從信號與系統到控制

單元：CT-FS性質-9

範例 - 利用 CT-FS 性質求得 CT-FS 係數

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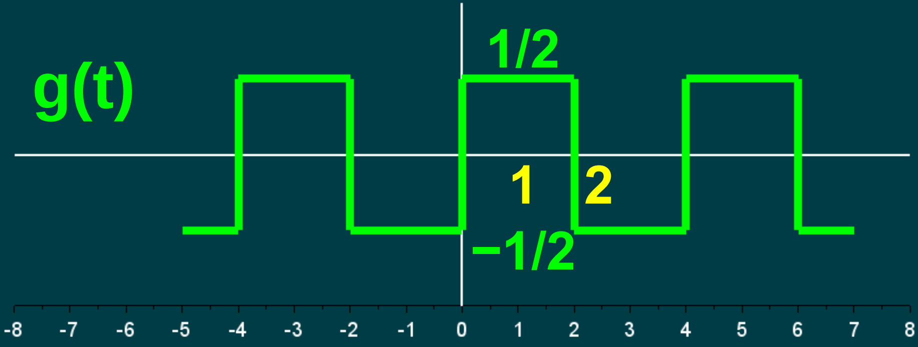
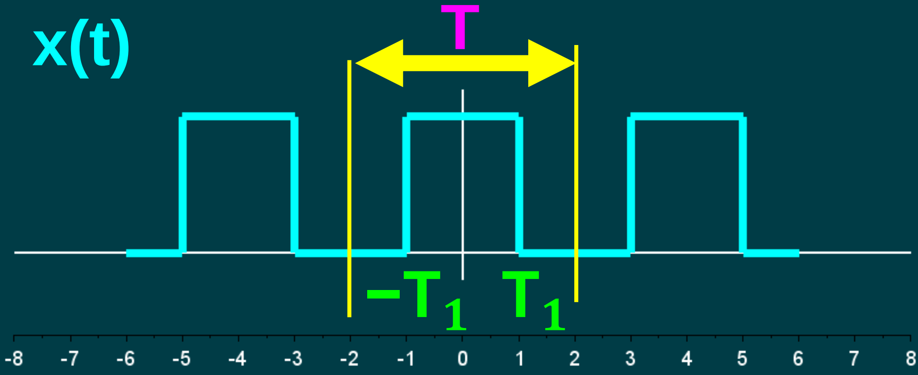
單元學習目標與大綱

- 舉一些例子，說明：
- 如何利用 CT-FS 性質求得 CT-FS 係數
- 偏移與平移的 方波信號 以及 三角波信號

連續時間週期方波函數





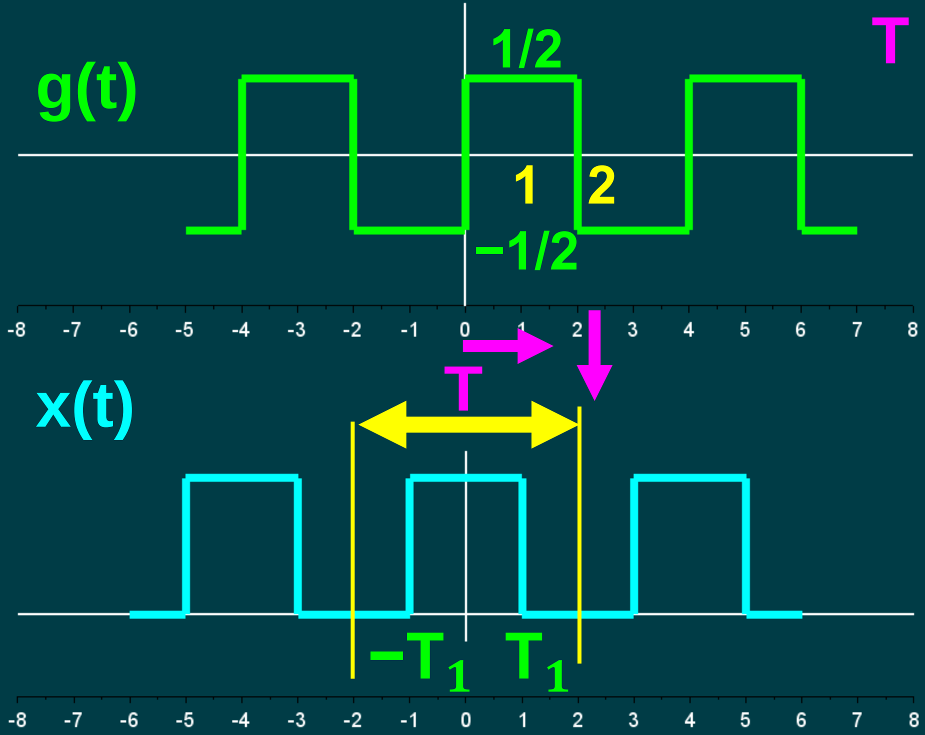
$$g(t) \xleftrightarrow{\text{FS}} g_k$$

