從信號與系統到控制

單元：離散F轉換-1

從 週期信號 到 非週期信號 的 特性改變

授課老師：連　豐　力

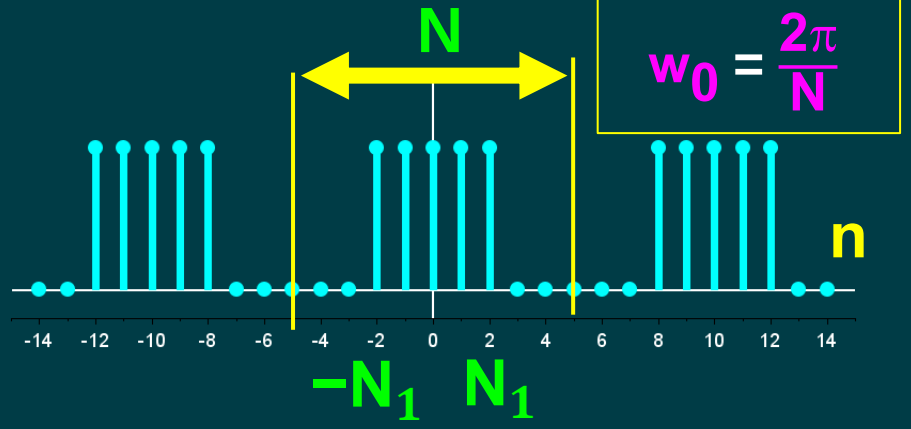
單元學習目標與大綱

- 複習 - 離散時間 週期方波 之 傅立葉級數
- 推演 - 不同週期的信號之 傅立葉級數差異
- 猜想 - 將 週期 變成 無限大 的 非週期信號之 傅立葉級數

離散時間週期方波函數的傅立葉級數

$$x[n] = \begin{cases} 1, & -N_1 \geq n > N_1 \\ 0, & \text{others in } \langle N \rangle \end{cases}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n]\, e^{-jkw_0 n}$$



$$w_0 = \frac{2\pi}{N}$$

$$a_k = \frac{1}{N}(2N_1 + 1)$$

$k = 0, \pm N, \pm 2N, \ldots$

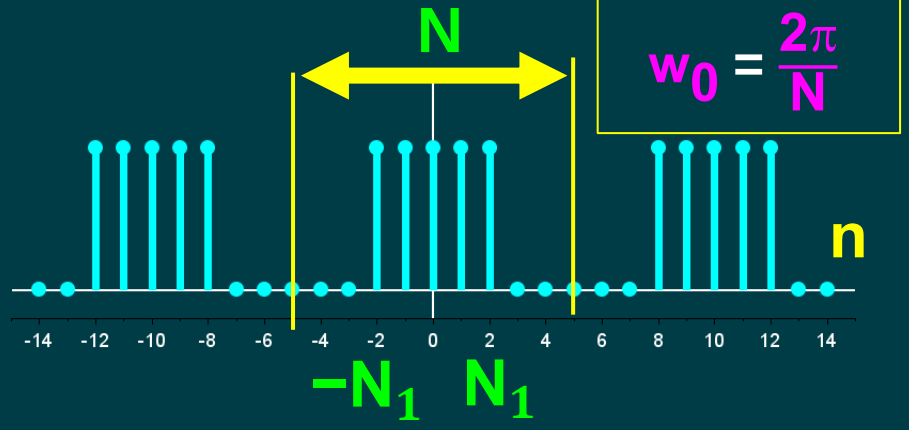
$$= \frac{1}{N} \frac{\sin(kw_0(N_1 + 1/2))}{\sin(kw_0/2)}$$

$k \neq 0, \pm N, \pm 2N, \ldots$

離散時間週期方波函數的傅立葉級數

$$x[n] = \begin{cases} 1, & -N_1 \geq n > N_1 \\ 0, & \text{others in } \langle N \rangle \end{cases}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n]\, e^{-jkw_0 n}$$



$$w_0 = \frac{2\pi}{N}$$

$$N\, a_k = N \frac{1}{N} (2N_1 + 1) = (2N_1 + 1) \qquad k = 0, \pm N, \pm 2N, \ldots$$

$$= N \frac{1}{N} \frac{\sin(kw_0(N_1 + 1/2))}{\sin(kw_0/2)} = \frac{\sin(kw_0(N_1 + 1/2))}{\sin(kw_0/2)} \qquad k \neq 0, \pm N, \pm 2N, \ldots$$

