# 從信號與系統到控制

## 單元：離散F轉換-8

## 傅立葉轉換範例 - 週期三角函數

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# 單元學習目標與大綱

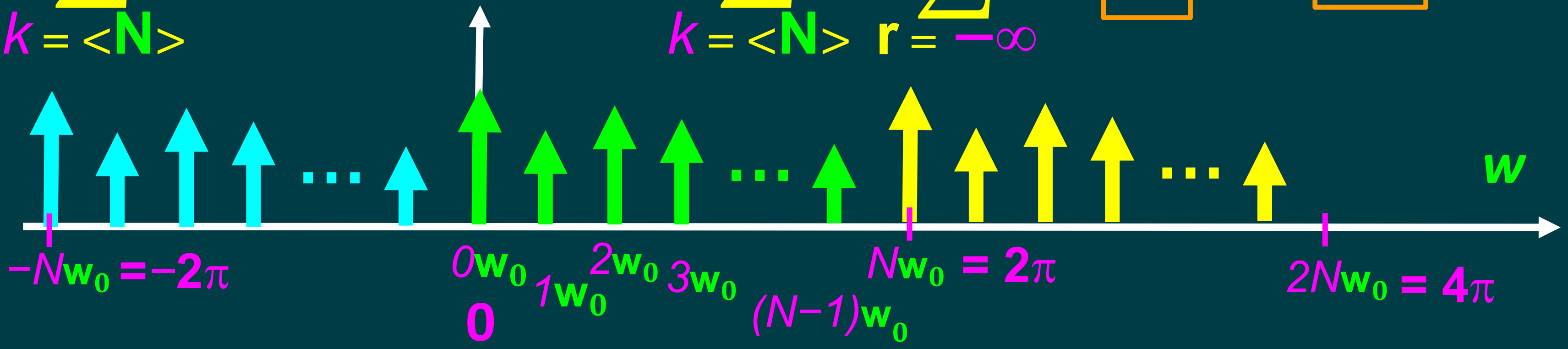
- 根據 傅立葉轉換 有關週期信號 的關係式
- 計算 週期三角函數 的 傅立葉轉換

# 週期信號的 傅立葉轉換 表示式

• 一個 週期信號 的 傅立葉轉換 的關係式：

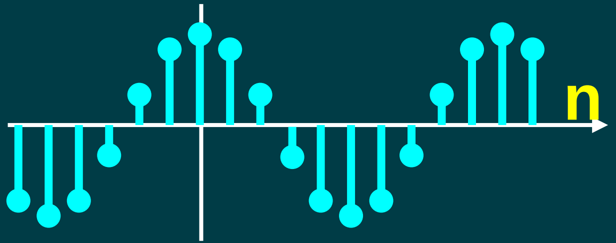
$$x[n] \overset{FT}{\longleftrightarrow} X(e^{jw})$$

$$= \sum_{k=\langle N\rangle} a_k e^{jkw_0 n} \qquad = \sum_{k=\langle N\rangle} \sum_{r=-\infty}^{+\infty} 2\pi\, a_k\, \delta(w - kw_0 - r2\pi)$$



# 週期三角函數 的 傅立葉轉換

$x[n] = \cos(k w_0 n)$




$= \frac{1}{2}\left(e^{j k w_0 n} + e^{-j k w_0 n}\right)$

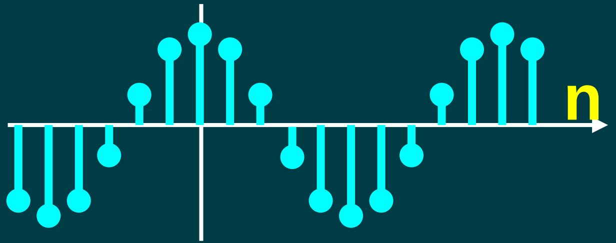
$\cos(s) = \frac{1}{2}(e^{js} + e^{-js})$

