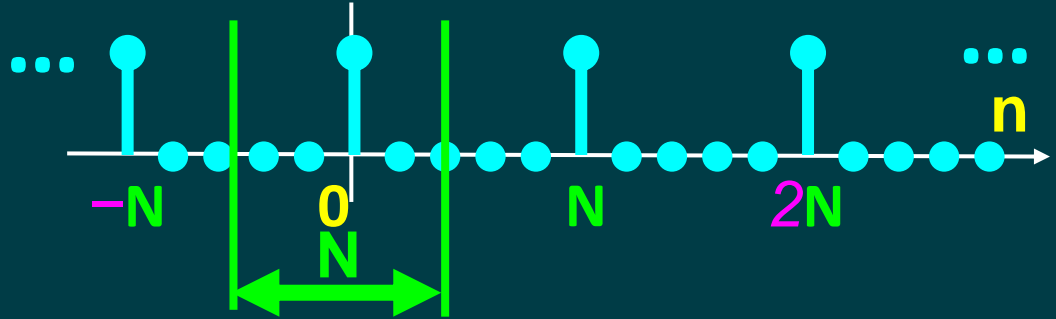
$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n}$$

$n = 0$

$$= \frac{1}{N} 1 \boxed{e^{-jk\omega_0 0}} = \frac{1}{N}$$

週期脈衝函數的傅立葉轉換

$$x[n] = \sum_{k=-\infty}^{+\infty} \delta[n - kN]$$



$$a_k = \frac{1}{N} \quad X(e^{j\omega}) = \sum_{k=\langle N \rangle} \sum_{r=-\infty}^{+\infty} 2\pi \boxed{a_k} \delta(\omega - k\omega_0 - r2\pi)$$

$$= \sum_{k=\langle N \rangle} \sum_{r=-\infty}^{+\infty} 2\pi \frac{1}{N} \delta(\omega - k\omega_0 - r2\pi)$$

週期脈衝函數的傅立葉轉換

$$X(e^{j\omega}) = \sum_{k=\langle N \rangle} \sum_{r=-\infty}^{+\infty} \left[2\pi \frac{1}{N} \right] \delta(\omega - k\omega_0 - r2\pi)$$

$$\omega_0 = \frac{2\pi}{N}$$

$$= \frac{2\pi}{N} \sum_{k=\langle N \rangle} \sum_{r=-\infty}^{+\infty} \delta\left(\omega - k\frac{2\pi}{N} - r2\pi\right)$$

$$2\pi = N \frac{2\pi}{N}$$

$$= \frac{2\pi}{N} \sum_{k=\langle N \rangle} \sum_{r=-\infty}^{+\infty} \delta\left(\omega - k\frac{2\pi}{N} - rN\frac{2\pi}{N}\right)$$