離散時間週期方波函數的傅立葉級數

$2N_1 + 1 = 5$

$N = 10$

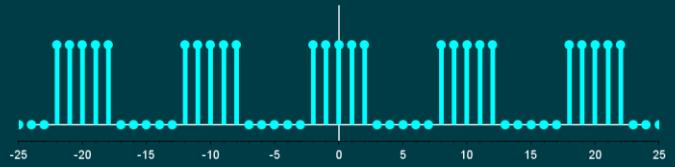
$$N a_k = (2N_1 + 1)$$

$$= \frac{\sin(kw_0(N_1 + 1/2))}{\sin(kw_0/2)}$$

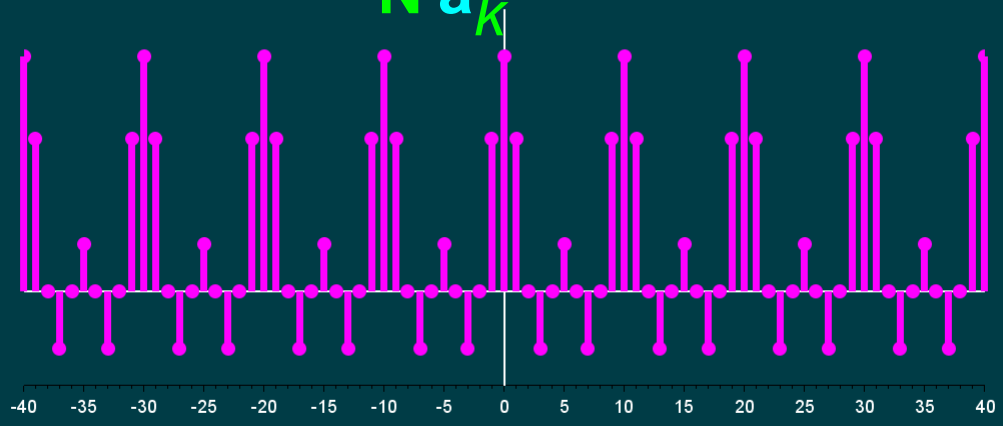
$k = 0, \pm N, \pm 2N, \ldots$

$k \neq 0, \pm N, \pm 2N, \ldots$

x[n]



$N a_k$



離散時間週期方波函數的傅立葉級數

$2N_1 + 1 = 5$

$N = 20$

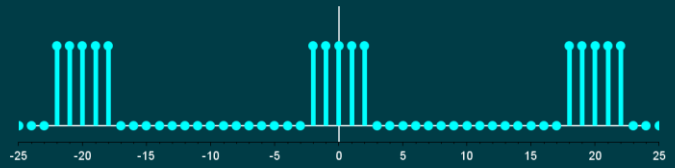
$$N a_k = (2N_1 + 1)$$

$$= \frac{\sin(kw_0(N_1 + 1/2))}{\sin(kw_0/2)}$$

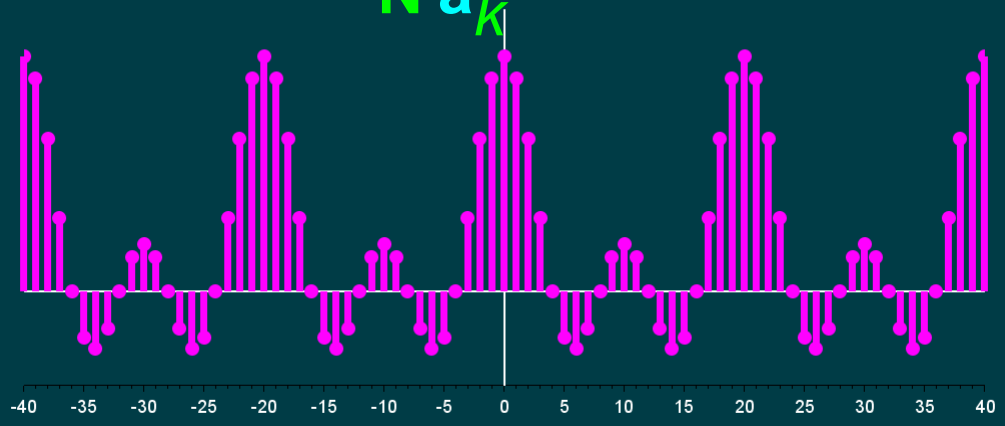
$k = 0, \pm N, \pm 2N, \ldots$

$k \neq 0, \pm N, \pm 2N, \ldots$

x[n]



