

從信號與系統到控制

單元：連續F轉換-3

傅立葉轉換 範例 - 指數函數

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單元學習目標與大綱

- 根據 **傅立葉轉換** 的公式與關係式
- 計算 **指數函數** 的 **傅立葉轉換**
- 瞭解 **傅立葉轉換** **不存在** 的範例

傅立葉轉換 的 表示式

$$x(t) \xleftrightarrow{\text{FT}} X(j\omega)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

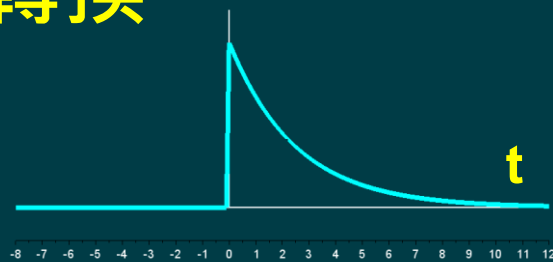
指數函數的傅立葉轉換

$$x(t) = e^{-a t} u(t) \quad a > 0$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

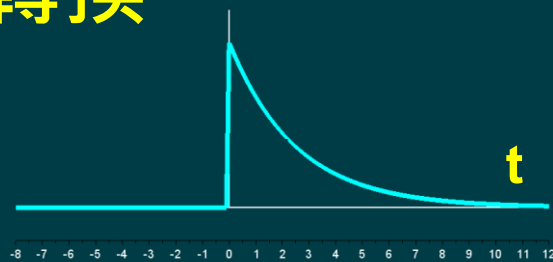
$$= \int_{-\infty}^{\infty} e^{-a t} u(t) e^{-j\omega t} dt$$

$$= \int_0^{\infty} e^{-a t} e^{-j\omega t} dt = \int_0^{\infty} e^{-(a + j\omega)t} dt$$



指數函數的傅立葉轉換

$$x(t) = e^{-a t} u(t) \quad a > 0$$



$$X(j\omega) = \int_0^{\infty} e^{-(a + j\omega)t} dt$$

$$= \frac{1}{-(a + j\omega)} e^{-(a + j\omega)t} \Big|_0^{\infty} = \frac{1}{(a + j\omega)}$$

$$= \frac{1}{-(a + j\omega)} e^{-(a + j\omega)\infty} - \frac{1}{-(a + j\omega)} e^{-(a + j\omega)0}$$

指數函數的傅立葉轉換

$$x(t) = e^{-at} u(t) \quad a > 0$$

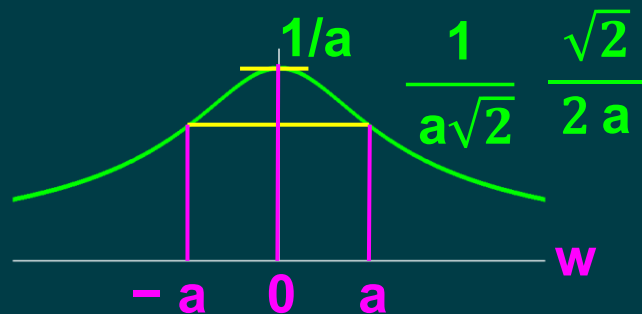
$$X(j\omega) = \frac{1}{(a + j\omega)}$$

$$|X(j\omega)| = \frac{1}{\sqrt{a^2 + \omega^2}}$$

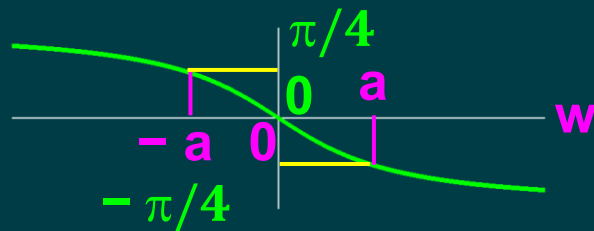
$$= \frac{1}{\sqrt{a^2 + \omega^2}}$$

$$\angle X(j\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

$$|X(j\omega)|$$



$$\angle X(j\omega)$$



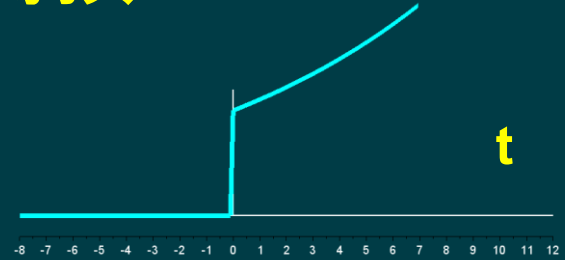
指數函數的傅立葉轉換

$$x(t) = e^{-a t} u(t) \quad a < 0$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-a t} u(t) e^{-j\omega t} dt$$

$$= \int_0^{\infty} e^{-a t} e^{-j\omega t} dt = \int_0^{\infty} e^{-(a + j\omega)t} dt$$



