從信號與系統到控制

單元：離散F轉換-4

傅立葉轉換 範例 - 指數函數

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單元學習目標與大綱

- 根據 傅立葉轉換 的公式與關係式
- 計算 指數函數 的 傅立葉轉換
- 瞭解 傅立葉轉換 不存在 的範例

傅立葉轉換 的 表示式

$$x[n] \overset{FT}{\longleftrightarrow} X(e^{jw})$$

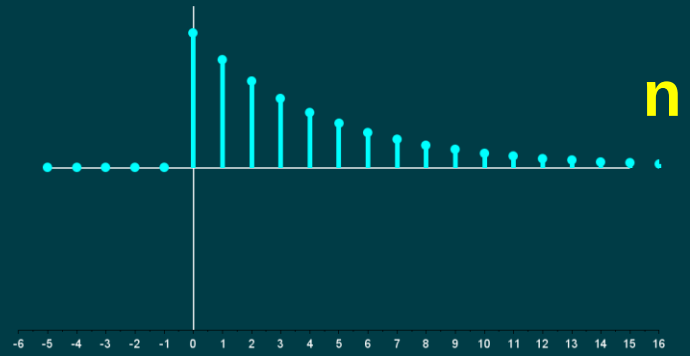
$$X(e^{jw}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-jwn}$$

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{jw})\, e^{jwn}\, dw$$

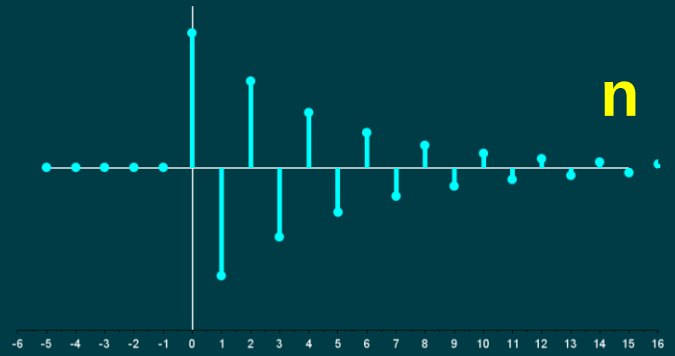
指數函數 的 傅立葉轉換

$x[n] = a^n u[n] \qquad |a| < 1$

$0 < a < 1$



$-1 < a < 0$



指數函數 的 傅立葉轉換

$$x[n] = a^n u[n] \qquad |a| < 1$$

$$X(e^{jw}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-jwn} = \sum_{n=-\infty}^{+\infty} a^n u[n]\, e^{-jwn}$$

$$= \sum_{n=0}^{+\infty} a^n e^{-jwn}$$

$$= \sum_{n=0}^{+\infty} (a\, e^{-jw})^n \qquad |a e^{-jw}| < 1 \qquad = \frac{1}{1 - (a\, e^{-jw})}$$

指數函數 的 傅立葉轉換

$$x[n] = a^n u[n] \qquad |a| < 1$$

$$X(e^{jw}) = \frac{1}{1 - (a\, e^{-jw})} = \frac{1}{1 - (a\cos(-w) + j\, a\sin(-w))}$$

$$|X(e^{jw})| = \frac{1}{\sqrt{[1 - a\cos(-w)]^2 + [-a\sin(-w)]^2}}$$

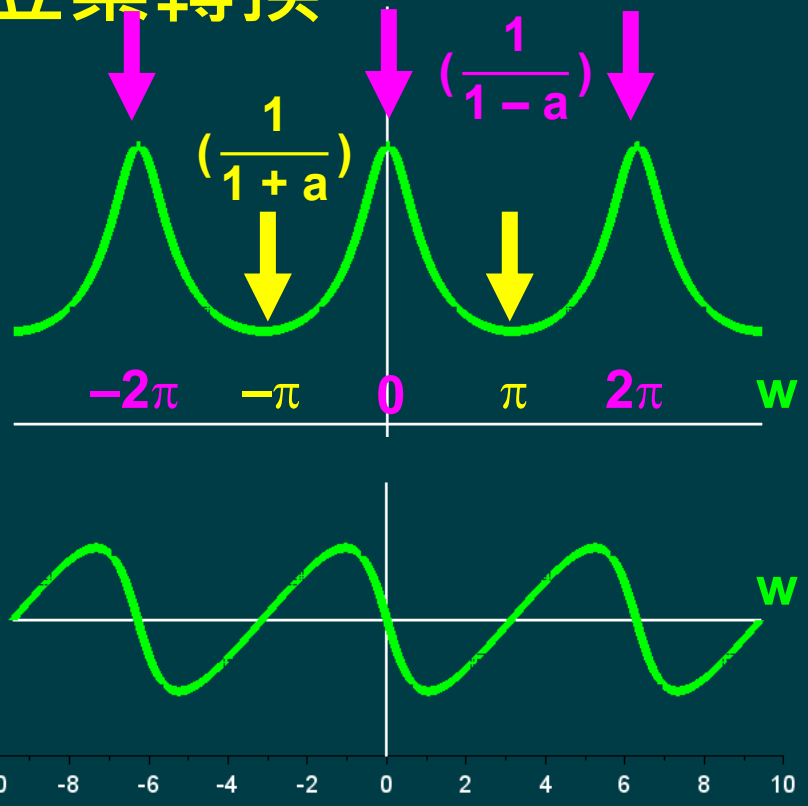
$$\angle X(e^{jw}) = -\tan^{-1}\left(\frac{-a\sin(-w)}{1 - a\cos(-w)}\right)$$

