從信號與系統到控制

單元：離散F級數-2

離散時間 三角函數 的 傅立葉級數 - 直覺

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單元學習目標與大綱

- 討論 離散時間 三角函數 的 傅立葉級數
- 直接比較 函數 與 傅立葉級數的係數

離散時間三角函數

$$x[n] = 1 + \sin\left(\frac{2\pi}{N} n\right) + 3\cos\left(\frac{2\pi}{N} n\right) + \cos\left(2\frac{2\pi}{N} n + \frac{\pi}{2}\right)$$

$$\cos(s) = \frac{1}{2}\left(e^{js} + e^{-js}\right)$$

$$\sin(s) = \frac{1}{2j}\left(e^{js} - e^{-js}\right)$$

$$= 1$$

$$+ \frac{1}{2j}\left(e^{j\frac{2\pi}{N}n} - e^{-j\frac{2\pi}{N}n}\right)$$

$$+ 3\,\frac{1}{2}\left(e^{j\frac{2\pi}{N}n} + e^{-j\frac{2\pi}{N}n}\right)$$

$$+ \frac{1}{2}\left(e^{j\left(2\frac{2\pi}{N}n + \frac{\pi}{2}\right)} + e^{-j\left(2\frac{2\pi}{N}n + \frac{\pi}{2}\right)}\right)$$

離散時間三角函數

$$x[n] = 1 + \frac{1}{2j}\left(e^{j\frac{2\pi}{N}n} - e^{-j\frac{2\pi}{N}n}\right) + \frac{3}{2}\left(e^{j\frac{2\pi}{N}n} + e^{-j\frac{2\pi}{N}n}\right)$$

$$+ \frac{1}{2}\left(e^{j(2\frac{2\pi}{N}n + \frac{\pi}{2})} + e^{-j(2\frac{2\pi}{N}n + \frac{\pi}{2})}\right)$$

$$= 1 + \left(\frac{3}{2} + \frac{1}{2j}\right) e^{j\frac{2\pi}{N}n} + \left(\frac{3}{2} - \frac{1}{2j}\right) e^{-j\frac{2\pi}{N}n}$$

$$+ \left(\frac{1}{2} e^{j\frac{\pi}{2}}\right) e^{j2\frac{2\pi}{N}n} + \left(\frac{1}{2} e^{-j\frac{\pi}{2}}\right) e^{-j2\frac{2\pi}{N}n}$$

離散時間三角函數

$$x[n] = \boxed{1} + \boxed{\left(\frac{3}{2} + \frac{1}{2j}\right)} e^{j\frac{2\pi}{N}n} + \boxed{\left(\frac{3}{2} - \frac{1}{2j}\right)} e^{-j\frac{2\pi}{N}n}$$

$$+ \boxed{\left(\frac{1}{2} e^{j\frac{\pi}{2}}\right)} e^{j2\frac{2\pi}{N}n} + \boxed{\left(\frac{1}{2} e^{-j\frac{\pi}{2}}\right)} e^{-j2\frac{2\pi}{N}n}$$

$$= a_0 + a_1 e^{j\frac{2\pi}{N}n} + a_{-1} e^{-j\frac{2\pi}{N}n}$$

$$+ a_2 e^{j2\frac{2\pi}{N}n} + a_{-2} e^{-j2\frac{2\pi}{N}n}$$

離散時間三角函數

$$e^{js} = \cos(s) + j\sin(s)$$

$$a_0 = 1$$

$$a_1 = \left(\frac{3}{2} + \frac{1}{2j}\right) = \frac{3}{2} - \frac{1}{2}j$$

$$a_{-1} = \left(\frac{3}{2} - \frac{1}{2j}\right) = \frac{3}{2} + \frac{1}{2}j$$

$$a_2 = \left(\frac{1}{2}e^{j\frac{\pi}{2}}\right) = \frac{1}{2}\left[\cos\left(\frac{\pi}{2}\right) + j\sin\left(\frac{\pi}{2}\right)\right] = \frac{1}{2}j$$

$$a_{-2} = \left(\frac{1}{2}e^{-j\frac{\pi}{2}}\right) = \frac{1}{2}\left[\cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right)\right] = -\frac{1}{2}j$$