$$g_k = \frac{1}{T}\int_T g(t)\, e^{-jkw_0 t}\, dt$$

$$\begin{cases} a_0 = \dfrac{2T_1}{T} \\ a_k = \dfrac{1}{k\pi}\sin\left(k\, 2\pi \dfrac{T_1}{T}\right) \end{cases}$$

連續時間週期方波函數

$$w_0 = \frac{2\pi}{4}$$



$T = 4 \quad T_1 = 1$

$$g(t) = x(t-1) - \frac{1}{2}$$

$$x(t) \xleftrightarrow{FS} a_k$$

$$x(t-1) \xleftrightarrow{FS} a_k\, e^{jkw_0(-1)}$$

$$a_k\, e^{-jk\left(\frac{\pi}{2}\right)}$$

連續時間週期方波函數

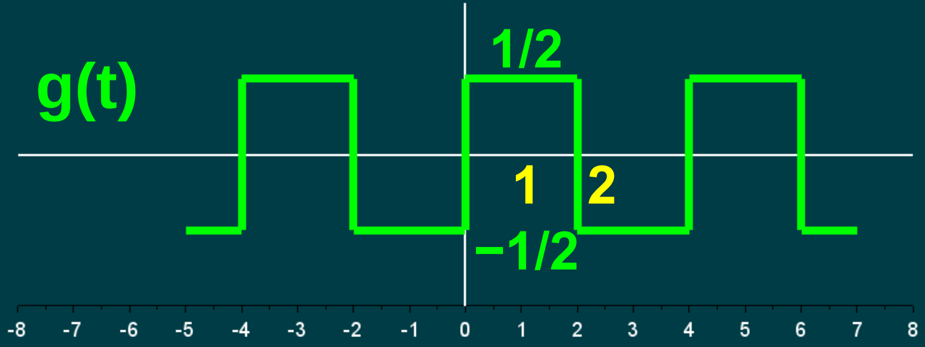
$$g(t) \overset{FS}{\longleftrightarrow} g_k$$

$$= x(t-1) - \frac{1}{2} \qquad = \begin{cases} \boxed{a_0} - \dfrac{1}{2} & k = 0 \\ \boxed{a_k}\, e^{-jk\left(\frac{\pi}{2}\right)} & k \neq 0 \end{cases}$$

