從信號與系統到控制

單元：DT-FS性質-6

利用 DT-FS 性質求得 DT-FS 係數

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單元學習目標與大綱

- 舉一個 方波信號 的例子，說明：
- 如何利用 DT-FS 性質 求得 DT-FS 係數

單元學習目標與大綱

- 舉一個 方波信號 的例子，說明：
- 如何利用 DT-FS 性質 求得 DT-FS 係數
- 也就是，不需要進行複雜的公式計算

離散時間週期方波函數

$$w_0 = \frac{2\pi}{N}$$

g[n]

$$g[n] \overset{FS}{\longleftrightarrow} c_k$$

$$c_k = \frac{1}{N} \sum_{n=\langle N \rangle} g[n]\, e^{-jkw_0 n}$$

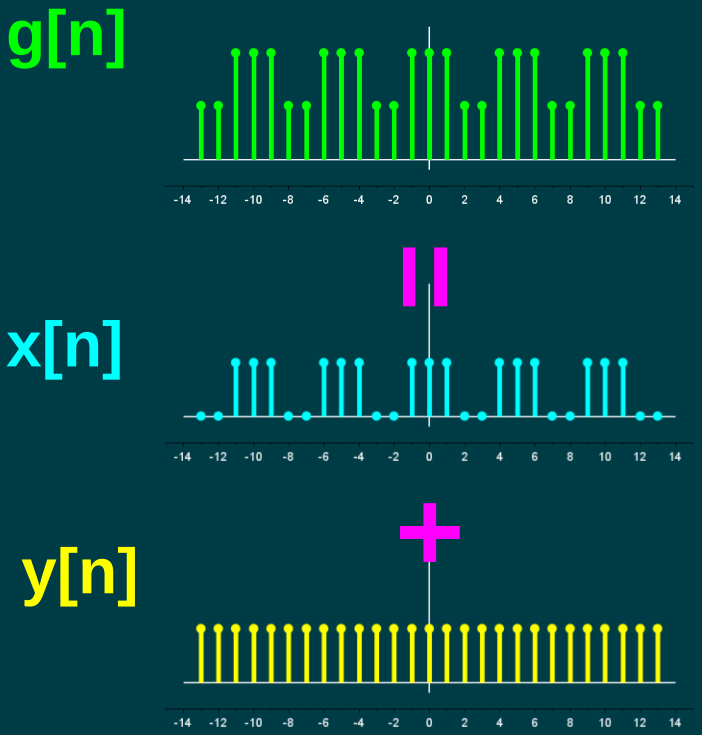
x[n]

$$a_k = \frac{1}{N}(2N_1+1) \qquad k = 0, \pm N, \pm 2N, \ldots$$

$$= \frac{1}{N}\frac{\sin(kw_0(N_1+1/2))}{\sin(kw_0/2)} \qquad k \neq 0, \pm N, \pm 2N, \ldots$$

離散時間週期方波函數

$$w_0 = \frac{2\pi}{N}$$

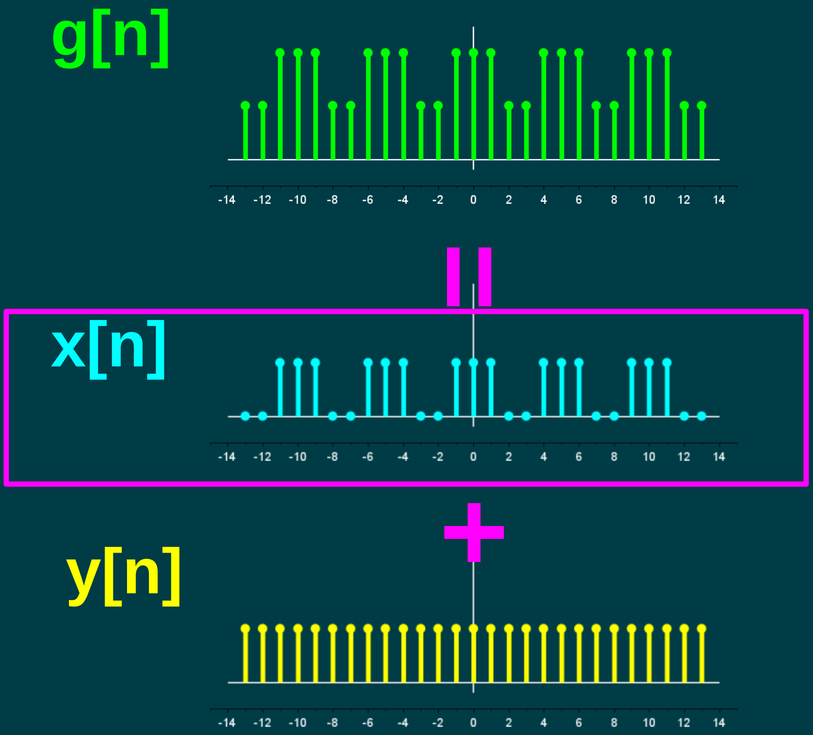


$$g[n] \xleftrightarrow{\text{FS}} c_k$$

$$g[n] = x[n] + y[n]$$

$$c_k = a_k + b_k$$

離散時間週期方波函數



$$w_0 = \frac{2\pi}{N}$$

$$a_k = \frac{1}{\boxed{N}}\left(2\boxed{N_1}+1\right) \qquad k = 0, \pm N, \pm 2N, \ldots$$

$$= \frac{1}{\boxed{N}} \frac{\sin\left(k\boxed{w_0}\left(\boxed{N_1}+1/2\right)\right)}{\sin\left(k\boxed{w_0}/2\right)} \qquad k \neq 0, \pm N, \pm 2N, \ldots$$

