

從信號與系統到控制

單元：離散F轉換-8

傅立葉轉換範例 - 週期三角函數

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單元學習目標與大綱

- 根據 傅立葉轉換 有關週期信號 的關係式
- 計算 週期三角函數 的 傅立葉轉換

週期信號的 傅立葉轉換 表示式

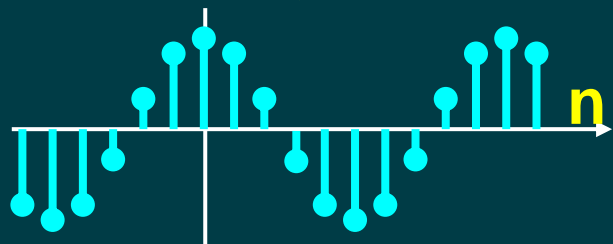
- 一個 週期信號 的 傅立葉轉換 的關係式：

$$\begin{aligned}
 & x[n] \xleftrightarrow{\text{FT}} X(e^{j\omega}) \\
 & = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} = \sum_{k=\langle N \rangle} \sum_{r=-\infty}^{+\infty} 2\pi a_k \delta(\omega - k\omega_0 - r2\pi)
 \end{aligned}$$

The diagram illustrates the Fourier Transform of a periodic signal. The top part shows the relationship between the discrete-time signal $x[n]$ and its discrete-time Fourier Transform $X(e^{j\omega})$. The middle part shows the signal as a sum of complex exponentials and the transform as a sum of impulses. The bottom part is a plot of the frequency spectrum with impulses at integer multiples of the fundamental frequency ω_0 . The impulses are colored cyan, green, and yellow, corresponding to the terms in the equations above. The x-axis is labeled ω and has tick marks at $-N\omega_0 = -2\pi$, 0 , $1\omega_0$, $2\omega_0$, $3\omega_0$, $(N-1)\omega_0$, $N\omega_0 = 2\pi$, and $2N\omega_0 = 4\pi$.

週期三角函數 的 傅立葉轉換

$$x[n] = \cos(k\omega_0 n)$$



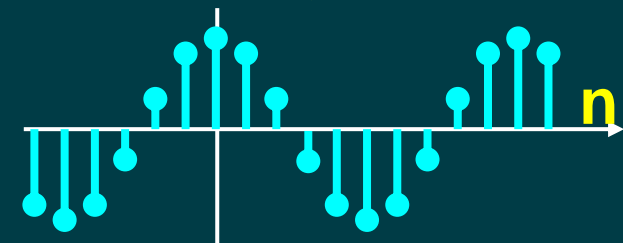
$$= \frac{1}{2} (e^{j k \omega_0 n} + e^{-j k \omega_0 n})$$

$$\cos(s) = \frac{1}{2} (e^{js} + e^{-js})$$

$$= \frac{1}{2} e^{j k \omega_0 n} + \frac{1}{2} e^{-j k \omega_0 n}$$

$$a_k = \frac{1}{2} \quad a_{-k} = \frac{1}{2}$$

週期三角函數的傅立葉轉換

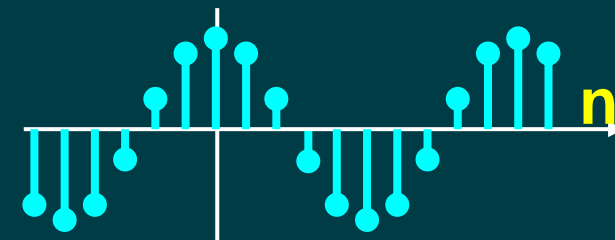
$$x[n] = \frac{1}{2} e^{jkw_0 n} + \frac{1}{2} e^{-jkw_0 n}$$


$$a_k = \frac{1}{2} \quad a_{-k} = \frac{1}{2}$$

$$X(e^{j\omega}) = \sum_{r=-\infty}^{+\infty} 2\pi \frac{1}{2} \delta(\omega - kw_0 - r2\pi) + \sum_{r=-\infty}^{+\infty} 2\pi \frac{1}{2} \delta(\omega + kw_0 - r2\pi)$$