$= \frac{1}{2} e^{j k w_0 n} + \frac{1}{2} e^{-j k w_0 n}$

$a_k = \frac{1}{2}$ $a_{-k} = \frac{1}{2}$

# 週期三角函數 的 傅立葉轉換

$$x[n] = \frac{1}{2} e^{jkw_0 n} + \frac{1}{2} e^{-jkw_0 n}$$



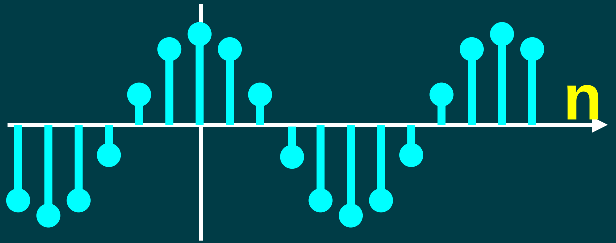

$$a_k = \frac{1}{2} \qquad a_{-k} = \frac{1}{2}$$

$$X(e^{jw}) = \sum_{r=-\infty}^{+\infty} 2\pi \frac{1}{2} \delta(w - kw_0 - r2\pi) + \sum_{r=-\infty}^{+\infty} 2\pi \frac{1}{2} \delta(w + kw_0 - r2\pi)$$

$$X(e^{jw}) = \sum_{k=\langle N \rangle} \sum_{r=-\infty}^{+\infty} 2\pi a_k \delta(w - kw_0 - r2\pi)$$

# 週期三角函數 的 傳立葉轉換

$$x[n] = \frac{1}{2} e^{j k w_0 n} + \frac{1}{2} e^{-j k w_0 n}$$

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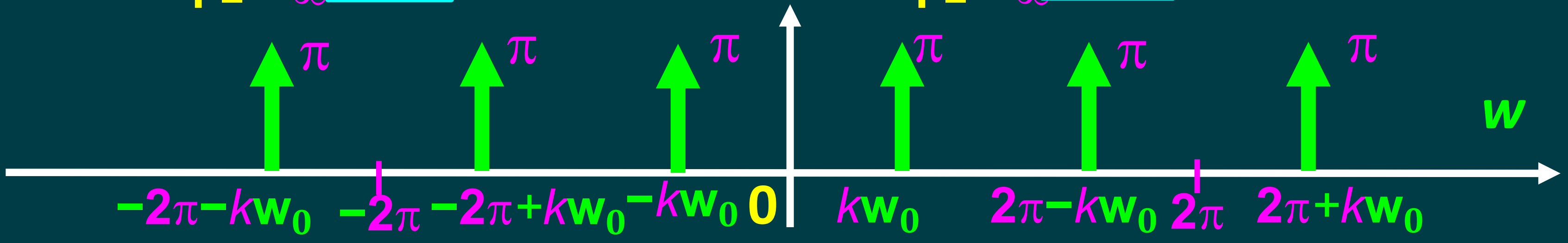

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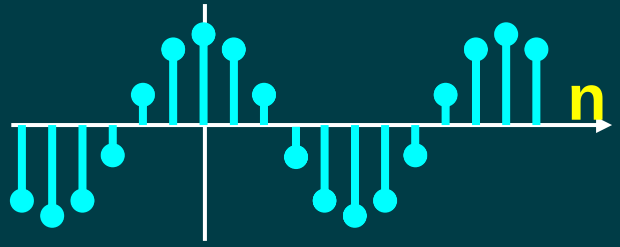
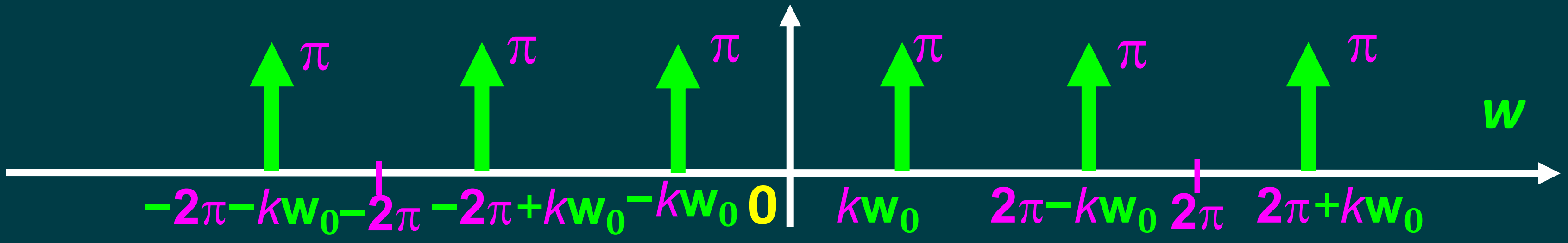
$$x[n] = \cos(k w_0 n)$$




$$X(e^{jw}) = \sum_{r=-\infty}^{+\infty} 2\pi \frac{1}{2} \delta(w - k w_0 - r2\pi) + \sum_{r=-\infty}^{+\infty} 2\pi \frac{1}{2} \delta(w + k w_0 - r2\pi)$$