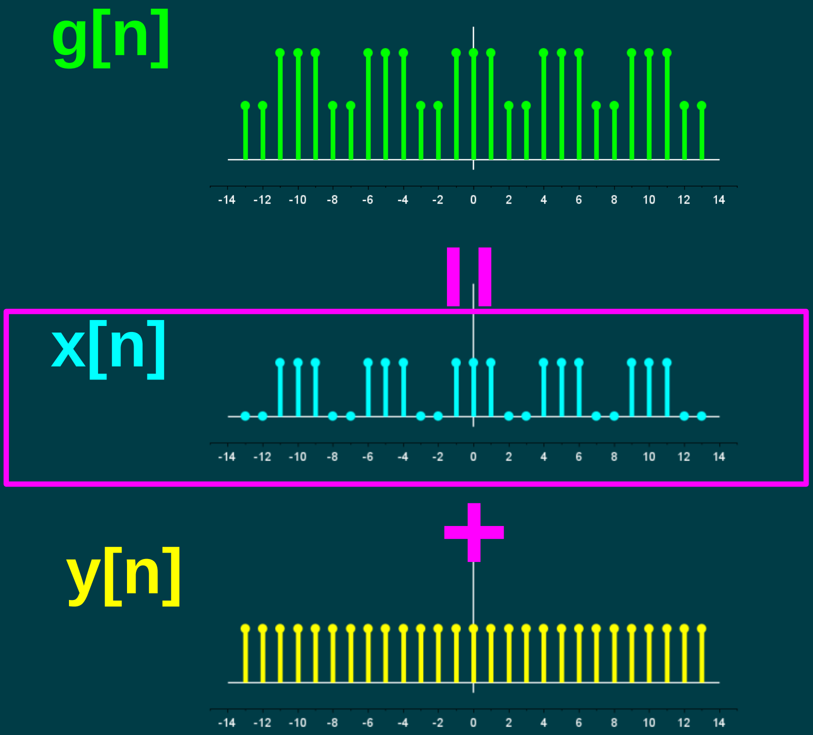
$$N = 5,\ N_1 = 1$$

$$a_k = \frac{1}{5}\left(2 \bullet 1 + 1\right) \qquad k = 0, \pm 5, \ldots$$

$$= \frac{1}{5} \frac{\sin\left(k\frac{2\pi}{5}\left(1+1/2\right)\right)}{\sin\left(k\frac{2\pi}{5}/2\right)} \qquad k \neq 0, \pm 5, \ldots$$

離散時間週期方波函數

$$w_0 = \frac{2\pi}{N}$$



$$a_k = \frac{1}{N}(2N_1+1) \qquad k = 0, \pm N, \pm 2N, \ldots$$

$$= \frac{1}{N}\,\frac{\sin(kw_0(N_1+1/2))}{\sin(kw_0/2)} \qquad k \neq 0, \pm N, \pm 2N, \ldots$$

$$N = 5,\ N_1 = 1$$

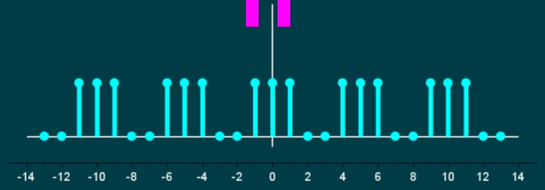
$$a_k = \frac{1}{5}(3) \qquad k = 0, \pm 5, \ldots$$

$$= \frac{1}{5}\,\frac{\sin(k\frac{3\pi}{5})}{\sin(k\frac{1\pi}{5})} \qquad k \neq 0, \pm 5, \ldots$$

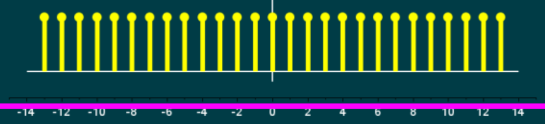
離散時間週期方波函數

g[n]

x[n]



y[n]



$$\boxed{w_0 = \frac{2\pi}{N}}$$

$$a_k = \boxed{\frac{1}{N}(2N_1+1) \qquad k = 0, \pm N, \pm 2N, \ldots}$$

$$= \boxed{\frac{1}{N}\frac{\sin(kw_0(N_1+1/2))}{\sin(kw_0/2)} \qquad k \neq 0, \pm N, \pm 2N, \ldots}$$

$$N = 5,\ N_1 = 2$$

$$b_k = \frac{1}{5}(2 \cdot 2 + 1) \qquad k = 0, \pm 5, \ldots$$

$$= \frac{1}{5}\frac{\sin\left(k\frac{2\pi}{5}(2+1/2)\right)}{\sin\left(k\frac{2\pi}{5}/2\right)} \qquad k \neq 0, \pm 5, \ldots$$