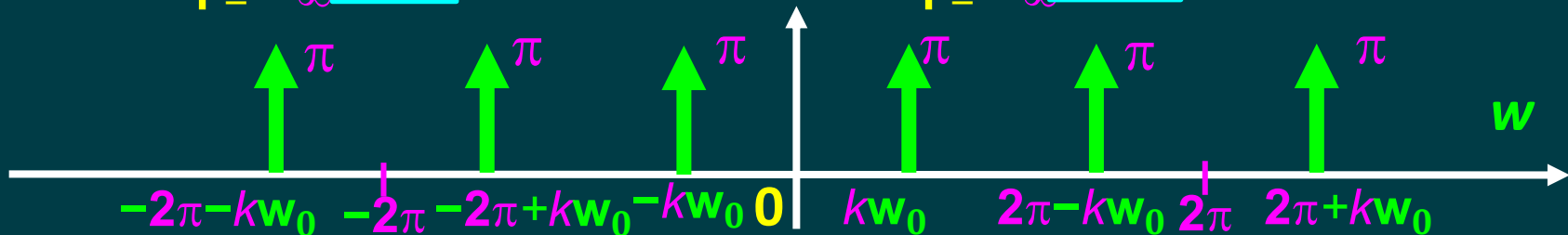
$$X(e^{j\omega}) = \sum_{k=\langle N \rangle} \sum_{r=-\infty}^{+\infty} 2\pi a_k \delta(\omega - kw_0 - r2\pi)$$

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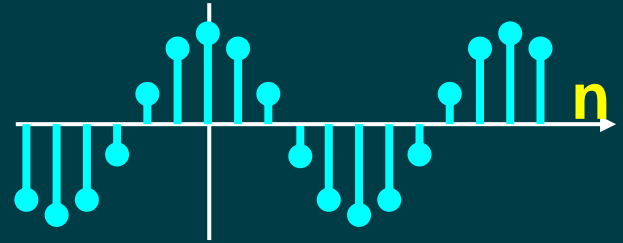
$$a_k = \frac{1}{2} \quad a_{-k} = \frac{1}{2}$$

$$X(e^{jw}) = \sum_{r=-\infty}^{+\infty} \frac{1}{2} \delta(w - kw_0 - r2\pi) + \sum_{r=-\infty}^{+\infty} \frac{1}{2} \delta(w + kw_0 - r2\pi)$$

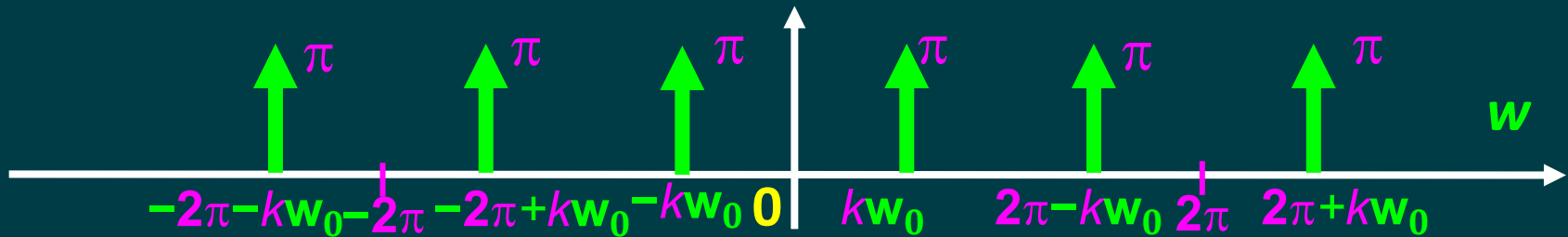


週期三角函數 的 傅立葉轉換

$$x[n] = \cos(kw_0 n)$$



$$X(e^{jw}) = \sum_{r=-\infty}^{+\infty} 2\pi \frac{1}{2} \delta(w - kw_0 - r2\pi) + \sum_{r=-\infty}^{+\infty} 2\pi \frac{1}{2} \delta(w + kw_0 - r2\pi)$$



週期三角函數 的 傅立葉轉換

$$\sin(s) = \frac{1}{2j} (e^{js} - e^{-js})$$

$$\begin{aligned} x[n] &= \sin(kw_0 n) \\ &= \frac{1}{2j} (e^{jkw_0 n} - e^{-jkw_0 n}) \\ &= \frac{1}{2j} e^{jkw_0 n} - \frac{1}{2j} e^{-jkw_0 n} \\ a_k &= \frac{1}{2j} \quad a_{-k} = -\frac{1}{2j} \end{aligned}$$