# 週期三角函數 的 傅立葉轉換

$$\sin(s) = \frac{1}{2j}\left(e^{js} - e^{-js}\right)$$

$$x[n] = \sin(k w_0 n)$$

$$= \frac{1}{2j}\left(e^{j k w_0 n} - e^{-j k w_0 n}\right)$$

$$= \boxed{\frac{1}{2j}}\, e^{j \boxed{k w_0} n} \boxed{-\frac{1}{2j}}\, e^{\boxed{-j k w_0} n}$$

$$a_k = \frac{1}{2j} \qquad a_{-k} = -\frac{1}{2j}$$

# 週期三角函數 的 傅立葉轉換

$$x[n] = \frac{1}{2j} e^{jkw_0 n} - \frac{1}{2j} e^{-jkw_0 n}$$

$$a_k = \frac{1}{2j} \qquad a_{-k} = -\frac{1}{2j}$$

$$X(e^{jw}) = \sum_{r=-\infty}^{+\infty} 2\pi \frac{1}{2j} \delta(w - kw_0 - r2\pi) + \sum_{r=-\infty}^{+\infty} 2\pi \frac{-1}{2j} \delta(w + kw_0 - r2\pi)$$

$$X(e^{jw}) = \sum_{k=\langle N \rangle} \sum_{r=-\infty}^{+\infty} 2\pi a_k \delta(w - kw_0 - r2\pi)$$

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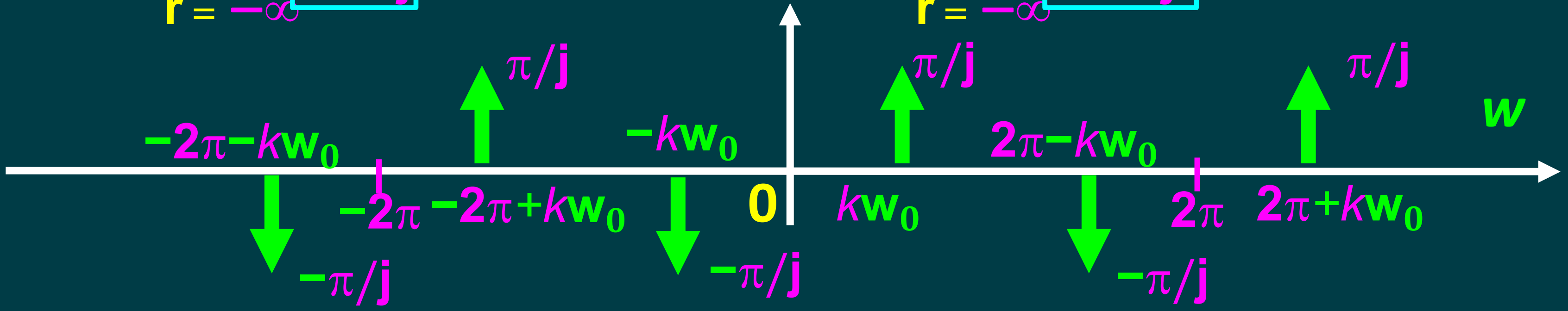
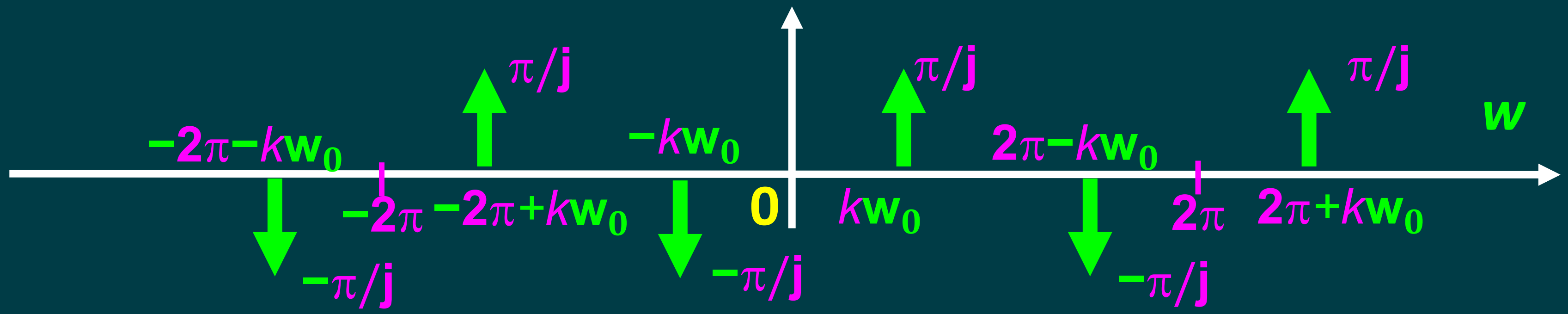


