從信號與系統到控制

單元：連續F轉換-1

從 週期信號 到 非週期信號 的 特性改變

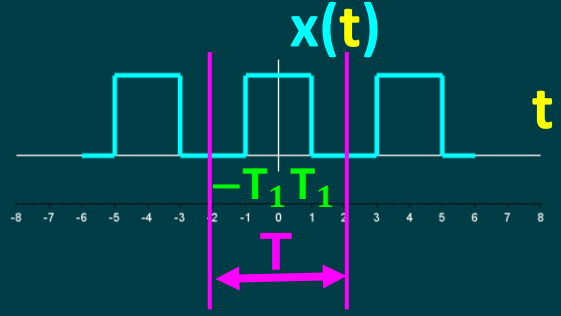
授課老師：連　豐　力

單元學習目標與大綱

- 複習 - 連續時間 週期方波 之 傅立葉級數
- 推演 - 不同週期的信號之 傅立葉級數差異
- 猜想 - 將 週期 變成 無限大 的 非週期信號 之 傅立葉級數

連續時間週期方波函數的傅立葉級數

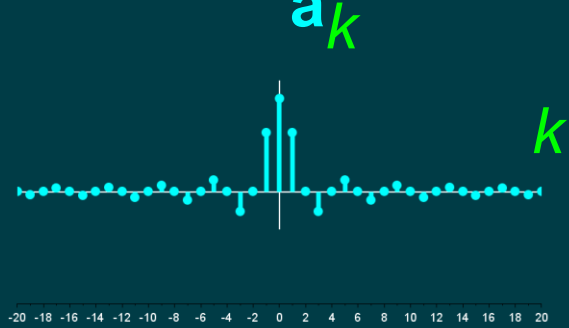
$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}$$



$$a_k = \frac{1}{T}\int_T x(t)\, e^{-j k w_0 t}\, dt$$

$$a_0 = \frac{2T_1}{T}$$

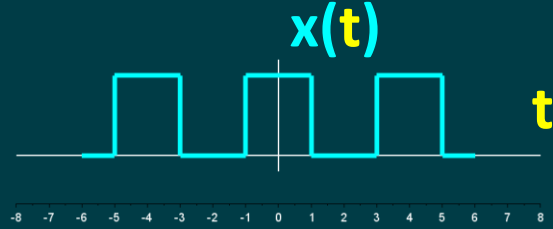
$$a_k = \frac{1}{k\pi}\sin\left(k\, 2\pi \frac{T_1}{T}\right)$$



連續時間週期方波函數的傅立葉級數

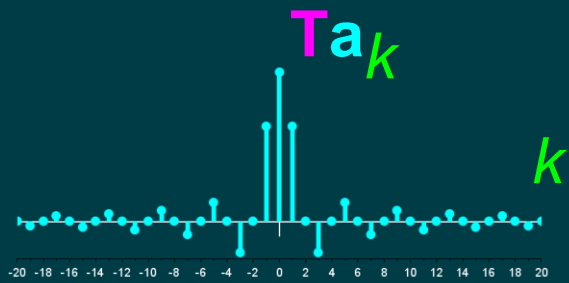
$$a_0 = \boxed{\frac{2T_1}{T}}$$

$$a_k = \boxed{\frac{1}{k\pi}\ \sin\left(k\ 2\pi\ \frac{T_1}{T}\right)}$$



$$T\,a_0 = T\,\frac{2T_1}{T} = 2\,T_1$$

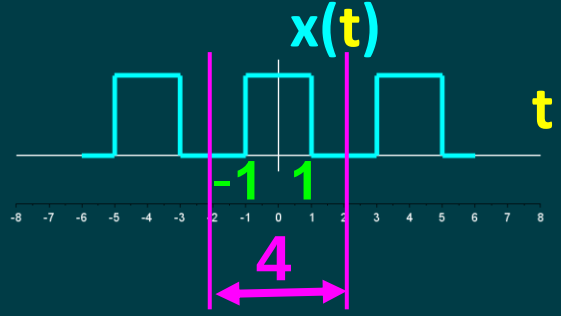
$$T\,a_k = \frac{T}{k\pi}\ \sin\left(k\ 2\pi\ \frac{T_1}{T}\right)$$