$N a_k$



離散時間週期方波函數的傅立葉級數

$2N_1 + 1 = 5$

$N = 40$

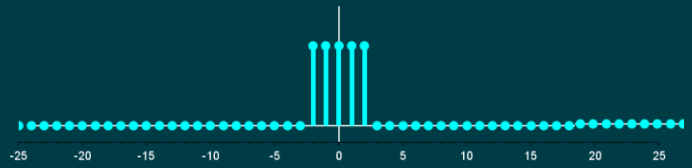
$$N a_k = (2N_1 + 1)$$

$$= \frac{\sin(k w_0 (N_1 + 1/2))}{\sin(k w_0 / 2)}$$

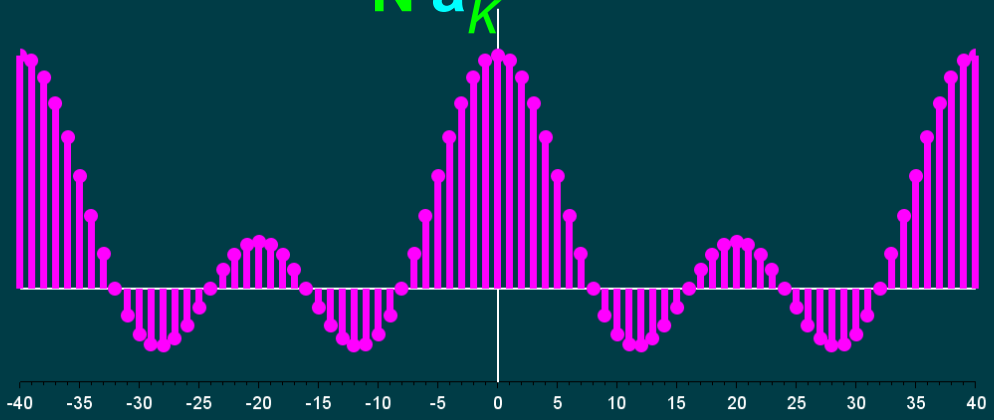
$k = 0, \pm N, \pm 2N, \ldots$

$k \neq 0, \pm N, \pm 2N, \ldots$

x[n]



$N a_k$



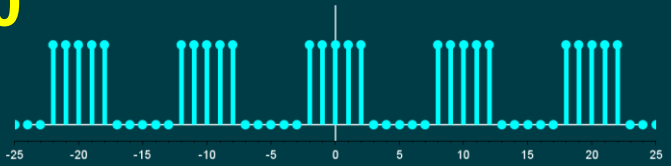
不同週期信號之傅立葉級數

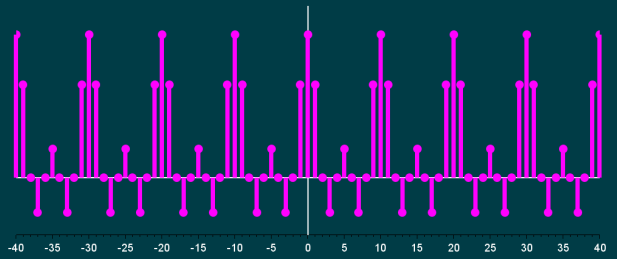
$2N_1 + 1 = 5$

x[n]