# 週期三角函數 的 傅立葉轉換

$$x[n] = \sin(k w_0 n)$$

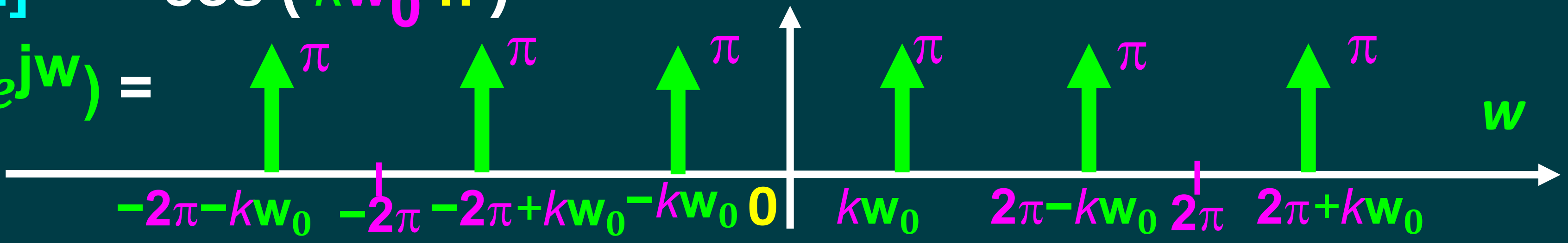
$$X(e^{jw}) = \sum_{r=-\infty}^{+\infty} 2\pi \frac{1}{2j} \delta(w - k w_0 - r2\pi) + \sum_{r=-\infty}^{+\infty} 2\pi \frac{-1}{2j} \delta(w + k w_0 - r2\pi)$$



# 週期三角函數 的 傅立葉轉換

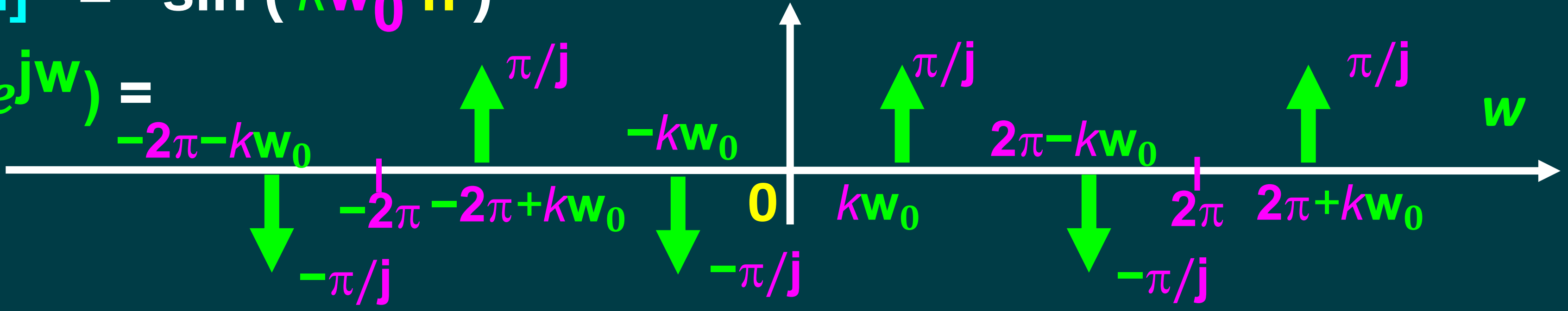
$x[n] = \cos(k w_0 n)$

$X(e^{jw}) =$




$x[n] = \sin(k w_0 n)$

$X(e^{jw}) =$



# 參考文獻

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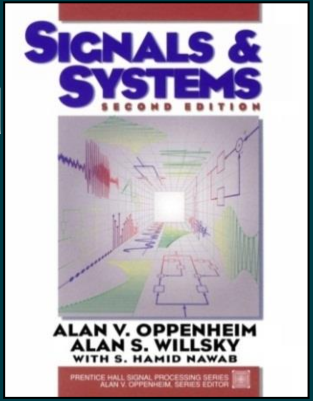
  Prentice Hall, 2nd Edition, 1997




- SciLab:
  Open source software for numerical computation
  http://www.scilab.org/