週期三角函數的傅立葉轉換

$$x[n] = \frac{1}{2j} e^{jkw_0 n} - \frac{1}{2j} e^{-jkw_0 n}$$

$$a_k = \frac{1}{2j} \quad a_{-k} = -\frac{1}{2j}$$

$$X(e^{j\omega}) = \sum_{r=-\infty}^{+\infty} 2\pi \frac{1}{2j} \delta(\omega - kw_0 - r2\pi) + \sum_{r=-\infty}^{+\infty} 2\pi \frac{-1}{2j} \delta(\omega + kw_0 - r2\pi)$$

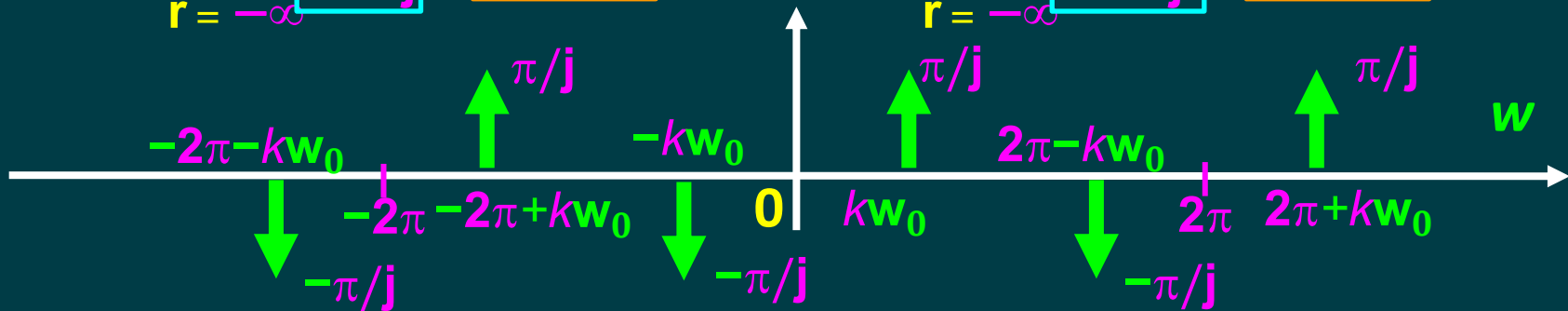
$$X(e^{j\omega}) = \sum_{k=\langle N \rangle} \sum_{r=-\infty}^{+\infty} 2\pi a_k \delta(\omega - kw_0 - r2\pi)$$

週期三角函數的傅立葉轉換

$$x[n] = \frac{1}{2j} e^{jkw_0 n} - \frac{1}{2j} e^{-jkw_0 n}$$

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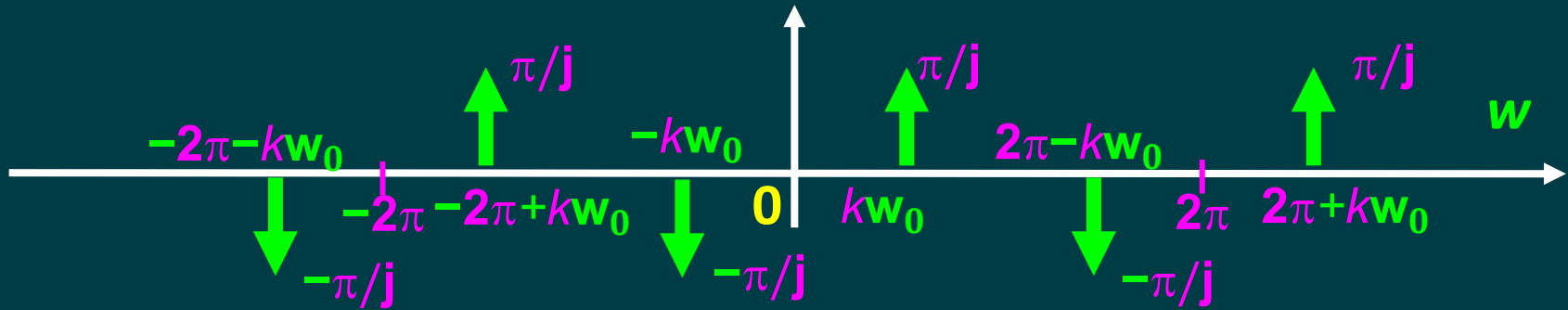
$$X(e^{jw}) = \sum_{r=-\infty}^{+\infty} \frac{1}{2j} \delta(w - kw_0 - r2\pi) + \sum_{r=-\infty}^{+\infty} \frac{-1}{2j} \delta(w + kw_0 - r2\pi)$$