連續時間週期方波函數的傅立葉級數

$$T\,a_0 = 2\,T_1$$

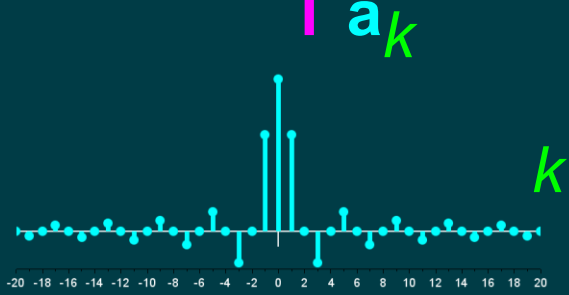
$$T\,a_k = \frac{T}{k\pi}\sin\left(k\ 2\pi\frac{T_1}{T}\right)$$



$$T = 4 \qquad T_1 = 1$$

$$T\,a_0 = 2$$

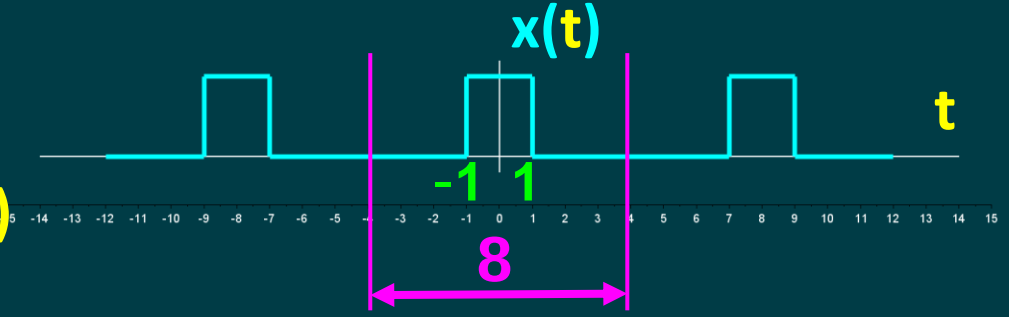
$$T\,a_k = \frac{4}{k\pi}\sin\left(k\ \frac{\pi}{2}\right)$$



