從信號與系統到控制

單元：連續F轉換-9

傅立葉轉換 範例 - 週期三角函數

授課老師：連　豐　力

單元學習目標與大綱

- 根據 傅立葉轉換 有關 週期信號 的關係式
- 計算 週期三角函數 的 傅立葉轉換

週期信號的 傅立葉轉換 表示式

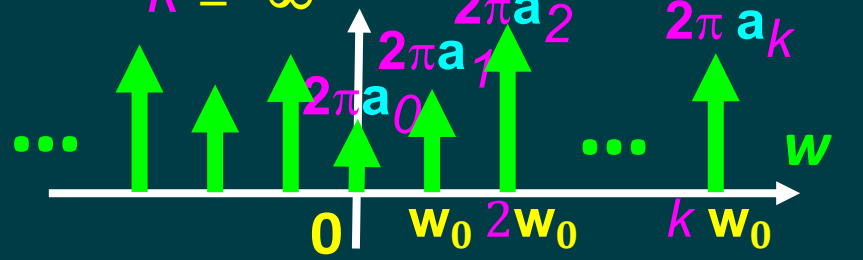
- 一個週期信號 的 傅立葉轉換 的關係式：

$$x(t) \xrightarrow{FT} X(jw)$$

$$x(t) \overset{FS}{=} \sum_{k=-\infty}^{+\infty} a_k e^{j k w_0 t}$$

$$X(jw) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(w - k w_0)$$

- 任意的週期信號



週期三角函數 的 傅立葉轉換

$$x(t) = \cos(k w_0 t)$$

$$= \frac{1}{2}\left(e^{j k w_0 t} + e^{-j k w_0 t} \right)$$

$$\cos(s) = \frac{1}{2}\left(e^{js} + e^{-js}\right)$$

$$= \boxed{\frac{1}{2}}\, e^{j \boxed{k w_0} t} \boxed{+ \frac{1}{2}}\, e^{\boxed{-j k w_0} t}$$

$$a_k = \frac{1}{2} \qquad a_{-k} = \frac{1}{2}$$

週期三角函數 的 傅立葉轉換

$$x(t) = \cos(k w_0 t)$$

$$= \frac{1}{2} e^{j k w_0 t} + \frac{1}{2} e^{-j k w_0 t}$$

$$a_k = \frac{1}{2} \qquad a_{-k} = \frac{1}{2}$$

$$X(jw) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(w - k w_0)$$

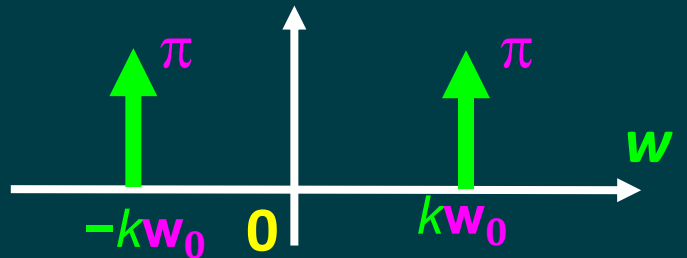
$$X(jw) = 2\pi \frac{1}{2} \delta(w - k w_0) + 2\pi \frac{1}{2} \delta(w + k w_0)$$

週期三角函數 的 傅立葉轉換

$x(t) = \cos(k w_0 t)$

$$X(jw) = \cancel{2\pi}\,\frac{1}{\cancel{2}}\,\delta(w - kw_0) + \cancel{2\pi}\,\frac{1}{\cancel{2}}\,\delta(w + kw_0)$$

$$= \pi\,\delta(w - kw_0) + \pi\,\delta(w + kw_0)$$



週期三角函數 的 傅立葉轉換

$$\sin(s) = \frac{1}{2j}\left(e^{js} - e^{-js}\right)$$

$$x(t) = \sin(k w_0 t)$$

$$= \frac{1}{2j}\left(e^{j k w_0 t} - e^{-j k w_0 t}\right)$$

$$= \boxed{\frac{1}{2j}}\, e^{j \boxed{k w_0} t} \boxed{- \frac{1}{2j}}\, e^{\boxed{-j k w_0} t}$$