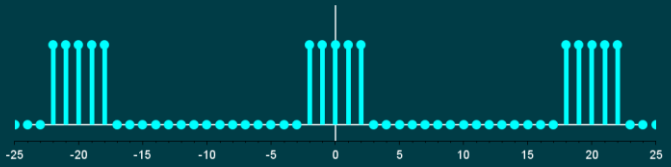
Na_k

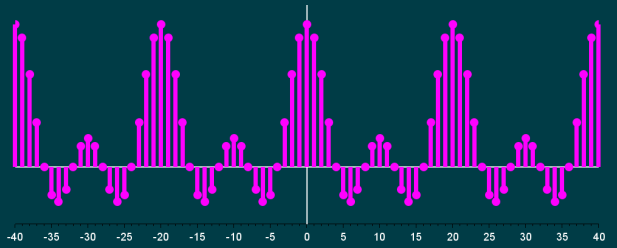
N = 10



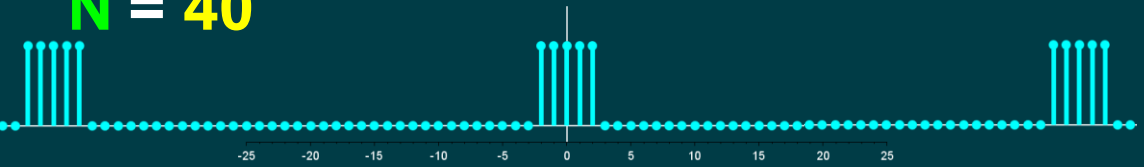


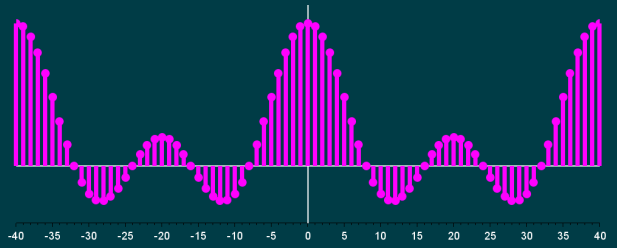
N = 20





N = 40





不同週期信號之傅立葉級數

$$w_0 = \frac{2\pi}{N}$$

N = 10

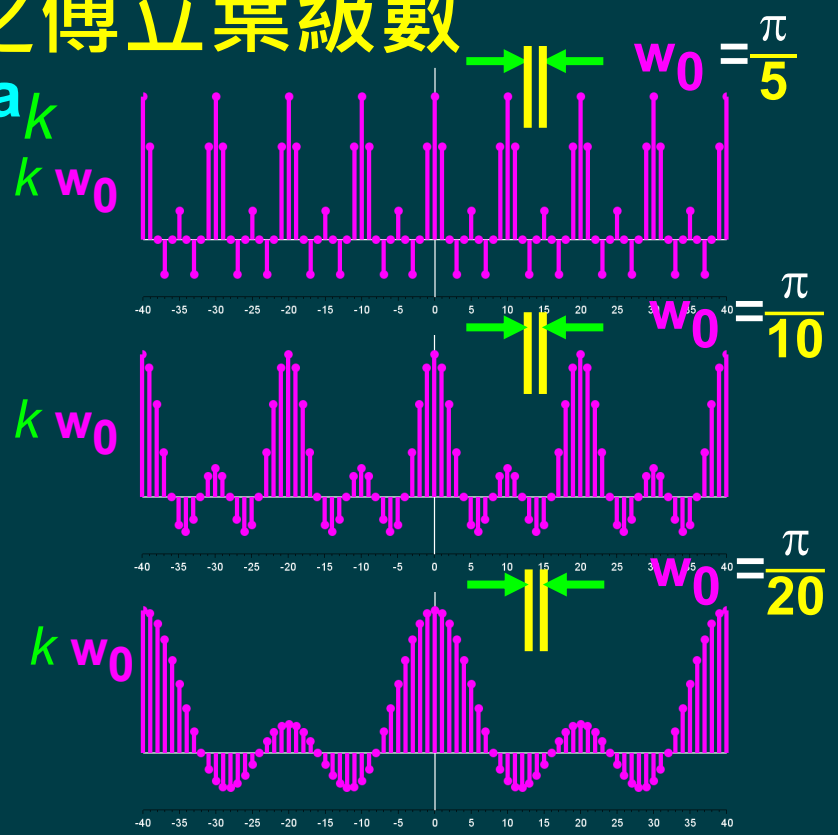
$$w_0 = \frac{2\pi}{10} = \frac{\pi}{5}$$