連續時間週期方波函數的傅立葉級數

$$T\,a_0 = 2\,T_1$$

$$T\,a_k = \frac{T}{k\pi}\sin\left(k\,2\pi\frac{T_1}{T}\right)$$

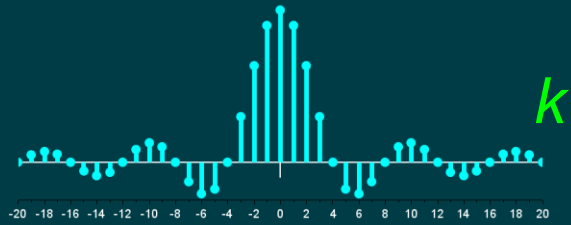


$T = 8 \qquad T_1 = 1$

$$T\,a_0 = 2$$

$$T\,a_k = \frac{8}{k\pi}\sin\left(k\,\frac{\pi}{4}\right)$$

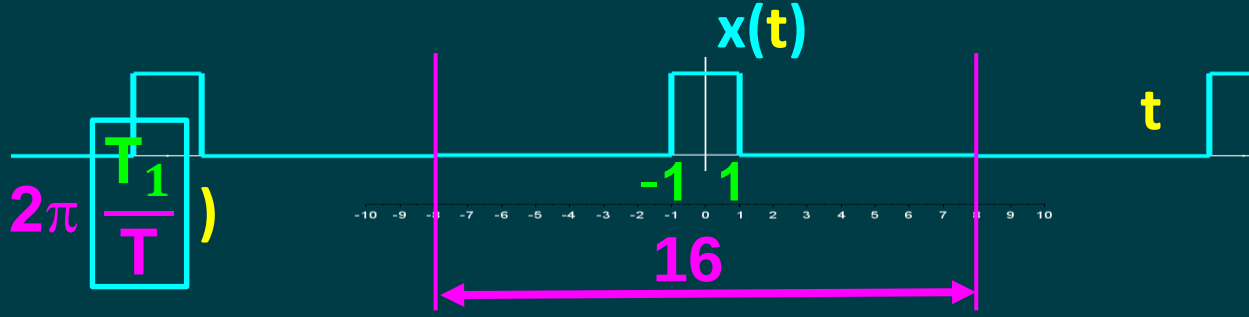
$T\,a_k$



連續時間週期方波函數的傅立葉級數

$$T\,a_0 = 2\,T_1$$

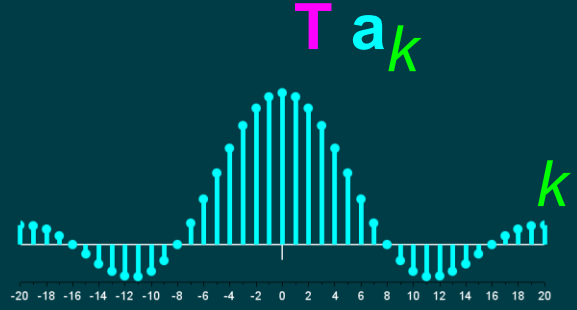
$$T\,a_k = \frac{T}{k\pi}\sin\left(k\,2\pi\frac{T_1}{T}\right)$$



$T = 16 \quad T_1 = 1$

$T\,a_0 = 2$

$$T\,a_k = \frac{16}{k\pi}\sin\left(k\,\frac{\pi}{8}\right)$$

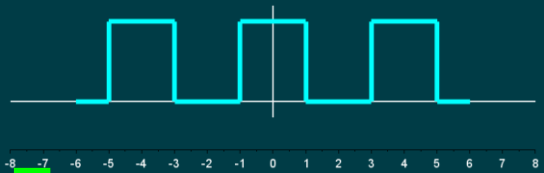


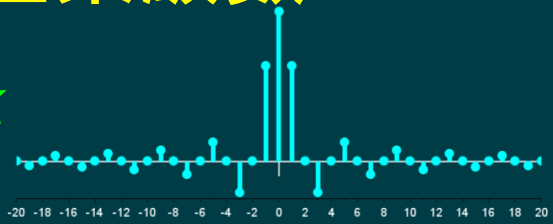
不同週期信號之傅立葉級數

$T = 4T_1$

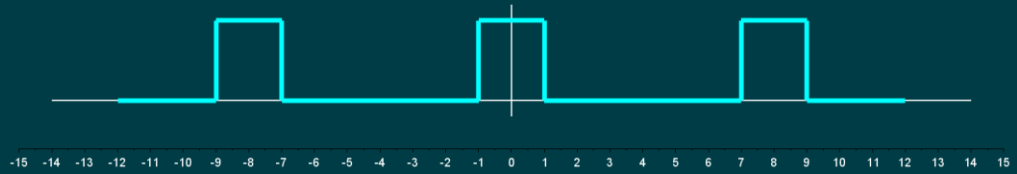
x(t)