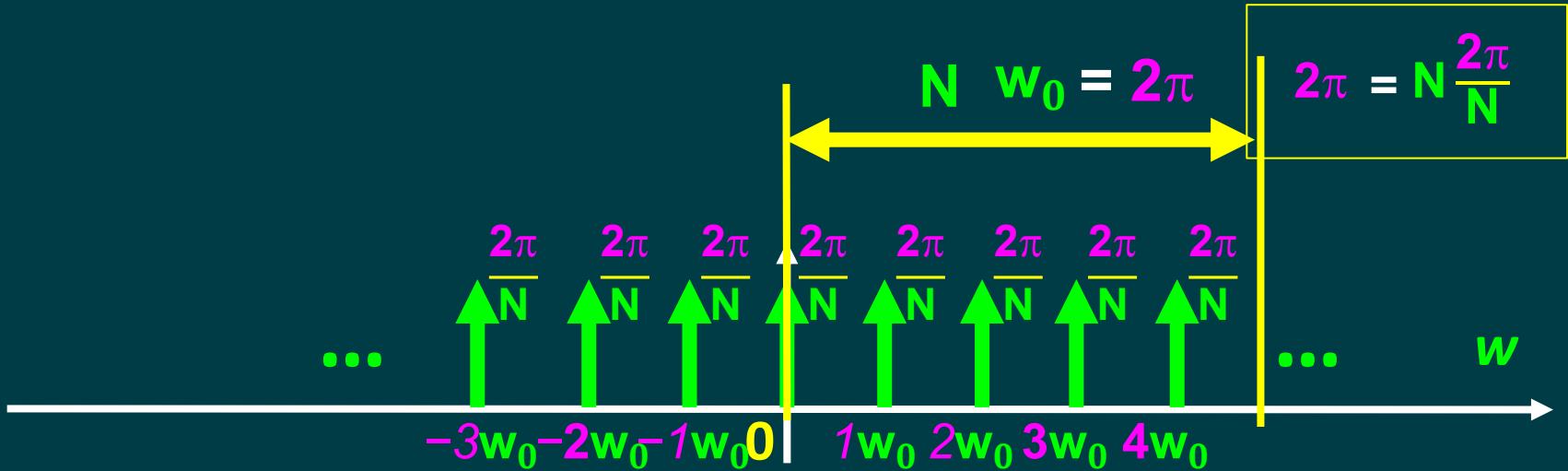
週期脈衝函數的傅立葉轉換

$$\begin{aligned} X(e^{j\omega}) &= \frac{2\pi}{N} \sum_{k=\langle N \rangle} \sum_{r=-\infty}^{+\infty} \delta\left(\omega - k\frac{2\pi}{N} - rN\frac{2\pi}{N}\right) \\ &= \frac{2\pi}{N} \sum_{k=\langle N \rangle} \sum_{r=-\infty}^{+\infty} \delta\left(\omega - (k + rN)\frac{2\pi}{N}\right) \\ &\quad m = k + rN \\ &= \frac{2\pi}{N} \sum_{m=-\infty}^{+\infty} \delta\left(\omega - (m)\frac{2\pi}{N}\right) \end{aligned}$$

週期脈衝函數的傅立葉轉換

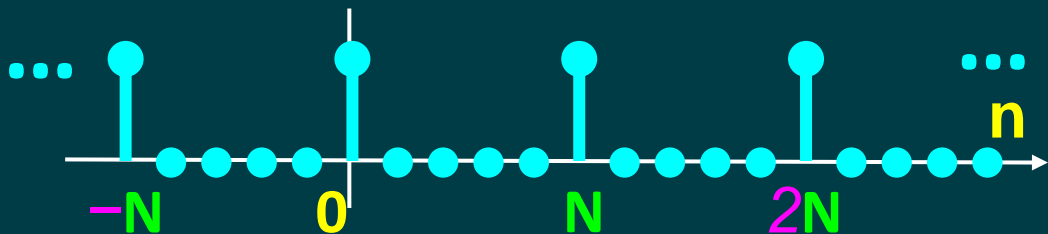
$$X(e^{j\omega}) = \frac{2\pi}{N} \sum_{m=-\infty}^{+\infty} \delta(\omega - (m) \frac{2\pi}{N})$$