N = 20

$$w_0 = \frac{2\pi}{20} = \frac{\pi}{10}$$

N = 40

$$w_0 = \frac{2\pi}{40} = \frac{\pi}{20}$$



不同週期信號之傅立葉級數

$w_0 = \frac{2\pi}{N}$

$N\,a_k$

$k\,w_0$

$N = 10$

$w_0 = \frac{2\pi}{10} = \frac{\pi}{5}$

$N\,w_0 = 2\pi$

$N = 20$

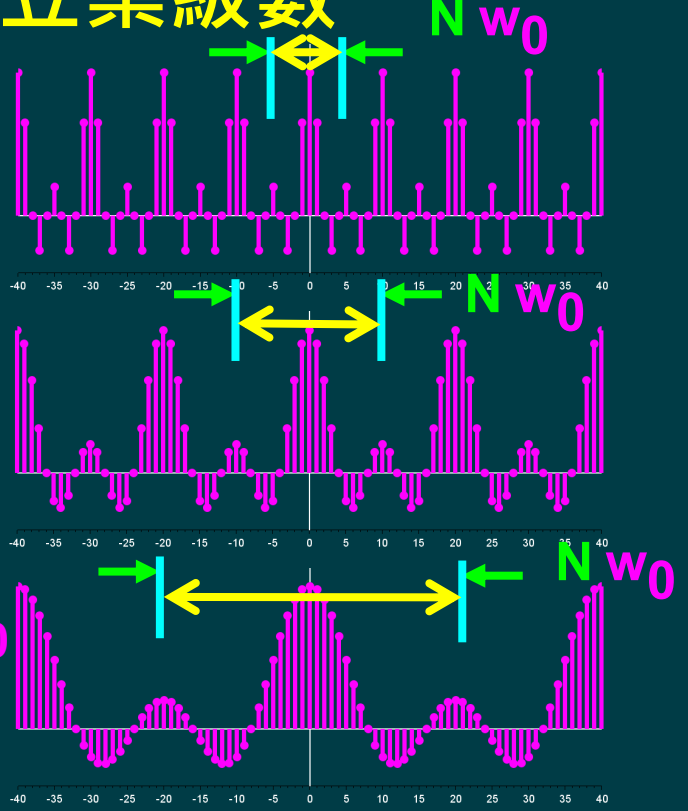
$w_0 = \frac{2\pi}{20} = \frac{\pi}{10}$