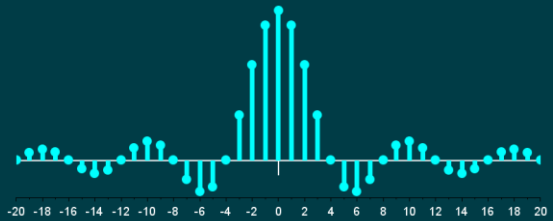
$T\,a_k$



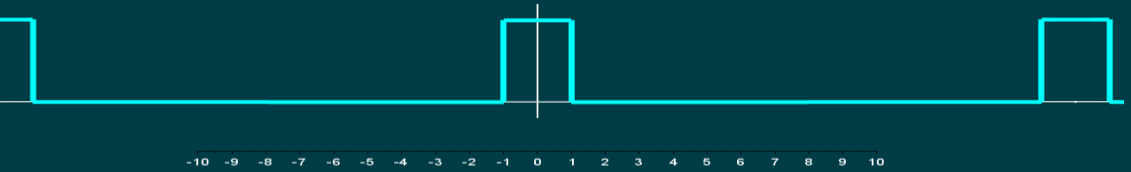


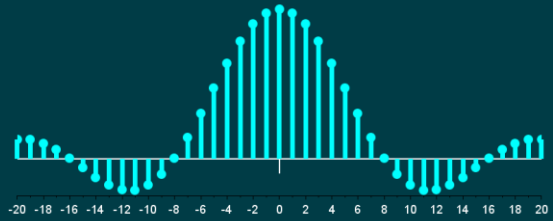
$T = 8T_1$





$T = 16T_1$





不同週期信號之傅立葉級數

$T = 4T_1$

$w_0 = \frac{2\pi}{T}$

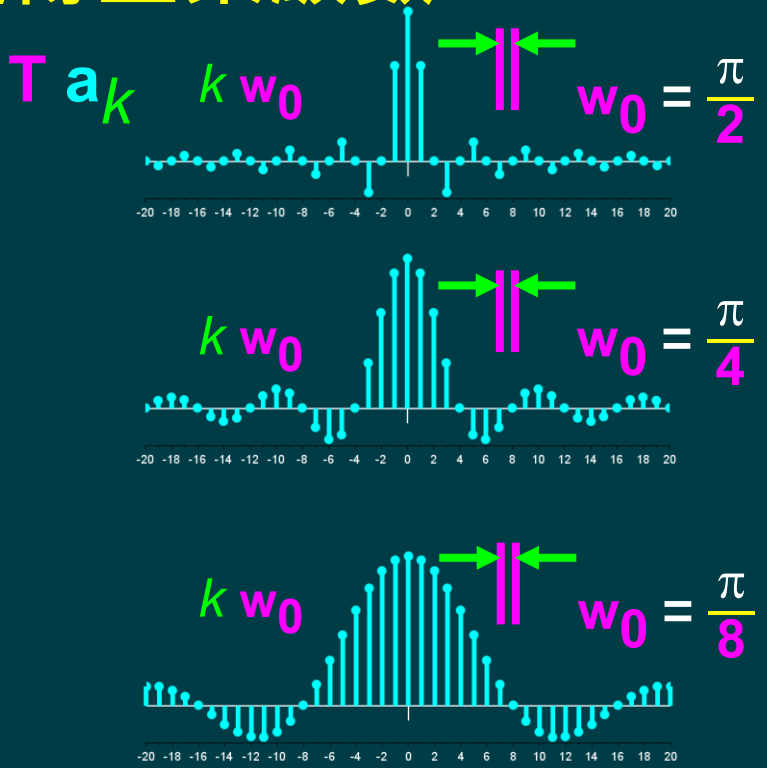
$w_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$T = 8T_1$