離散時間三角函數

N = 10

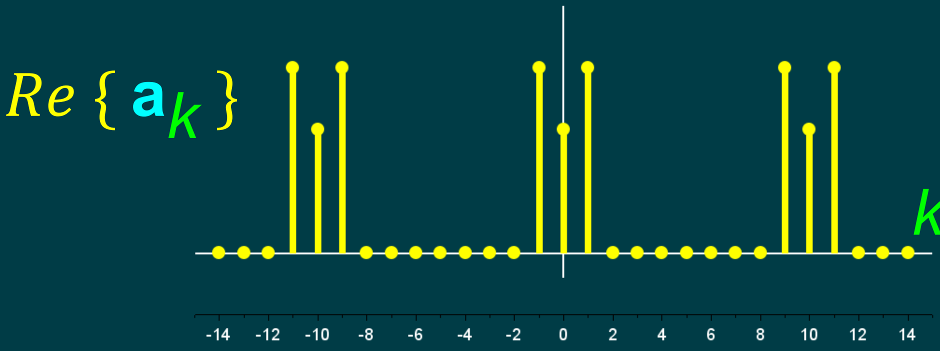
$$a_0 = 1$$

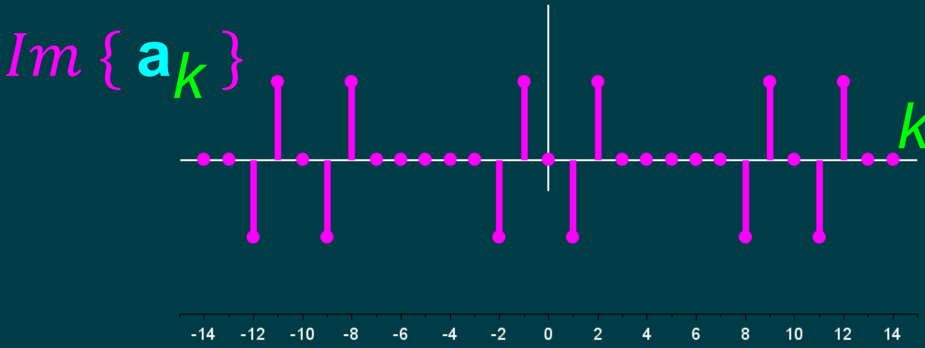
$$a_1 = \frac{3}{2} - \frac{1}{2}j$$

$$a_{-1} = \frac{3}{2} + \frac{1}{2}j$$

$$a_2 = \frac{1}{2}j$$

$$a_{-2} = -\frac{1}{2}j$$





離散時間三角函數

N = 10

$$\mathrm{Re} + j\,\mathrm{Im} = \mathrm{Mag}\; e^{j\,\mathrm{Ang}}$$

$$a_0 = \boxed{1}$$

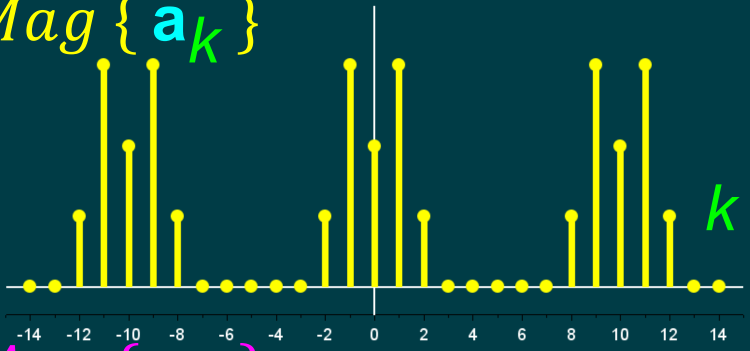
$$a_1 = \frac{3}{2} - \frac{1}{2}j = 1.58\, e^{j(-0.32)}$$