$$T_1 = 1$$
$$T = 4$$

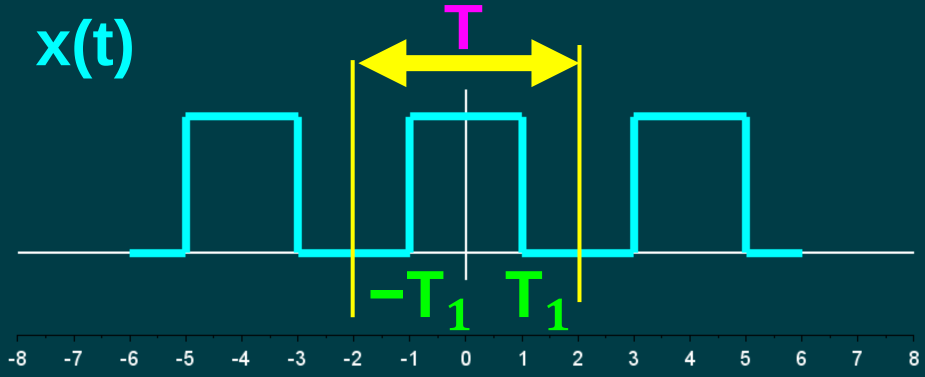
$$\begin{cases} \boxed{a_0 = \dfrac{2T_1}{T}} \\ \boxed{a_k = \dfrac{1}{k\pi} \sin\left(k\, 2\pi \dfrac{T_1}{T}\right)} \end{cases} \qquad = \begin{cases} \dfrac{1}{2} - \dfrac{1}{2} = 0 & k = 0 \\ \dfrac{\sin(k\pi/2)}{k\pi}\, e^{-j\frac{k\pi}{2}} & k \neq 0 \end{cases}$$

連續時間週期方波函數



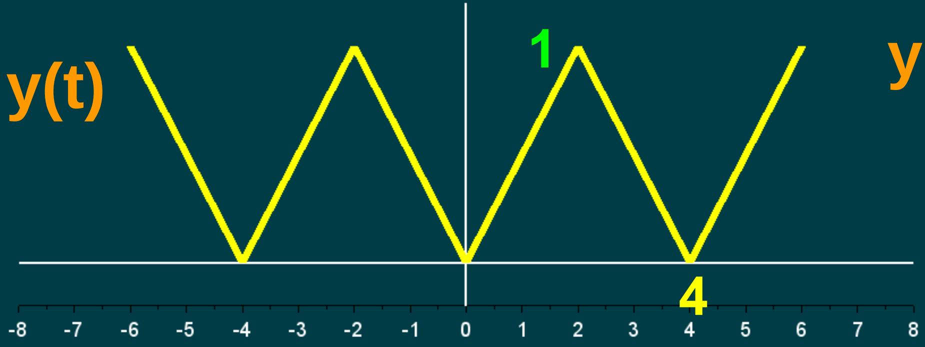
$$\begin{cases} \frac{1}{2}-\frac{1}{2}=0 & k=0 \\ \frac{\sin(k\pi/2)}{k\pi}\, e^{-j\frac{k\pi}{2}} & k\neq 0 \end{cases}$$

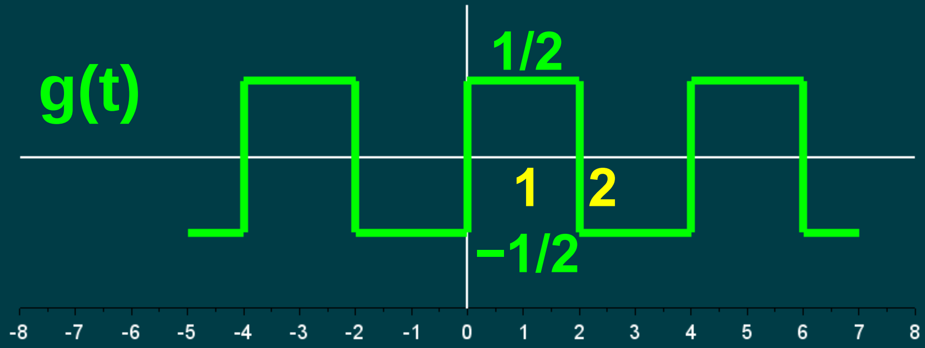
$$g(t) = x(t-1) - \frac{1}{2}$$



$$\begin{cases} a_0 = \frac{2T_1}{T} \\ a_k = \frac{1}{k\pi}\sin\left(k\,2\pi\,\frac{T_1}{T}\right) \end{cases}$$