指數函數 的 傅立葉轉換

$0 < a < 1$

$$|X(e^{jw})| = \frac{1}{\sqrt{[\,1 - a\cos(w)\,]^2 + [\,a\sin(w)\,]^2}}$$

$$\angle X(e^{jw}) = -\tan^{-1}\left(\frac{a\sin(w)}{1 - a\cos(w)}\right)$$

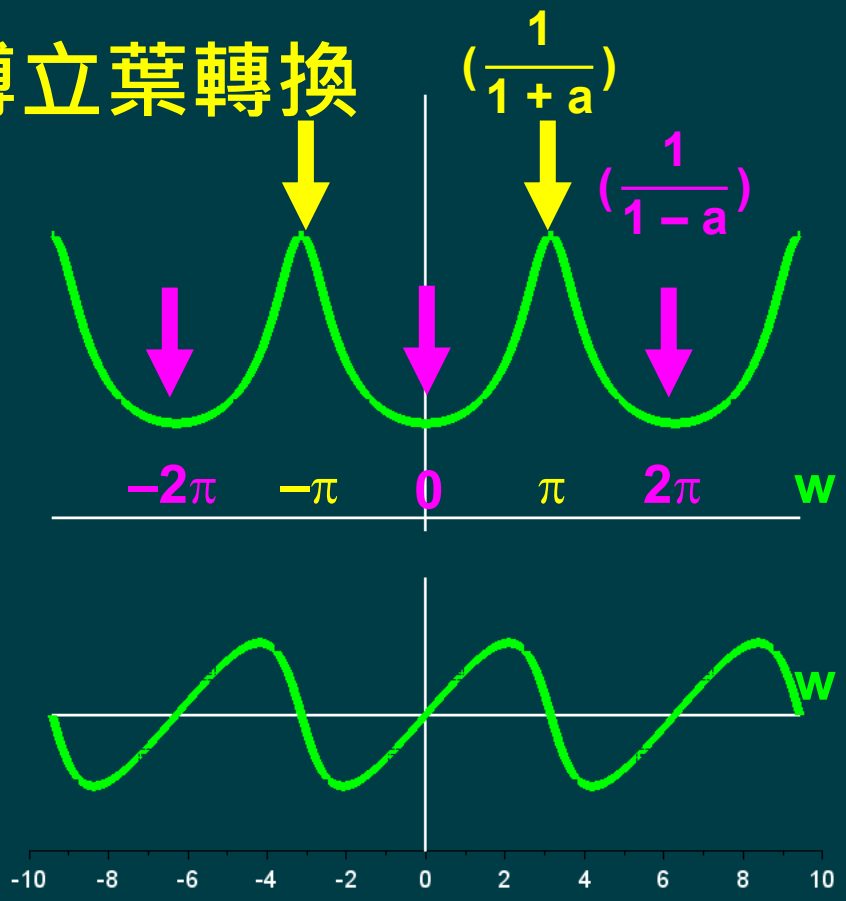


指數函數 的 傅立葉轉換

$-1 < a < 0$

$$|X(e^{jw})| = \frac{1}{\sqrt{[1 - a\cos(w)]^2 + [a\sin(w)]^2}}$$

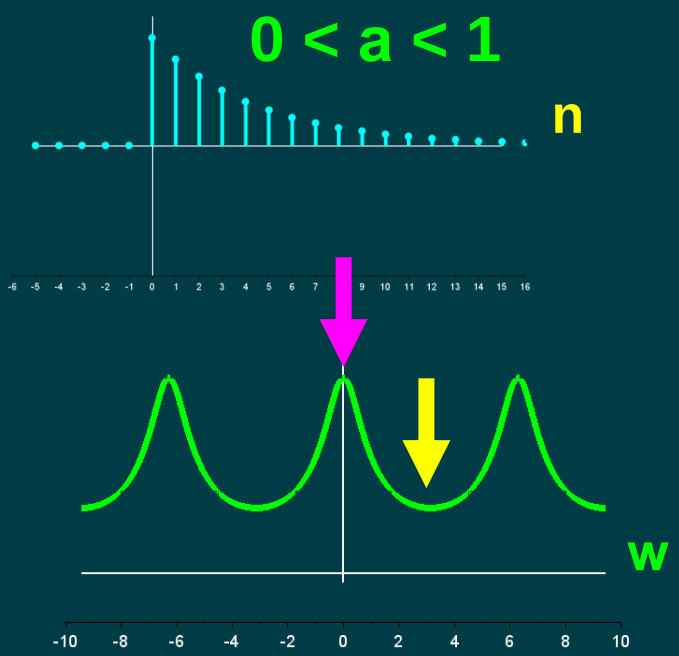
$$\angle X(e^{jw}) = -\tan^{-1}\left(\frac{a\sin(w)}{1 - a\cos(w)}\right)$$