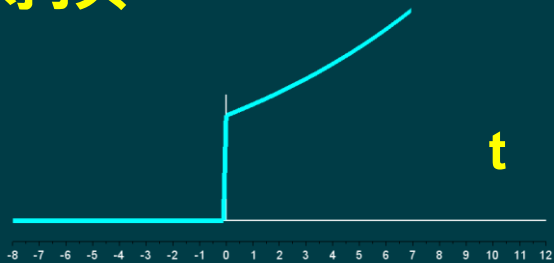
指數函數的傅立葉轉換

$$x(t) = e^{-a t} u(t) \quad a < 0$$

$$X(j\omega) = \int_0^{\infty} e^{-(a + j\omega)t} dt$$

$$= \frac{1}{-(a + j\omega)} e^{-(a + j\omega)t} \Big|_0^{\infty}$$

$$= \frac{1}{-(a + j\omega)} e^{-(a + j\omega)\infty} - \frac{1}{-(a + j\omega)} e^{-(a + j\omega)0}$$



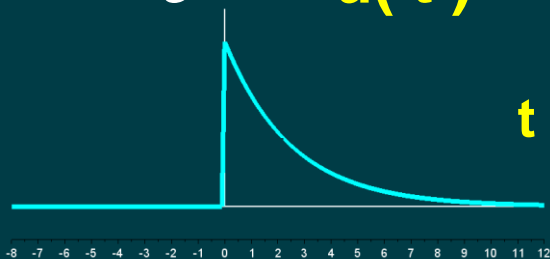
- 此積分結果不存在
- 沒有傅立葉轉換

指數函數的傅立葉轉換

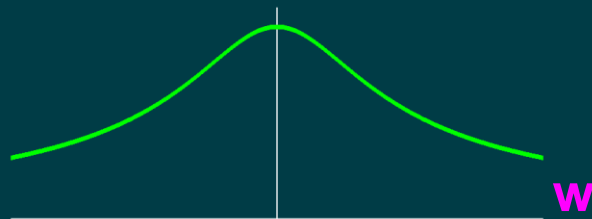
$$x(t) = e^{-a t} u(t)$$

$$a > 0$$

$$X(j\omega) = \frac{1}{(a + j\omega)}$$



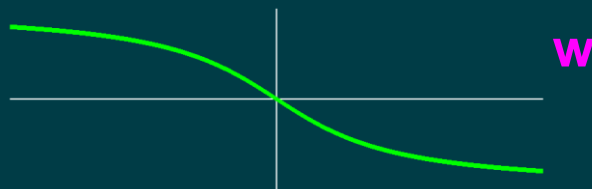
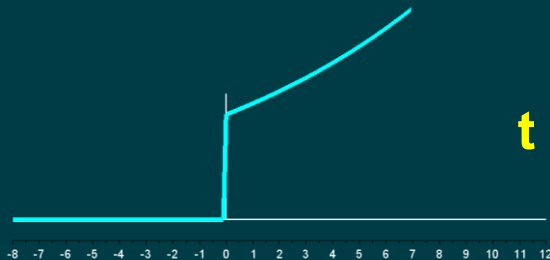
$$|X(j\omega)|$$



$$x(t) = e^{-a t} u(t)$$

$$a < 0$$

$$\nexists X(j\omega)$$



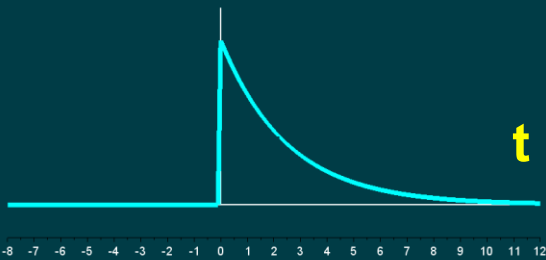
- 傅立葉轉換不存在

指數函數的傅立葉轉換

$$e^{-a t} u(t) \quad a > 0$$

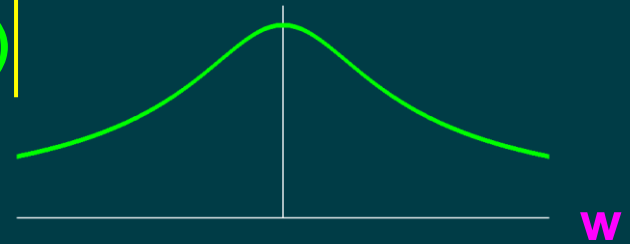
FT

$$\frac{1}{(a + j\omega)}$$

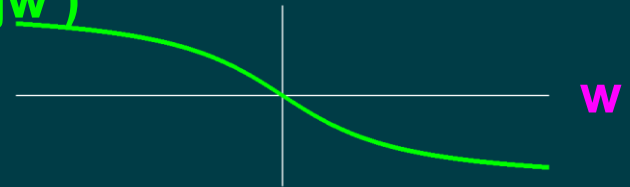


$$|X(j\omega)|$$

FT

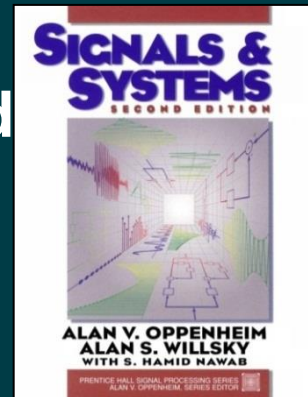


$$\angle X(j\omega)$$



參考文獻

- Alan V. Oppenheim, Alan S. Willsky, S. Hamid
Signals & Systems,
Prentice Hall, 2nd Edition, 1997



- **SciLab:**
Open source software for numerical computation
<http://www.scilab.org/>