$w_0 = \frac{2\pi}{8} = \frac{\pi}{4}$

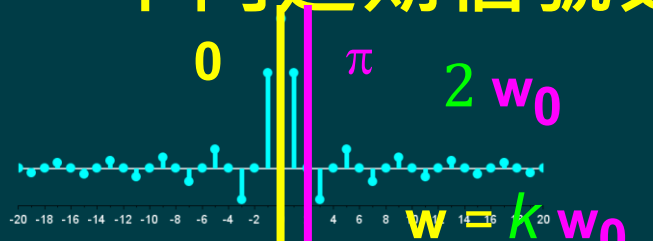
$T = 16T_1$

$w_0 = \frac{2\pi}{16} = \frac{\pi}{8}$

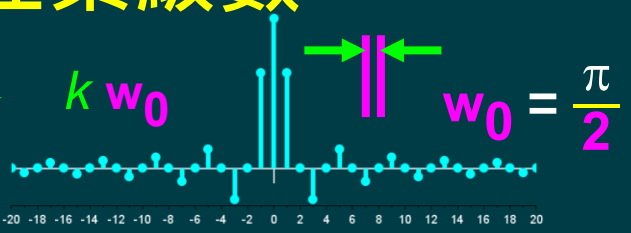


不同週期信號之傅立葉級數

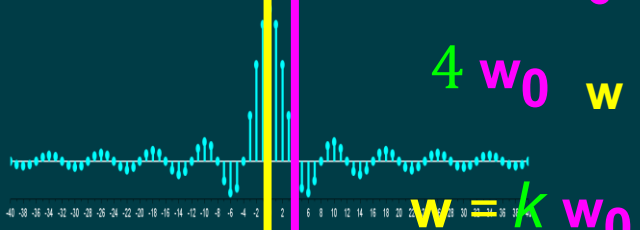
$T = 4\,T_1$

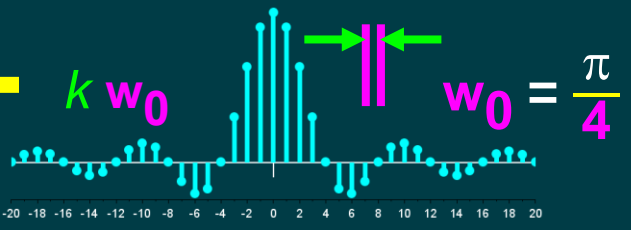


T a_k

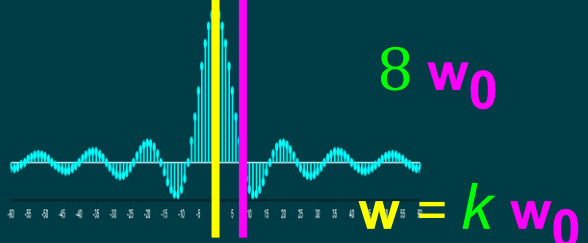


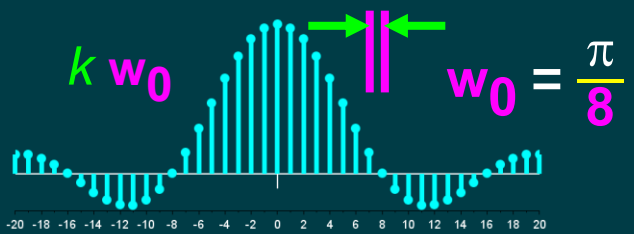
$T = 8\,T_1$





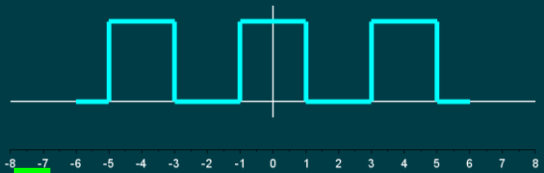
$T = 16\,T_1$



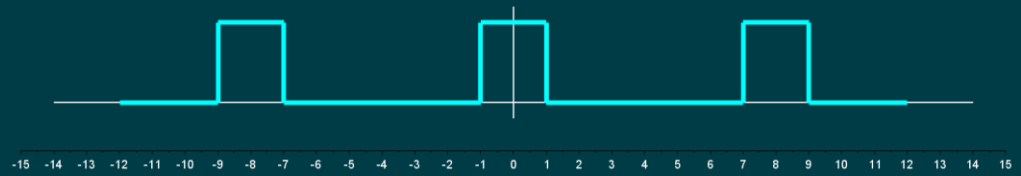


不同週期信號之傅立葉級數

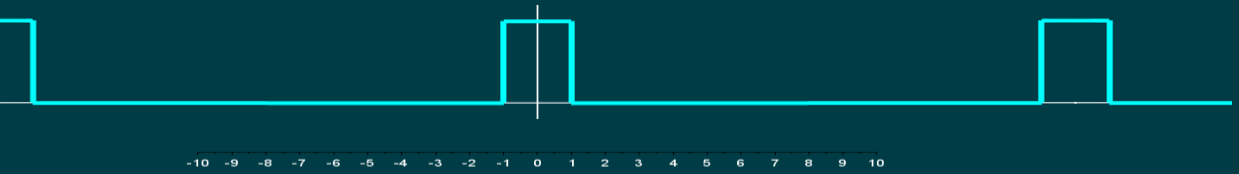
$T = 4\,T_1$



$T = 8\,T_1$

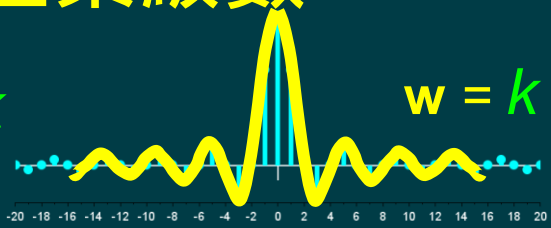


$T = 16\,T_1$

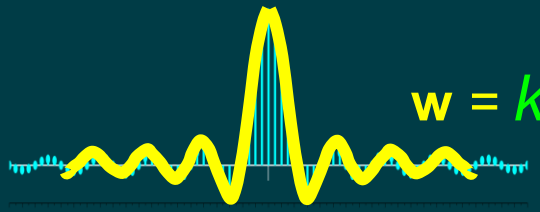


x(t)