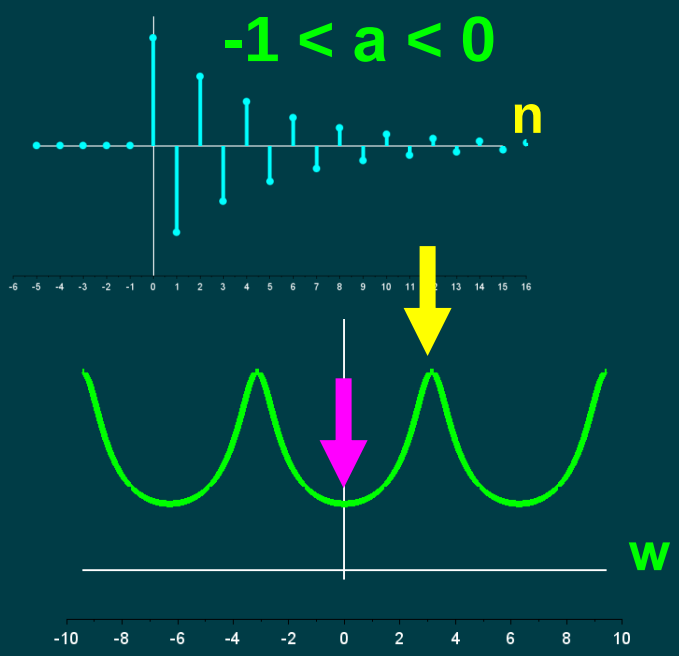
指數函數 的 傅立葉轉換

$x[n] = a^n u[n] \qquad |a| < 1$

$0 < a < 1$



$-1 < a < 0$



發散的 指數函數 的 傅立葉轉換

$$x[n] = a^n u[n] \qquad |a| > 1$$

$$X(e^{jw}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-jwn} = \sum_{n=-\infty}^{+\infty} a^n u[n] e^{-jwn}$$

$$= \sum_{n=0}^{+\infty} a^n e^{-jwn}$$

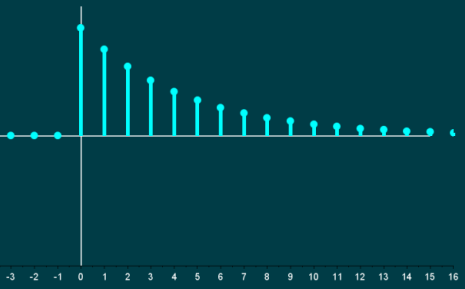
$$= \sum_{n=0}^{+\infty} (a e^{-jw})^n \qquad |a e^{-jw}| > 1 \qquad = \infty$$