週期三角函數 的 傅立葉轉換

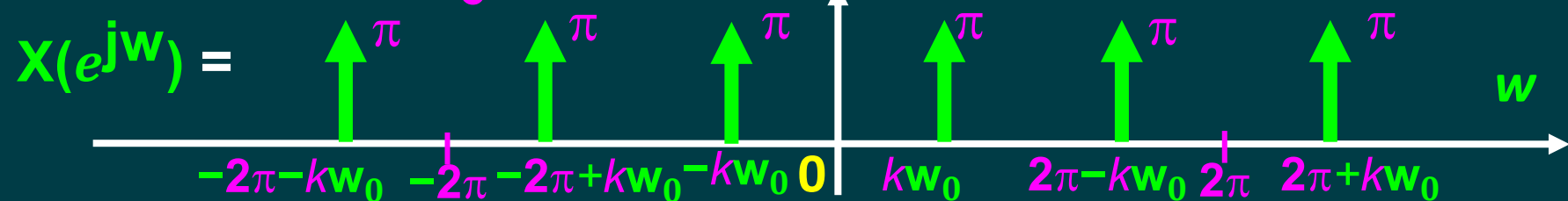
$$x[n] = \sin(k\omega_0 n)$$

$$X(e^{j\omega}) = \sum_{r=-\infty}^{+\infty} 2\pi \frac{1}{2j} \delta(\omega - k\omega_0 - r2\pi) + \sum_{r=-\infty}^{+\infty} 2\pi \frac{-1}{2j} \delta(\omega + k\omega_0 - r2\pi)$$

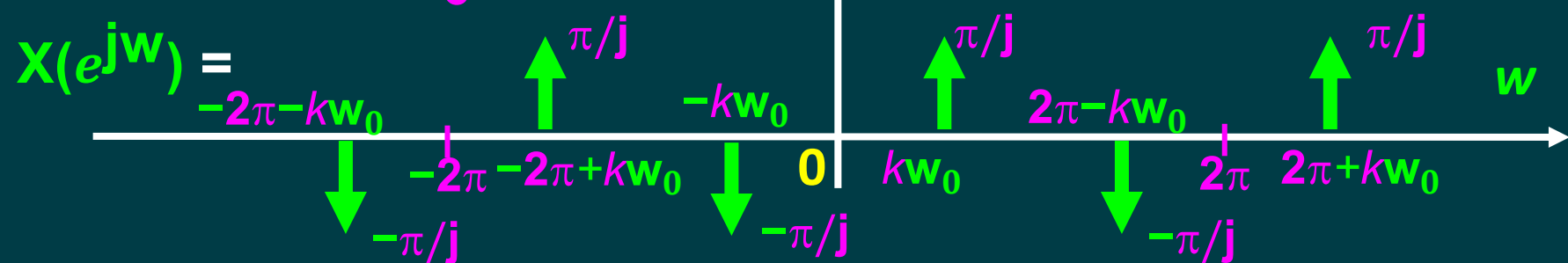


週期三角函數的傅立葉轉換

$$x[n] = \cos(k\omega_0 n)$$

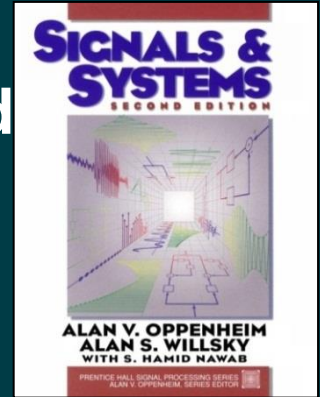


$$x[n] = \sin(k\omega_0 n)$$



參考文獻

- Alan V. Oppenheim, Alan S. Willsky, S. Hamid
Signals & Systems,
Prentice Hall, 2nd Edition, 1997



- **SciLab:**
Open source software for numerical computation
<http://www.scilab.org/>