

從信號與系統到控制

單元：連續摺積-2

連續脈衝響應與摺積計算

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單元學習目標與大綱

- 線性 非時變 系統 – 基本定義
- 連續 脈衝響應 – 輸入輸出操作
- 連續 摺積計算 – 摺積積分公式推導

線性非時變系統 (LTI system)

- 線性 (Linear)

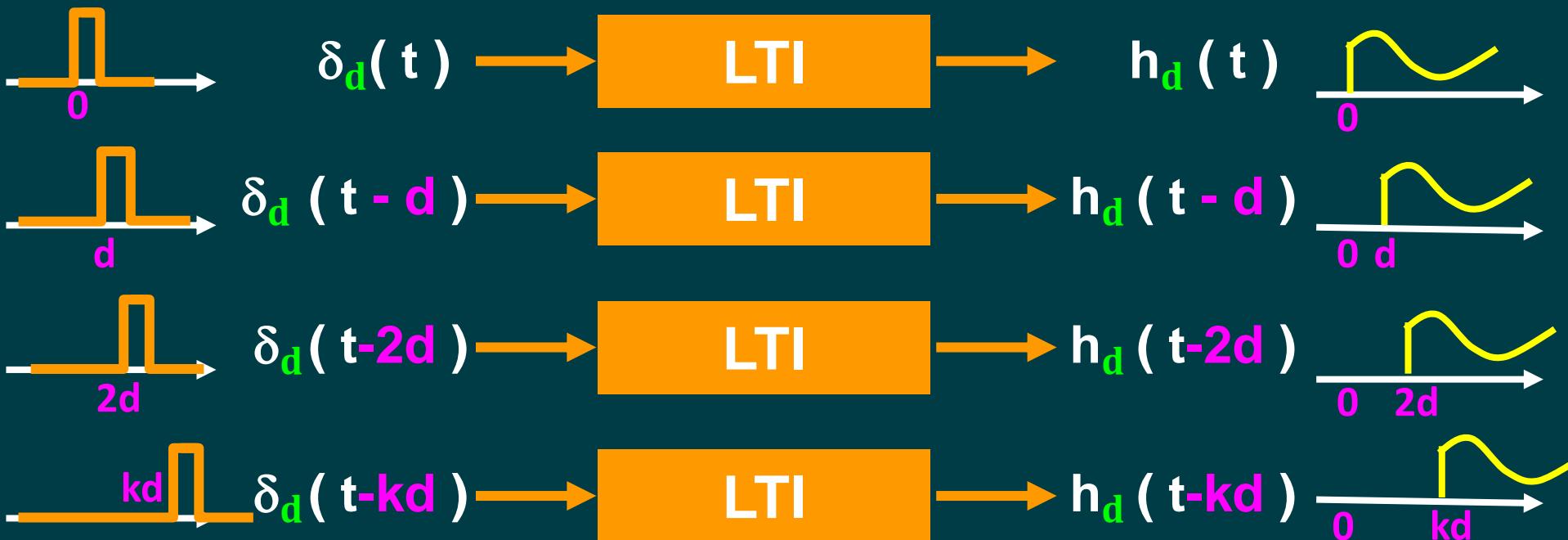


線性非時變系統 (LTI system)

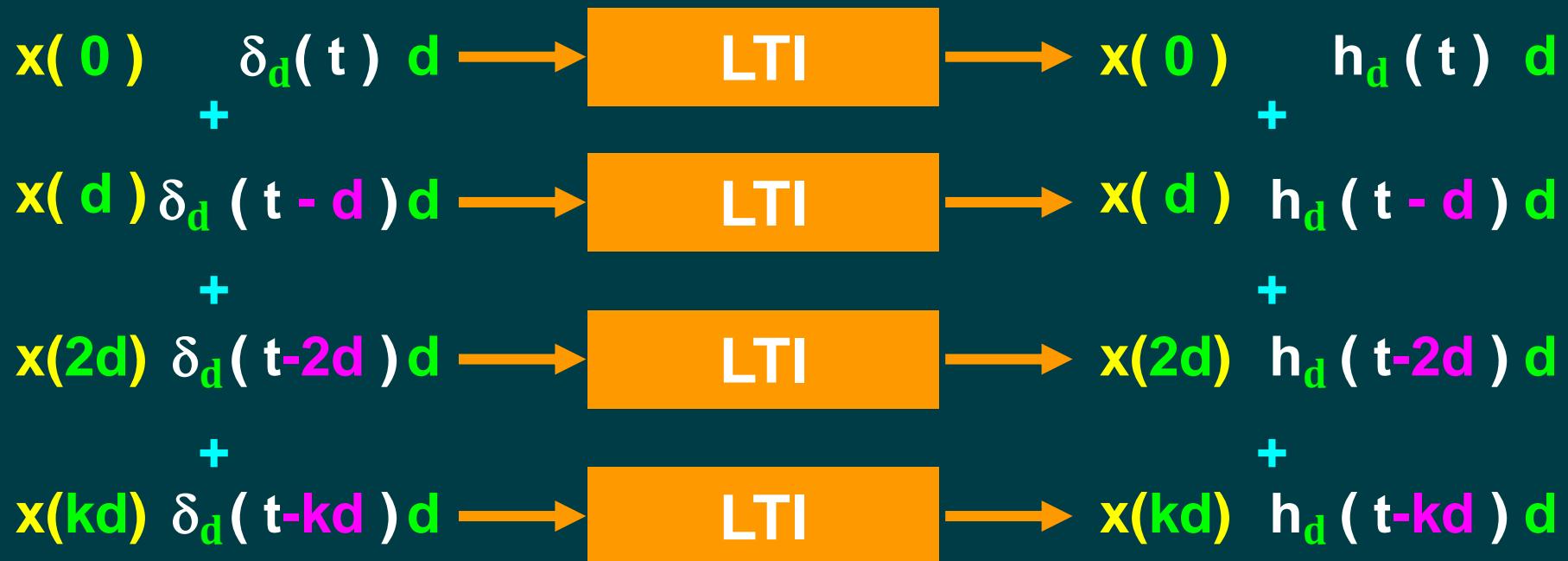
- 非時變 (Time-Invariant)