$$a_k = \frac{1}{2j} \qquad a_{-k} = -\frac{1}{2j}$$

$$X(jw) = \sum_{k=-\infty}^{+\infty} 2\pi\, a_k\, \delta(w - k w_0)$$

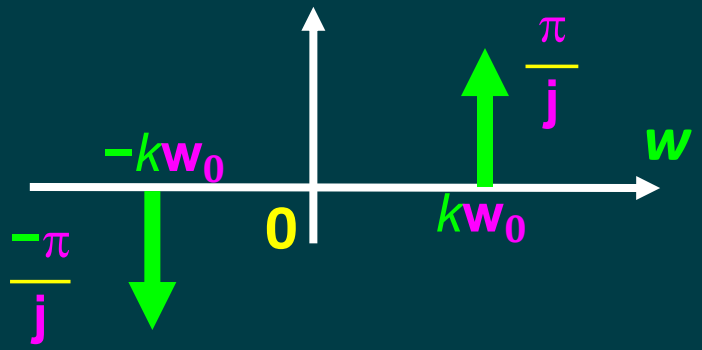
$$X(jw) = 2\pi \frac{1}{2j} \delta(w - k w_0) + 2\pi \frac{-1}{2j} \delta(w + k w_0)$$

週期三角函數 的 傅立葉轉換

$$x(t) = \sin(k w_0 t)$$

$$X(jw) = 2\pi \frac{1}{2j} \delta(w - k w_0) + 2\pi \frac{-1}{2j} \delta(w + k w_0)$$

$$= \frac{\pi}{j} \delta(w - k w_0) + \frac{-\pi}{j} \delta(w + k w_0)$$

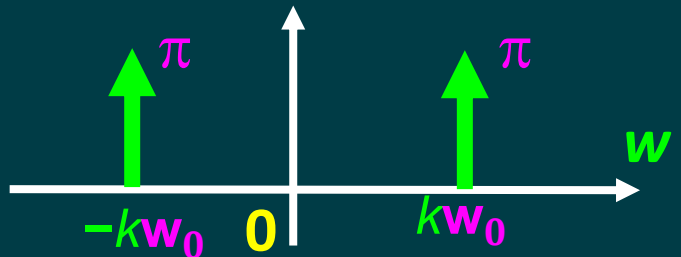


週期三角函數 的 傅立葉轉換

$x(t) = \cos(k w_0 t)$

FS $a_k = \frac{1}{2} \quad a_{-k} = \frac{1}{2}$

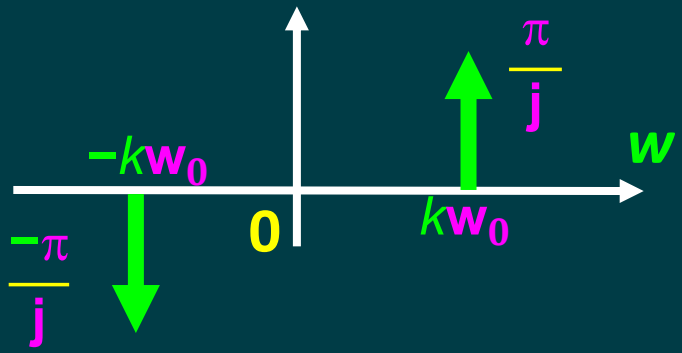
FT $X(jw) = \pi\delta(w - k w_0) + \pi\delta(w + k w_0)$



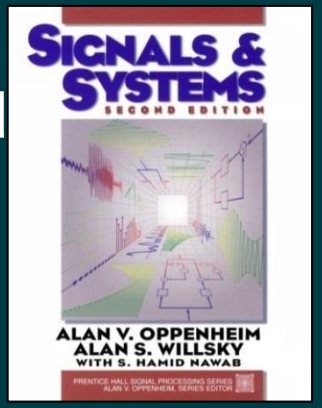
$x(t) = \sin(k w_0 t)$

$a_k = \frac{1}{2j} \quad a_{-k} = -\frac{1}{2j}$

$X(jw) = \frac{\pi}{j}\delta(w - k w_0) - \frac{\pi}{j}\delta(w + k w_0)$



參考文獻



- Alan V. Oppenheim, Alan S. Willsky, S. Hamid
  Signals & Systems,
  Prentice Hall, 2nd Edition, 1997

- **SciLab**:
  Open source software for numerical computation
  http://www.scilab.org/