從信號與系統到控制

單元：連續F級數-3

連續時間 三角函數 的 傅立葉級數 - 公式

授課老師：連　豐　力

單元學習目標與大綱

- 討論 連續時間 三角函數 的 傅立葉級數
- 利用公式 求得 傅立葉級數 的係數

連續時間三角函數

$$x(t) = 1 + \sin(w_0 t) + 2\cos(w_0 t) + \cos\left(2w_0 t + \frac{\pi}{4}\right)$$

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jkw_0 t}$$

$$a_k = \frac{1}{T}\int_T \boxed{x(t)}\, e^{-jkw_0 t}\, dt$$

$$a_k = \frac{1}{T}\int_T \left(1+\sin(w_0 t)+2\cos(w_0 t)+\cos\left(2w_0 t+\frac{\pi}{4}\right)\right) e^{-jkw_0 t} dt$$

連續時間三角函數

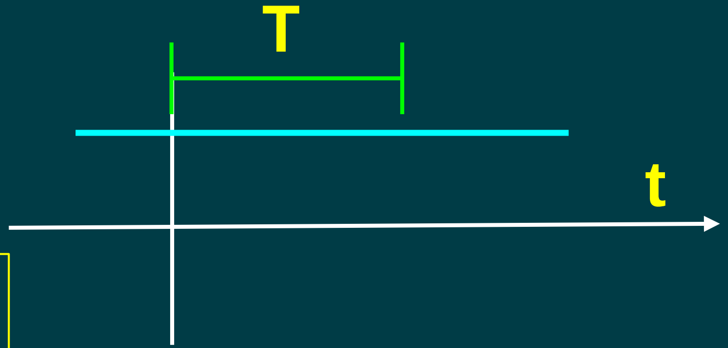
$$a_k = \frac{1}{T}\int_T \left(1+\sin(w_0 t)+2\cos(w_0 t)+\cos\left(2w_0 t+\frac{\pi}{4}\right)\right) e^{-\boxed{jk}w_0 t}\,dt$$

$$k = 0$$

$$a_0 = \frac{1}{T}\int_T \left(\boxed{1}+\boxed{\sin(w_0 t)}+\boxed{2\cos(w_0 t)}+\boxed{\cos\left(2w_0 t+\frac{\pi}{4}\right)}\right)\ 1\ dt$$

三角函數一個週期的積分

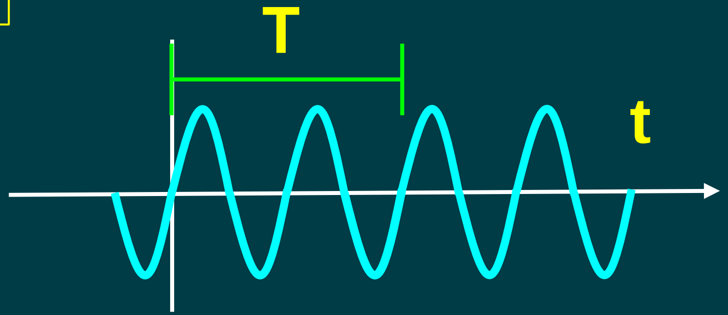
$$\int_T 1 \; dt = T$$



$$\boxed{w_0 = \frac{2\pi}{T}}$$

$$\int_T \sin(k\, w_0\, t) \; dt = 0$$

$$\int_T \cos(k\, w_0\, t) \; dt = 0$$



連續時間三角函數

$e^{js} = \cos(s) + j\sin(s)$

$$a_k = \frac{1}{T}\int_T \left(1+\sin(w_0t)+2\cos(w_0t)+\cos\left(2w_0t+\frac{\pi}{4}\right)\right)e^{-jkw_0t}dt$$

$$k = 0$$

$$a_0 = \frac{1}{T}\int_T \left(\boxed{1}+\boxed{\sin(w_0t)}+\boxed{2\cos(w_0t)}+\boxed{\cos\left(2w_0t+\frac{\pi}{4}\right)}\right)\ 1\ dt$$

$$= \frac{1}{T}\ T = 1$$

連續時間三角函數

$$e^{js} = \cos(s) + j\sin(s)$$

$$a_k = \frac{1}{T}\int_T \left(1+\sin(w_0 t)+2\cos(w_0 t)+\cos\left(2w_0 t+\frac{\pi}{4}\right)\right)e^{-\boxed{jk}w_0 t}dt$$

$$k = 1$$

$$a_1 = \frac{1}{T}\int_T \left(\boxed{1}+\boxed{\sin(w_0 t)}+\boxed{2\cos(w_0 t)}+\boxed{\cos\left(2w_0 t+\frac{\pi}{4}\right)}\right)\boxed{e^{-j1w_0 t}}dt$$

三角函數與指數函數一個週期的積分

$$\int_T e^{jk w_0 t}\, dt = 0$$

$$\int_T \cos(\boxed{m} w_0 t)\, e^{j\boxed{n} w_0 t}\, dt = \frac{1}{2} T \qquad m = n$$

$$= 0 \qquad m \neq n$$

$$\int_T \sin(\boxed{m} w_0 t)\, e^{j\boxed{n} w_0 t}\, dt = j\, \frac{1}{2} T \qquad m = n$$

$$= 0 \qquad m \neq n$$

連續時間三角函數

$$e^{js} = \cos(s) + j\sin(s)$$

$$a_k = \frac{1}{T}\int_T \left(1+\sin(w_0t)+2\cos(w_0t)+\cos(2w_0t+\frac{\pi}{4})\right)e^{-jkw_0t}dt$$

$$k = 1$$

$$a_1 = \frac{1}{T}\int_T \left(\boxed{1}+\boxed{\sin(w_0t)}+\boxed{2\cos(w_0t)}+\boxed{\cos(2w_0t+\frac{\pi}{4})}\right)\boxed{e^{-j1w_0t}}dt$$

$$= \frac{1}{T}\left(2\,\frac{1}{2}\,T - j\,\frac{1}{2}\,T\right) \qquad = 1 - j\frac{1}{2}$$