連續時間週期三角波函數





$$y(t) \overset{FS}{\longleftrightarrow} b_k$$

$$b_k = \frac{1}{T}\int_T y(t)\, e^{-jkw_0 t}\, dt$$

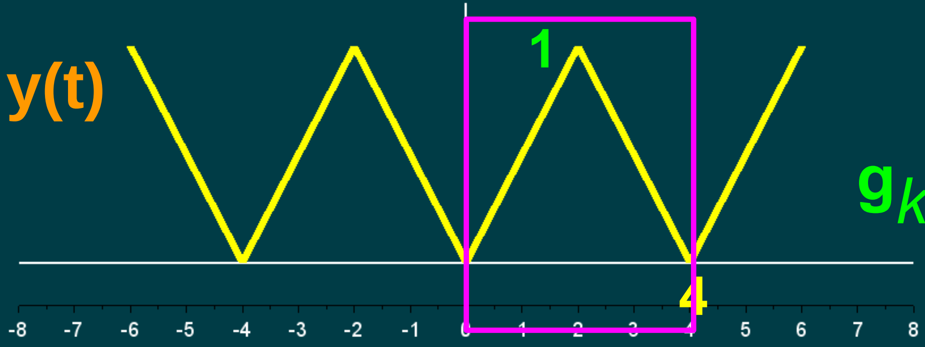
$$\frac{d}{dt} y(t) = g(t)$$

$$jkw_0 \; b_k = g_k$$

$$b_k = g_k \;/\; jkw_0$$

連續時間週期三角波函數

$$w_0 = \frac{2\pi}{4}$$



$$g_k = \begin{cases} \frac{1}{2} - \frac{1}{2} = 0 & k = 0 \\ \boxed{\frac{\sin(k\pi/2)}{k\pi} e^{-j\frac{k\pi}{2}}} & k \neq 0 \end{cases}$$

$$b_k = \frac{g_k}{j\,k\,w_0} = \frac{g_k}{j\,k\,\pi/2}$$

$$= \boxed{\frac{2\,\boxed{g_k}}{j\,k\,\pi}}$$

$$= \begin{cases} \frac{1}{4}\,\frac{4*1}{2} = \frac{1}{2} & k = 0 \\ \frac{2\sin(k\pi/2)}{j\,(k\pi)^2}\, e^{-j\frac{k\pi}{2}} & k \neq 0 \end{cases}$$

利用 CT-FS 性質求得 CT-FS 係數

$$g(t) \xleftrightarrow{FS} g_k$$

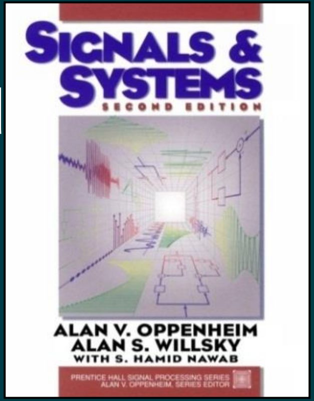
$$= x(t-1) - \frac{1}{2} \qquad = \begin{cases} a_0 - \frac{1}{2} & k = 0 \\ a_k\, e^{-jk\left(\frac{\pi}{2}\right)} & k \neq 0 \end{cases}$$

$$g(t) \xleftrightarrow{FS} g_k = b_k \; jkw_0$$

$$= \frac{d}{dt} y(t) \qquad b_k = g_k \;/\; jkw_0$$

參考文獻

- Alan V. Oppenheim, Alan S. Willsky, S. Hamid
  Signals & Systems,
  Prentice Hall, 2nd Edition, 1997



- SciLab:
  Open source software for numerical computation
  http://www.scilab.org/