離散時間週期方波函數

$$w_0 = \frac{2\pi}{N}$$

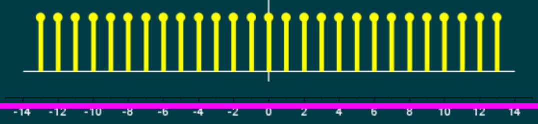
g[n]

=

x[n]

+

y[n]



$$a_k = \frac{1}{N}(2N_1 + 1) \qquad k = 0, \pm N, \pm 2N, \ldots$$

$$= \frac{1}{N}\,\frac{\sin(k w_0 (N_1 + 1/2))}{\sin(k w_0 /2)} \qquad k \neq 0, \pm N, \pm 2N, \ldots$$

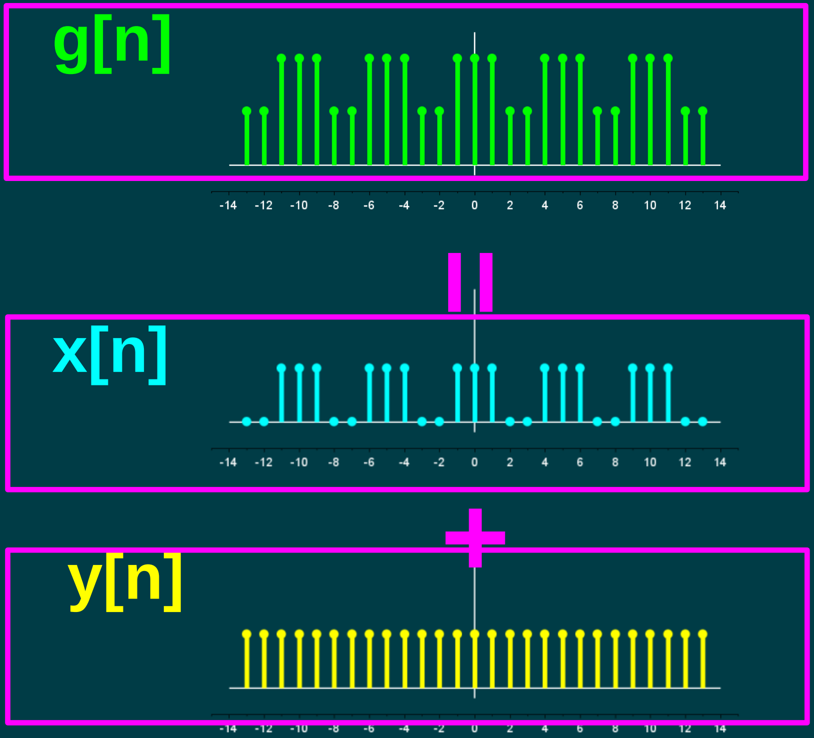
$$N = 5,\ N_1 = 2$$

$$b_k = \frac{1}{5}(5) = 1 \qquad k = 0, \pm 5, \ldots$$

$$= \frac{1}{5}\,\frac{\sin(k\frac{5\pi}{5})}{\sin(k\frac{1\pi}{5})} = 0 \qquad k \neq 0, \pm 5, \ldots$$

離散時間週期方波函數

$$w_0 = \frac{2\pi}{N}$$



$$a_k = \frac{1}{5}(3) \qquad k = 0, \pm 5, \dots$$

$$= \frac{1}{5}\,\frac{\sin\left(k\frac{3\pi}{5}\right)}{\sin\left(k\frac{1\pi}{5}\right)} \qquad k \neq 0, \pm 5, \dots$$

$$b_k = 1 \qquad k = 0, \pm 5, \dots$$

$$= 0 \qquad k \neq 0, \pm 5, \dots$$

$$c_k = \frac{3}{5} + 1 = \frac{8}{5} \qquad k = 0, \pm 5, \dots$$

$$= \frac{1}{5}\,\frac{\sin\left(k\frac{3\pi}{5}\right)}{\sin\left(k\frac{1\pi}{5}\right)} \qquad k \neq 0, \pm 5, \dots$$

離散傅立葉級數性質

$$g[n] \xleftrightarrow{FS} c_k$$

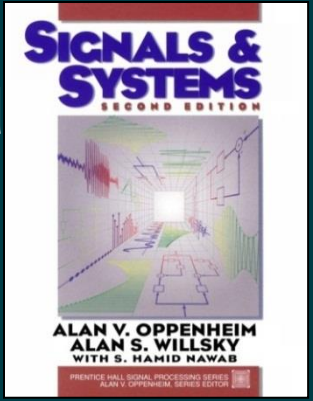
$$x[n] \xleftrightarrow{FS} a_k$$

$$y[n] \xleftrightarrow{FS} b_k$$

$$g[n] = x[n] + y[n]$$

$$c_k = a_k + b_k$$

參考文獻



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  Prentice Hall, 2nd Edition, 1997

- **SciLab:**
  Open source software for numerical computation
  http://www.scilab.org/