連續時間三角函數

$$e^{js} = \cos(s) + j\sin(s)$$

$$a_k = \frac{1}{T}\int_T \left(1+\sin(w_0 t)+2\cos(w_0 t)+\cos\left(2w_0 t+\frac{\pi}{4}\right)\right)e^{-j\boxed{k}w_0 t}dt$$

$$k = 2$$

$$a_2 = \frac{1}{T}\int_T \left(\boxed{1}+\boxed{\sin(w_0 t)}+\boxed{2\cos(w_0 t)}+\boxed{\cos\left(2w_0 t+\frac{\pi}{4}\right)}\right)\boxed{e^{-j2w_0 t}}dt$$

三角函數與指數函數一個週期的積分

$$\int_T e^{jk\omega_0 t}\,dt = 0$$

$$\int_T \cos(m\omega_0 t)\, e^{jn\omega_0 t}\,dt = \frac{1}{2}T \qquad m = n$$

$$= 0 \qquad m \neq n$$

$$\int_T \sin(m\omega_0 t)\, e^{jn\omega_0 t}\,dt = j\frac{1}{2}T \qquad m = n$$

$$= 0 \qquad m \neq n$$

連續時間三角函數

$$e^{js} = \cos(s) + j\sin(s)$$

$$a_k = \frac{1}{T}\int_T \left(1+\sin(w_0t)+2\cos(w_0t)+\cos(2w_0t+\frac{\pi}{4})\right)e^{-jkw_0t}dt$$

$$k = 2$$

$$a_2 = \frac{1}{T}\int_T \left(\boxed{1}+\boxed{\sin(w_0t)}+\boxed{2\cos(w_0t)}+\boxed{\boxed{\cos(2w_0t+\frac{\pi}{4})}}\right)\boxed{e^{\boxed{-j2w_0t}}}dt$$

$$= \frac{1}{T}\ \frac{1}{2}T\ \ e^{j\frac{\pi}{4}} = \frac{1}{2}\left[\cos(\frac{\pi}{4}) + j\sin(\frac{\pi}{4})\ \right] = \frac{\sqrt{2}}{4} + j\frac{\sqrt{2}}{4}$$

連續時間三角函數

$a_0 = 1$

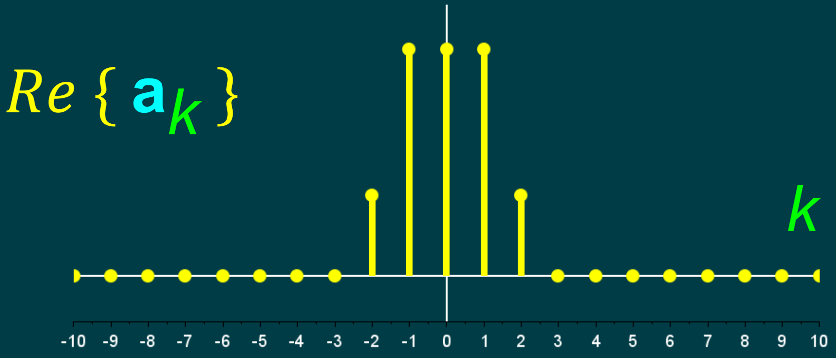
$a_1 = 1 - j\frac{1}{2}$

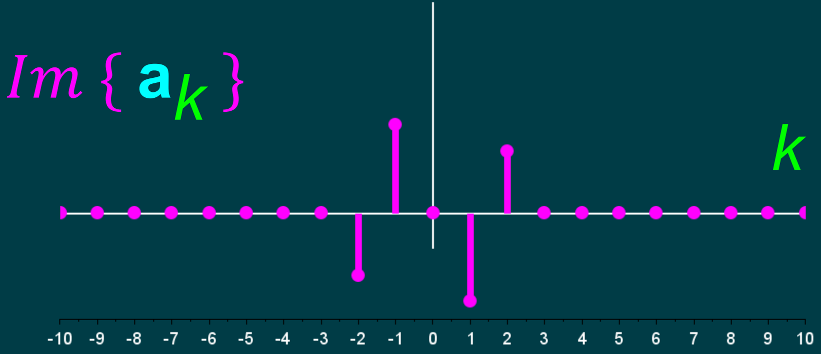
$a_2 = \frac{\sqrt{2}}{4}(1 + j)$

$a_{-1} = 1 + \frac{1}{2}j$

$a_{-2} = \frac{\sqrt{2}}{4}(1 - j)$

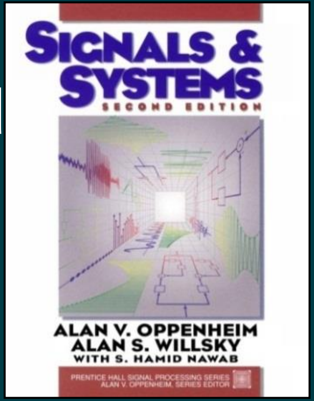
$k \neq 0, \pm 1, \pm 2 \quad a_k = 0$





參考文獻

- Alan V. Oppenheim, Alan S. Willsky, S. Hamid
  Signals & Systems,
  Prentice Hall, 2nd Edition, 1997



- SciLab:
  Open source software for numerical computation
  http://www.scilab.org/