連續脈衝響應 (Impulse Response)



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連續脈衝響應 (Impulse Response)

$$\dots + x(0) \delta_d(t) d$$

$$+ x(d) \delta_d(t-d) d$$

$$+ x(2d) \delta_d(t-2d) d$$

$$+ \dots$$

$$+ x(kd) \delta_d(t-kd) d + \dots$$

$$= \sum_{k=-\infty}^{+\infty} x(kd) \delta_d(t-kd) d$$

$$= x_d(t)$$



$$\dots + x(0) h_d(t) d$$

$$+ x(d) h_d(t-d) d$$

$$+ x(2d) h_d(t-2d) d$$

$$+ \dots$$

$$+ x(kd) h_d(t-kd) d$$

$$+ \dots$$

$$= \sum_{k=-\infty}^{+\infty} x(kd) h_d(t-kd) d$$

$$= y_d(t)$$

連續脈衝響應 (Impulse Response)



$$= \sum_{k=-\infty}^{+\infty} x(kd) \delta_d(t-kd) d = \sum_{k=-\infty}^{+\infty} x(kd) h_d(t-kd) d$$

$$d \rightarrow 0$$

$$x(t) \quad \begin{aligned} \tau &= kd \\ d &= d\tau \end{aligned}$$
$$= \int_{-\infty}^{+\infty} x(\tau) \delta(t - \tau) d\tau$$

$$d \rightarrow 0$$

$$y(t) \quad \begin{aligned} \tau &= kd \\ d &= d\tau \end{aligned}$$
$$= \int_{-\infty}^{+\infty} x(\tau) h(t - \tau) d\tau$$

摺積積分 (Convolution Integral)



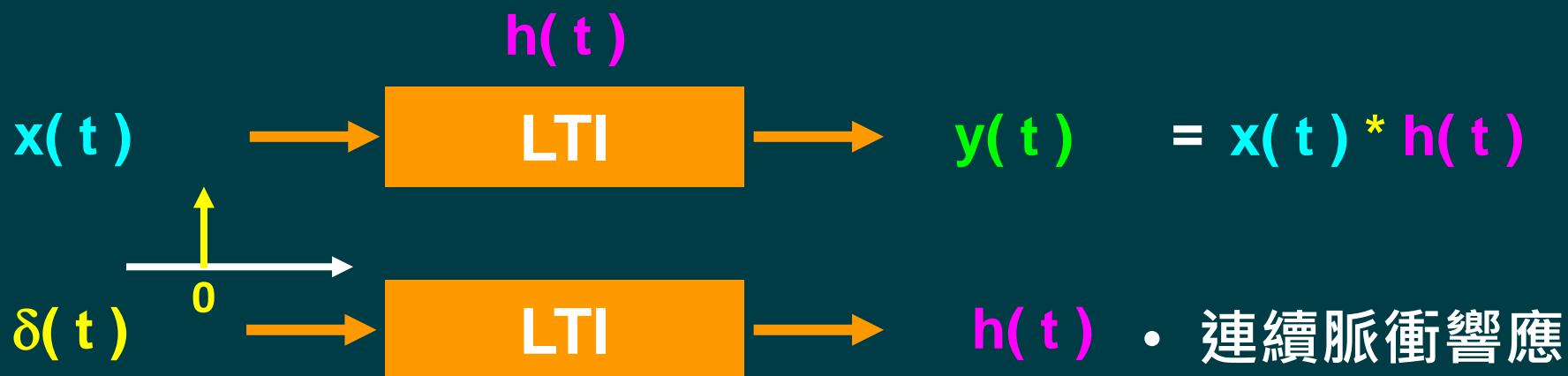
$$y(t) = \int_{-\infty}^{+\infty} x(\tau) h(t - \tau) d\tau = x(t) * h(t)$$

- 連續摺積計算 - 摺積積分 (Convolution Integral)

摺積積分 (Convolution Integral)

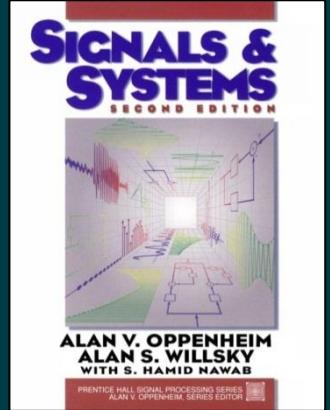
$$x(t) * h(t) = y(t)$$

The diagram illustrates the convolution operation between two signals, $x(t)$ and $h(t)$. It shows two wavy lines representing the signals. The first signal, $x(t)$, is shown with a blue arrow below it. The second signal, $h(t)$, is shown with a red arrow below it. The result of their convolution is a green wavy line, labeled $y(t)$, with a black arrow below it.



參考文獻

- Alan V. Oppenheim, Alan S. Willsky, S. Hamid,
Signals & Systems,
Prentice Hall, 2nd Edition, 1997



- **SciLab:**
Open source software for numerical computation
<http://www.scilab.org/>