$$\omega_0 = \frac{2\pi}{N}$$



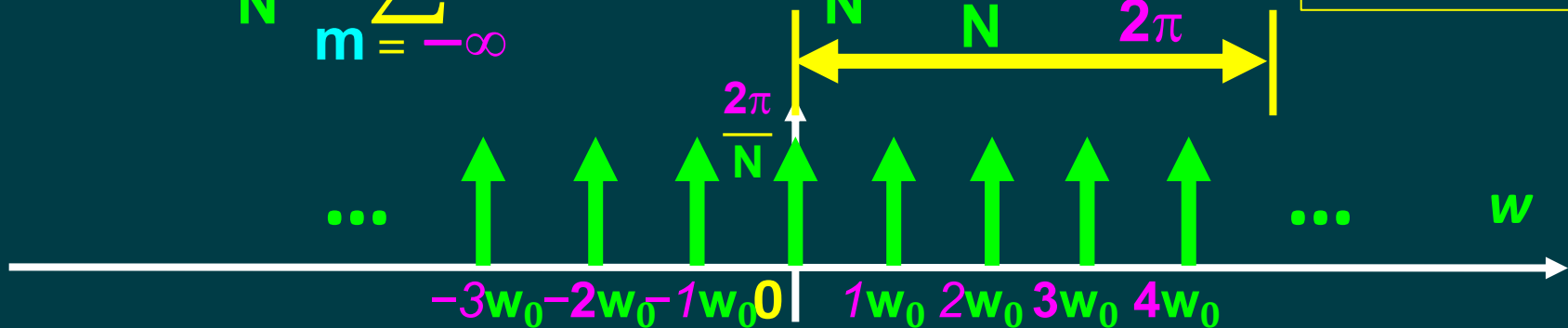
週期脈衝函數的傅立葉轉換

$$x[n] = \sum_{k=-\infty}^{+\infty} \delta[n - kN]$$



$$X(e^{j\omega}) = \frac{2\pi}{N} \sum_{m=-\infty}^{+\infty} \delta\left(\omega - (m) \frac{2\pi}{N}\right)$$

$$\omega_0 = \frac{2\pi}{N}$$



週期脈衝函數的傅立葉轉換

$$\sum_{k=-\infty}^{+\infty} \delta[n - kN]$$

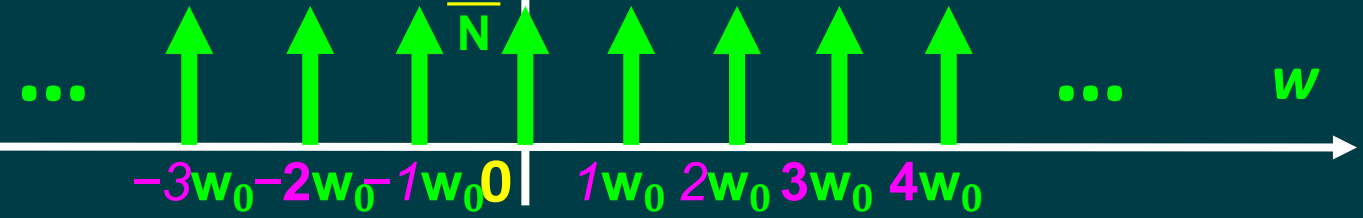
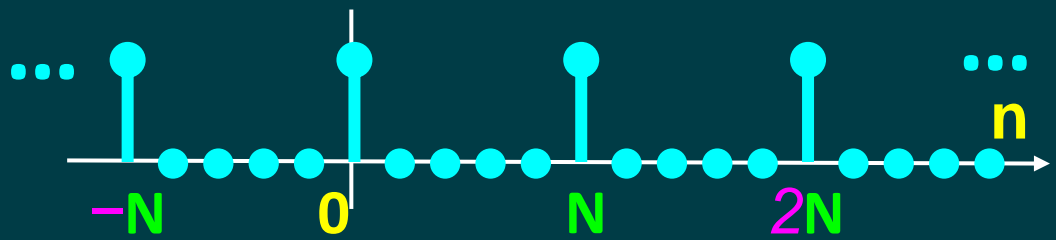
$$\delta[n - kN]$$

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$$\frac{2\pi}{N} \sum_{m=-\infty}^{+\infty} \delta\left(\omega - m \frac{2\pi}{N}\right)$$

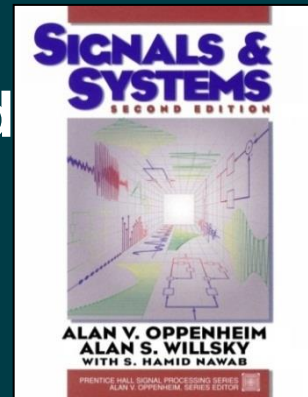
$$\delta\left(\omega - m \frac{2\pi}{N}\right)$$

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參考文獻

- Alan V. Oppenheim, Alan S. Willsky, S. Hamid
Signals & Systems,
Prentice Hall, 2nd Edition, 1997



- **SciLab:**
Open source software for numerical computation
<http://www.scilab.org/>