$$a_{-1} = \frac{3}{2} + \frac{1}{2}j = 1.58\, e^{j(0.32)}$$

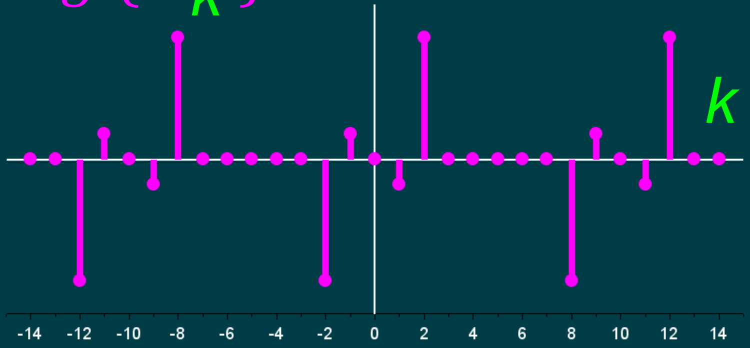
$$a_2 = \frac{1}{2}j = 0.50\, e^{j(1.57)}$$

$$a_{-2} = -\frac{1}{2}j = 0.50\, e^{j(-1.57)}$$

$Mag\{a_k\}$



$Ang\{a_k\}$

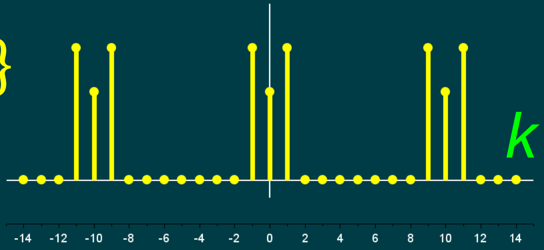


離散時間三角函數

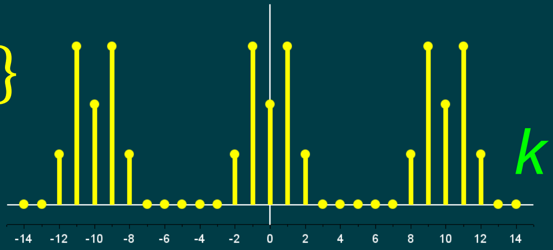
N = 10

$$x[n] = 1 + \sin\left(\frac{2\pi}{N} n\right) + 3\cos\left(\frac{2\pi}{N} n\right) + \cos\left(2\frac{2\pi}{N} n + \frac{\pi}{2}\right)$$

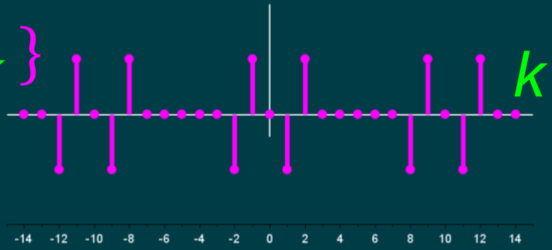
$Re\{a_k\}$



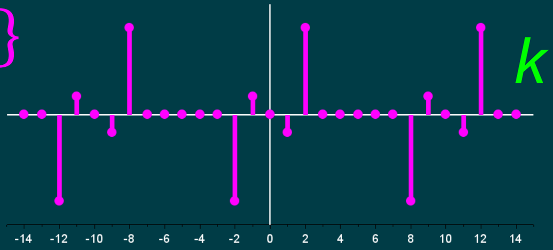
$Mag\{a_k\}$



$Im\{a_k\}$



$Ang\{a_k\}$



參考文獻

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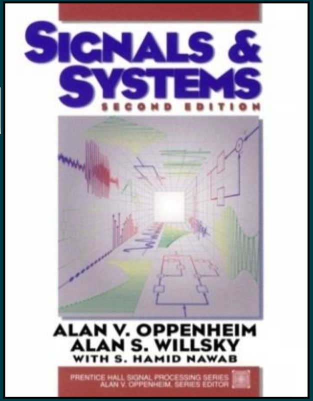
  Prentice Hall, 2nd Edition, 1997



- SciLab:
  Open source software for numerical computation
  http://www.scilab.org/