$N\,w_0 = 2\pi$

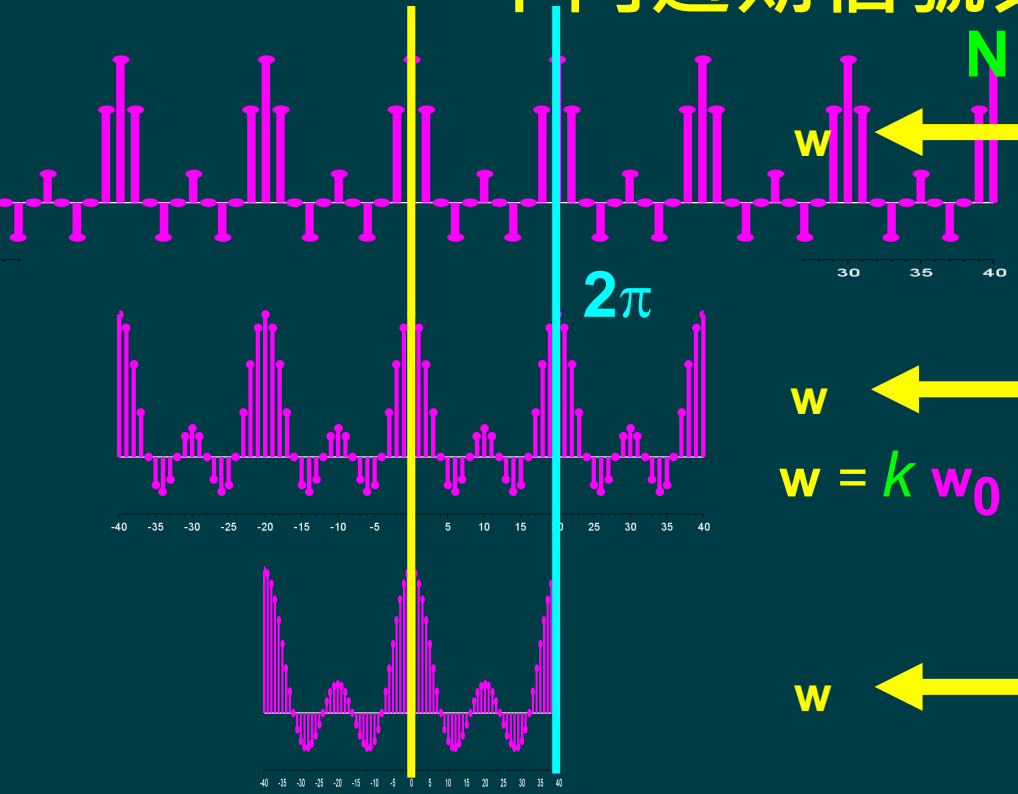
$N = 40$

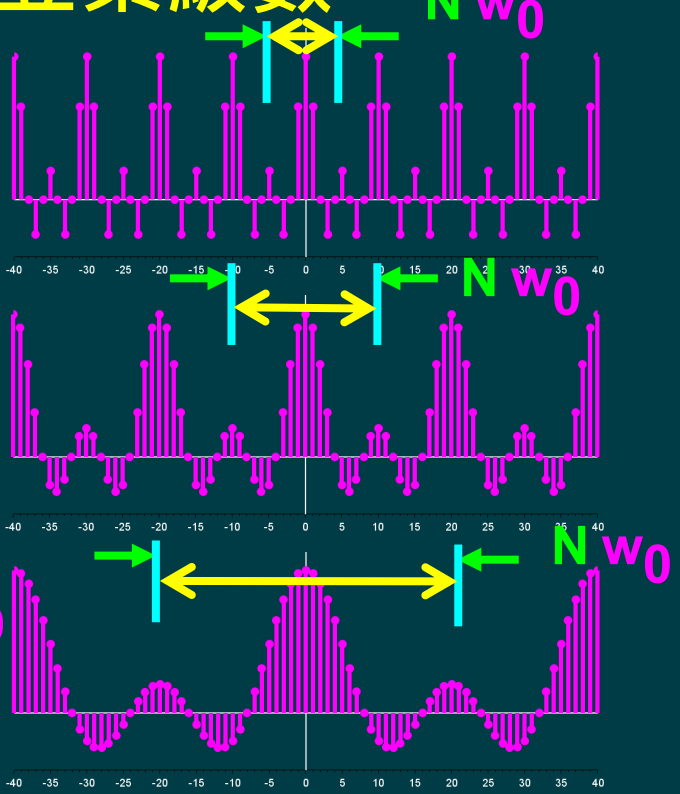
$w_0 = \frac{2\pi}{40} = \frac{\pi}{20}$

$N\,w_0 = 2\pi$



不同週期信號之傅立葉級數





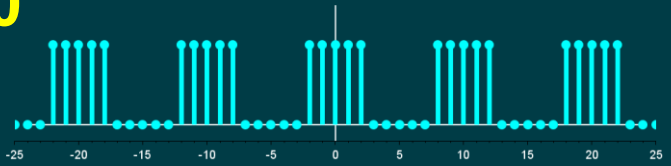
不同週期信號之傅立葉級數

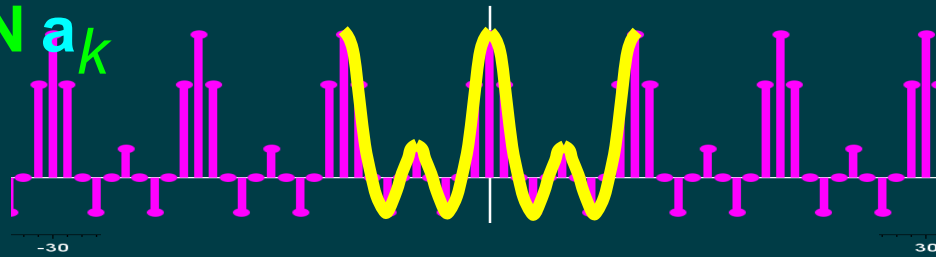
$2N_1 + 1 = 5$

x[n]

