從信號與系統到控制

單元：連續F級數-4

連續時間 週期方波 的 傅立葉級數

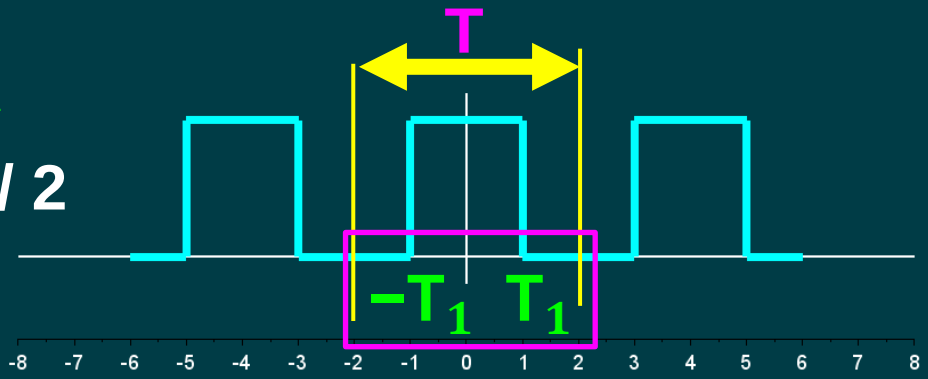
授課老師：連　豐　力

單元學習目標與大綱

- 討論 連續時間 週期性 方波函數
  的傅立葉級數

連續時間週期方波函數

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}$$



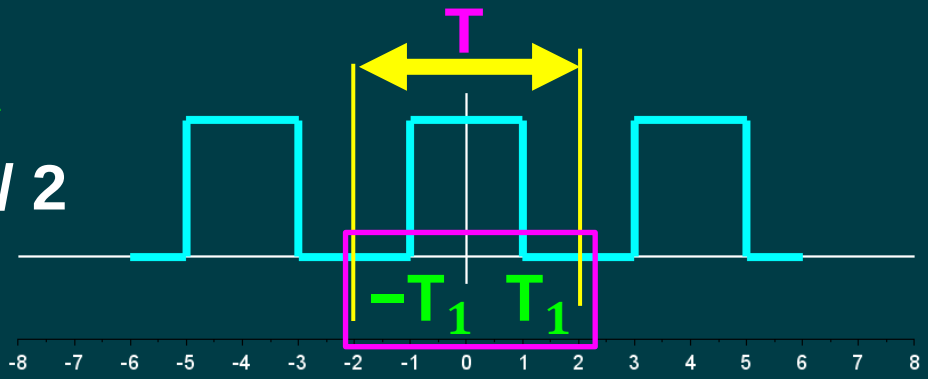
$$a_k = \frac{1}{T}\int_T x(t)\, e^{-j k w_0 t}\, dt$$

$k = 0$

$$a_0 = \frac{1}{T}\int_T x(t)\, e^{-j\,0\,w_0 t}\, dt = \frac{1}{T}\int_{-T_1}^{T_1} 1\ dt = \frac{2T_1}{T}$$

連續時間週期方波函數

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}$$



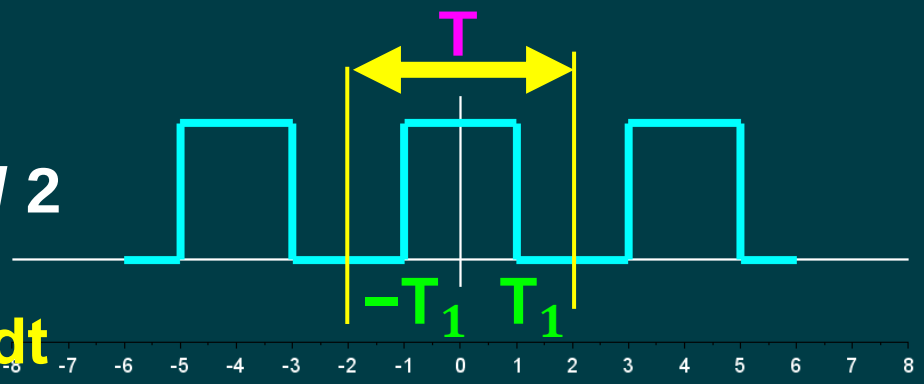
$$a_k = \frac{1}{T}\int_T x(t)\, e^{-j k w_0 t}\, dt$$

$k \neq 0$

$$a_k = \frac{1}{T}\int_T x(t)\, e^{-j k w_0 t}\, dt = \frac{1}{T}\int_{-T_1}^{T_1} e^{-j k w_0 t}\, dt$$

連續時間週期方波函數

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}$$



$$a_k = \frac{1}{T}\int_{-T_1}^{T_1} e^{-jk w_0 t}\,dt$$

$$= \frac{1}{T}\,\frac{1}{-jk w_0}\, e^{-jk w_0 t}\Big|_{-T_1}^{T_1}$$

$$= \frac{1}{jk w_0 T}\left[\, e^{jk w_0 T_1} - e^{-jk w_0 T_1} \,\right]$$

$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk w_0 t}\,dt$$

連續時間週期方波函數

$$a_k = \frac{1}{j\,k\,w_0 T} \cdot \frac{2j}{2j} \left[e^{jkw_0T_1} - e^{-jkw_0T_1} \right]$$