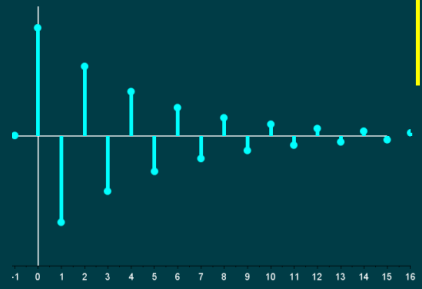
- 沒有傅立葉轉換

指數函數 的 傅立葉轉換

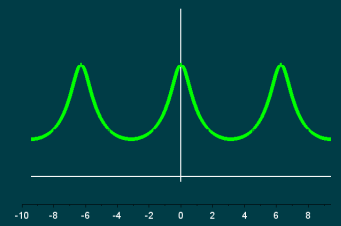
$x[n] = a^n u[n]$ $\boxed{|a| < 1}$ $X(e^{jw}) = \dfrac{1}{1 - (a e^{-jw})}$

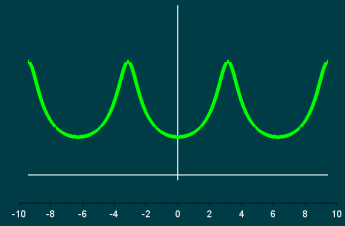
$0 < a < 1$ $-1 < a < 0$



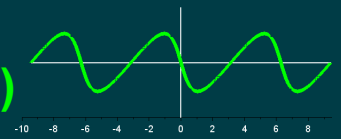


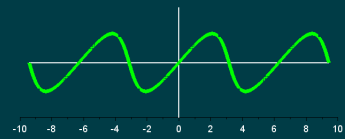
$|X(e^{jw})|$





$\angle X(e^{jw})$





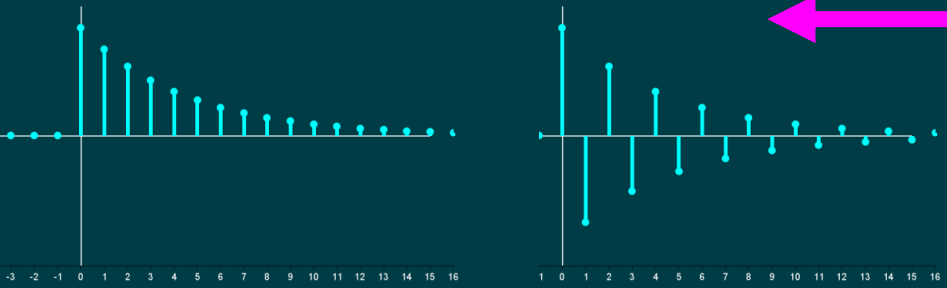
$x[n] = a^n u[n]$ $\boxed{|a| > 1}$

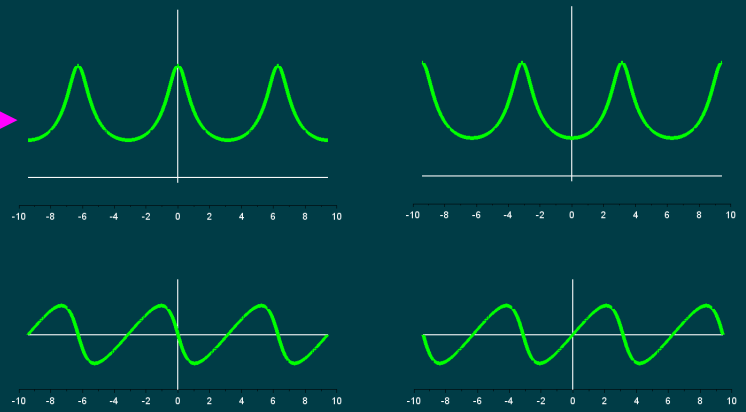
- 傅立葉轉換不存在

指數函數 的 傅立葉轉換

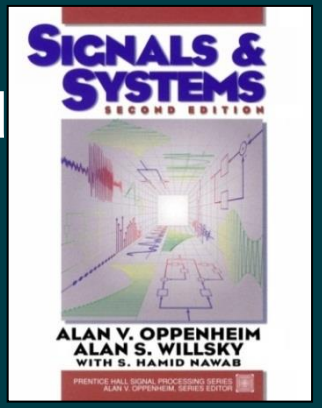
$a^n u[n]$ $\quad |a| < 1 \quad$ FT $\quad \frac{1}{1 - (a e^{-jw})}$

$0 < a < 1$ $\qquad -1 < a < 0$ $\qquad$ FT





參考文獻



- Alan V. Oppenheim, Alan S. Willsky, S. Hamid
  Signals & Systems,
  Prentice Hall, 2nd Edition, 1997

- **SciLab**:
  Open source software for numerical computation
  http://www.scilab.org/