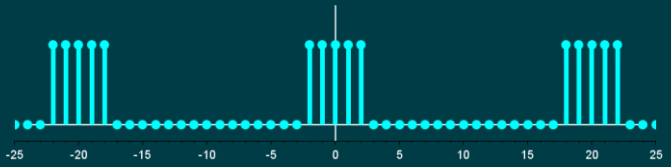
$N a_k$

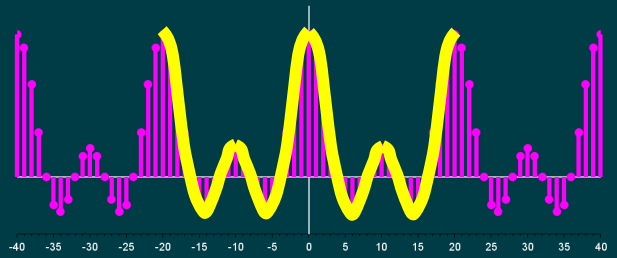
N = 10



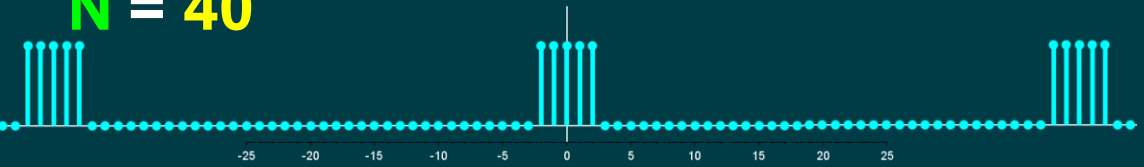


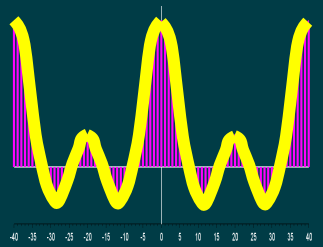
N = 20





N = 40





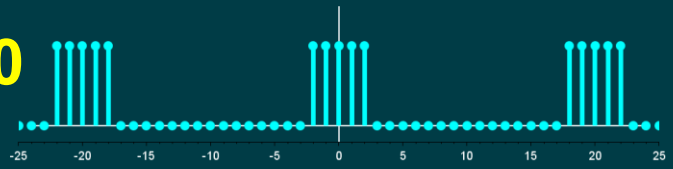
不同週期信號之傅立葉級數

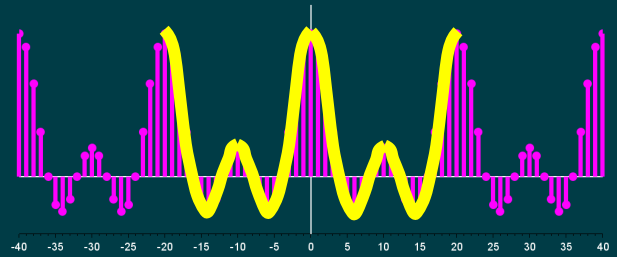
$2N_1 + 1 = 5$

x[n]

$N a_k$

$N = 20$

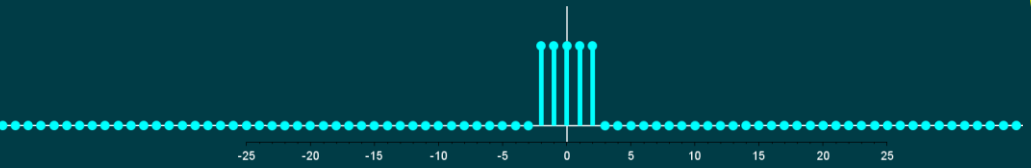


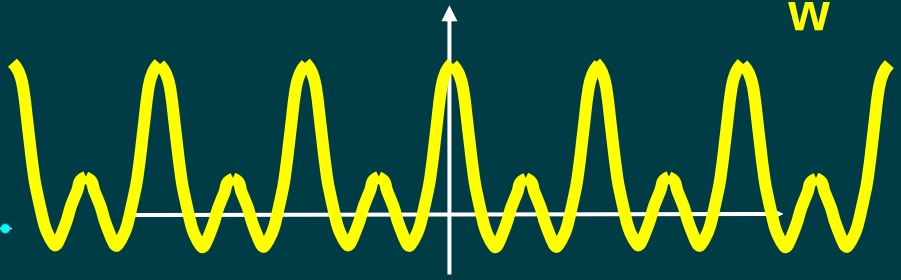


$N \rightarrow \infty$

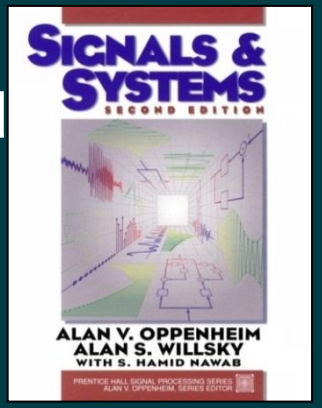
$w_0 = \frac{2\pi}{N}$

$w_0 \rightarrow 0$





參考文獻



- Alan V. Oppenheim, Alan S. Willsky, S. Hamid
  Signals & Systems,
  Prentice Hall, 2nd Edition, 1997

- **SciLab**:
  Open source software for numerical computation
  http://www.scilab.org/