$$= \frac{2j}{j\,k\,w_0 T} \sin(k\,w_0\,T_1)$$

$$= \frac{2}{k\,w_0 T} \sin(k\,w_0\,T_1)$$

$$= \frac{2}{k\frac{2\pi}{T}T} \sin(k\,w_0\,T_1)$$

$$\sin(s) = \frac{1}{2j} (e^{js} - e^{-js})$$

$$w_0 = \frac{2\pi}{T}$$

連續時間週期方波函數

$$a_k = \frac{2}{k\frac{2\pi}{T}T} \sin(k w_0 T_1)$$

$$= \frac{1}{k\pi} \sin(k w_0 T_1)$$

$$= \frac{1}{k\pi} \sin(k \frac{2\pi}{T} T_1)$$

$$= \frac{1}{k\pi} \sin(k \ 2\pi \boxed{\frac{T_1}{T}})$$

$$w_0 = \frac{2\pi}{T}$$

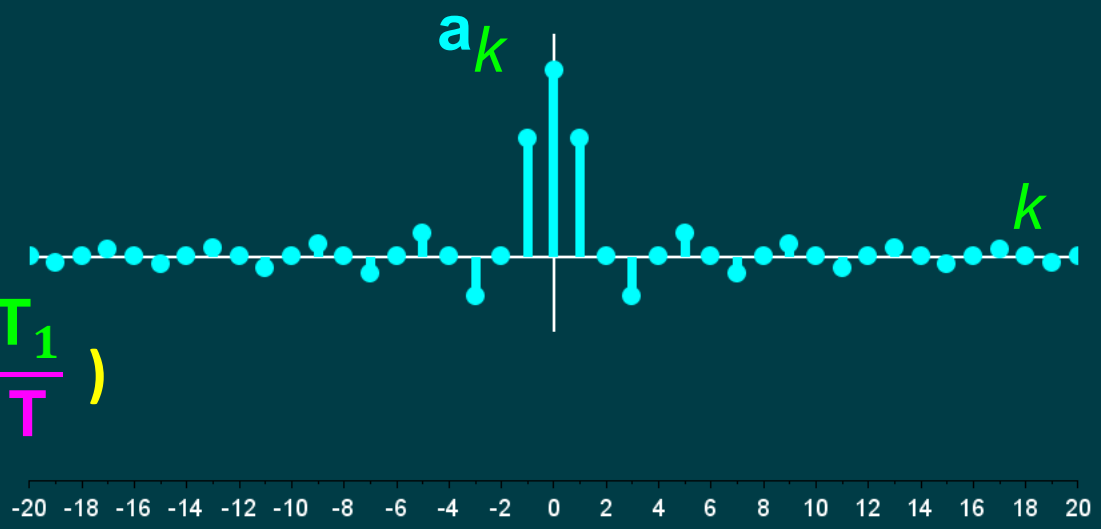
連續時間週期方波函數

$$a_0 = \frac{2T_1}{T}$$

$$a_k = \frac{1}{k\pi} \sin\left(k \, 2\pi \frac{T_1}{T}\right)$$

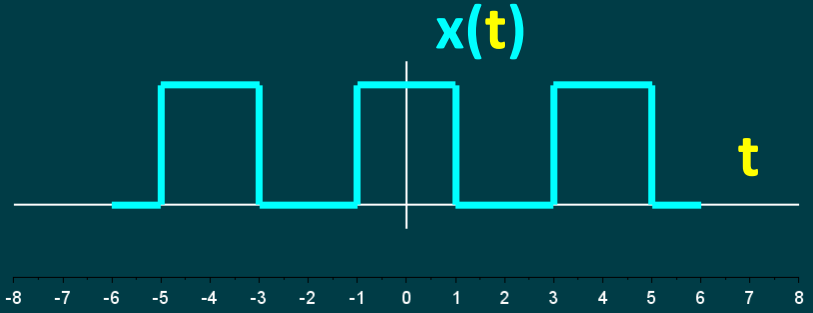
$T = 4$

$T_1 = 1$



連續時間週期方波函數

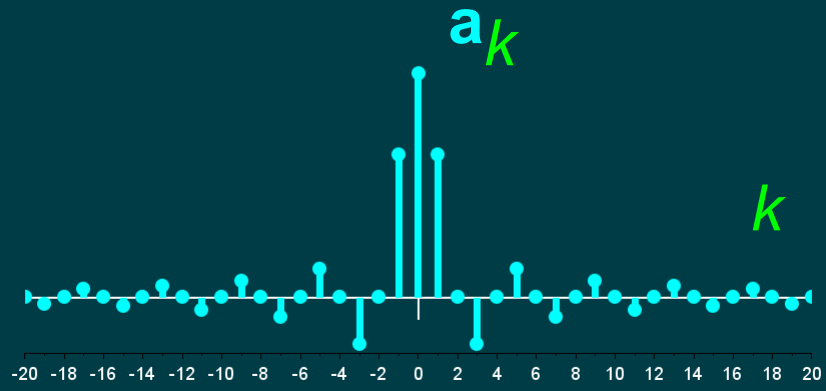
$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}$$



$$a_k = \frac{1}{T}\int_T x(t)\, e^{-j k w_0 t}\, dt$$

$$a_0 = \frac{2T_1}{T}$$

$$a_k = \frac{1}{k\pi}\sin\left(k\ 2\pi\ \frac{T_1}{T}\right)$$



參考文獻

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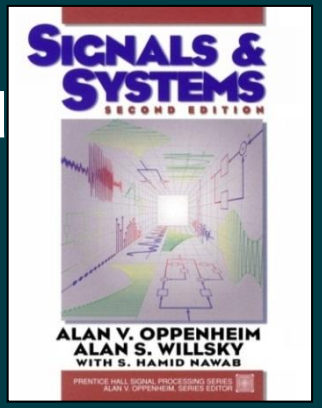

- **SciLab**:
  Open source software for numerical computation
  http://www.scilab.org/