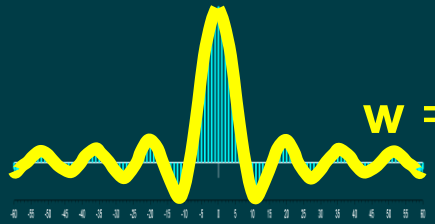
$T\,a_k$



$w = k\,w_0$



$w = k\,w_0$

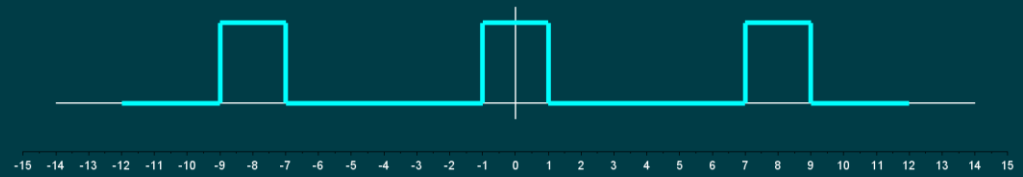


$w = k\,w_0$

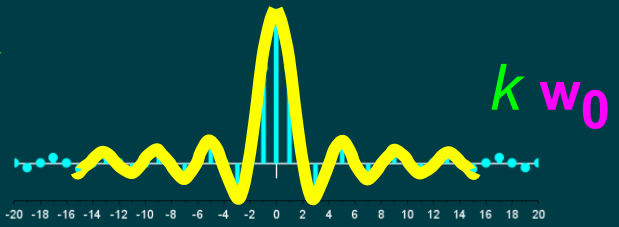
週期變成無限大的信號之傅立葉級數

$T = 8\,T_1$

x(t)



$T\,a_k$



$k\,w_0$

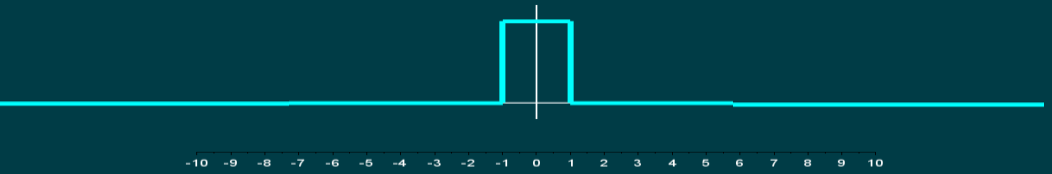
$T \to \infty$

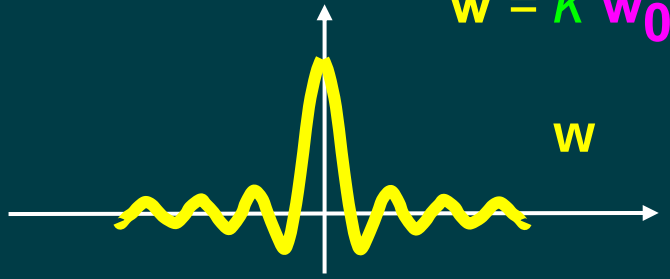
$w_0 = \frac{2\pi}{T}$

$w_0 \to 0$

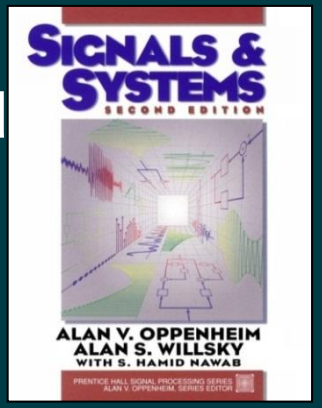
$w_0 \to 0$

$w = k\,w_0$





參考文獻



- Alan V. Oppenheim, Alan S. Willsky, S. Hamid
  Signals & Systems,
  Prentice Hall, 2nd Edition, 1997

- **SciLab**:
  Open source software for numerical computation
  http://www.scilab.org/