從信號與系統到控制

單元：連續摺積-2

連續脈衝響應與摺積計算

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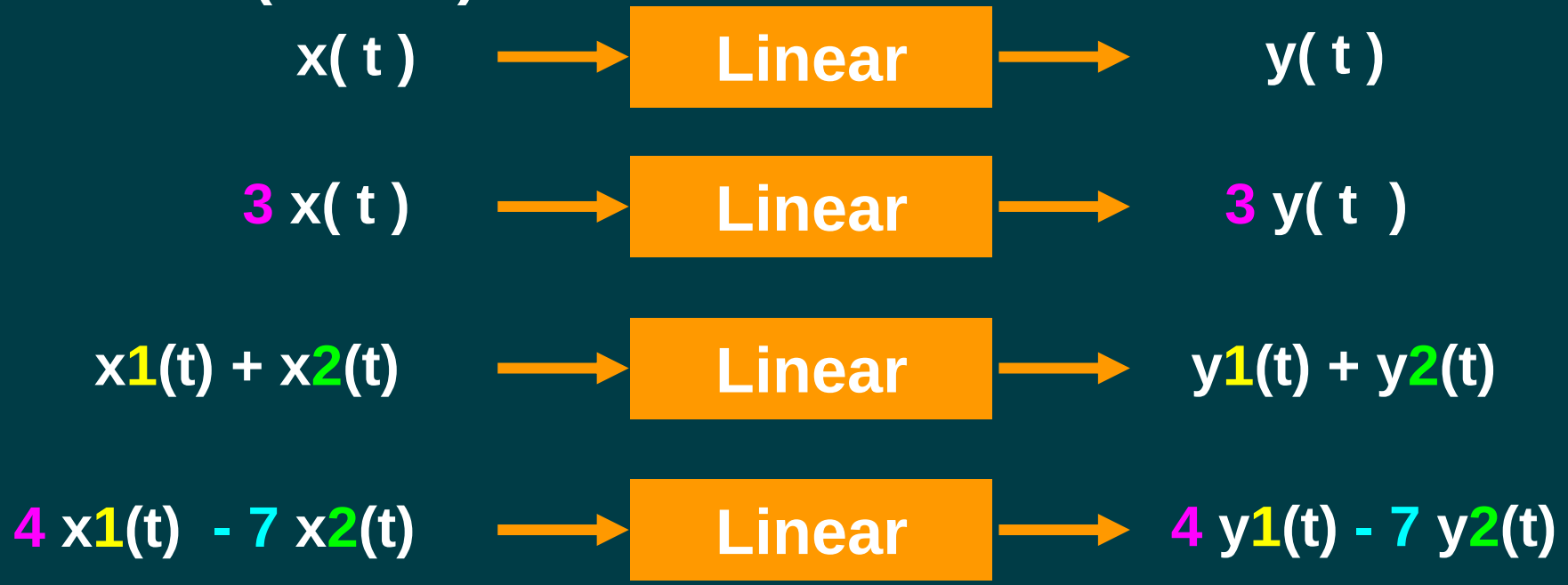
單元學習目標與大綱

- 線性 非時變 系統 – 基本定義
- 連續 脈衝響應 – 輸入輸出操作
- 連續 摺積計算 – 摺積積分公式推導

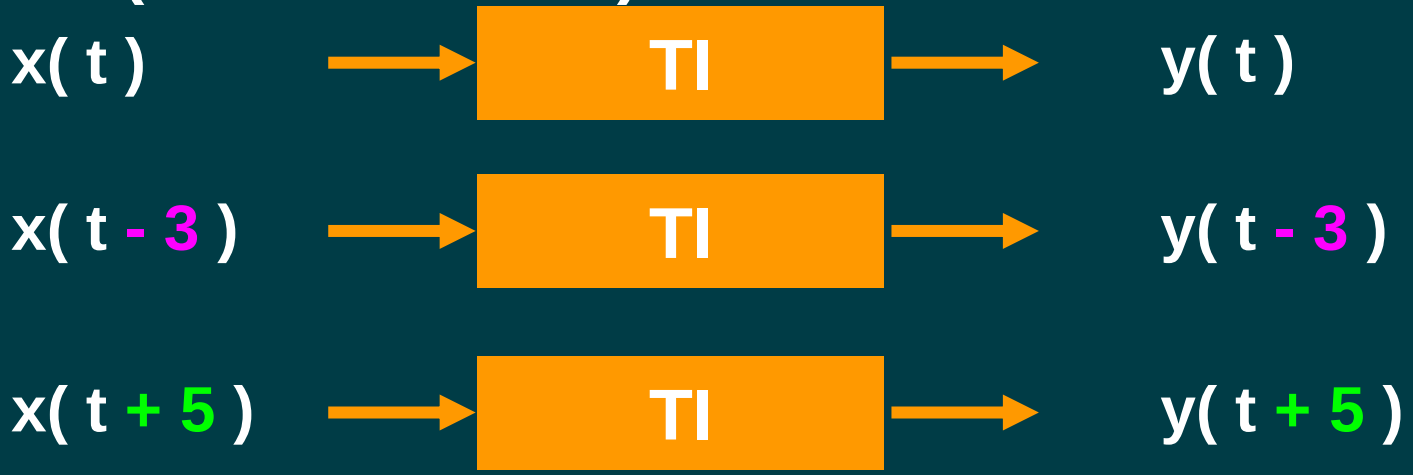
線性非時變系統 (LTI system)

- 線性 (Linear)

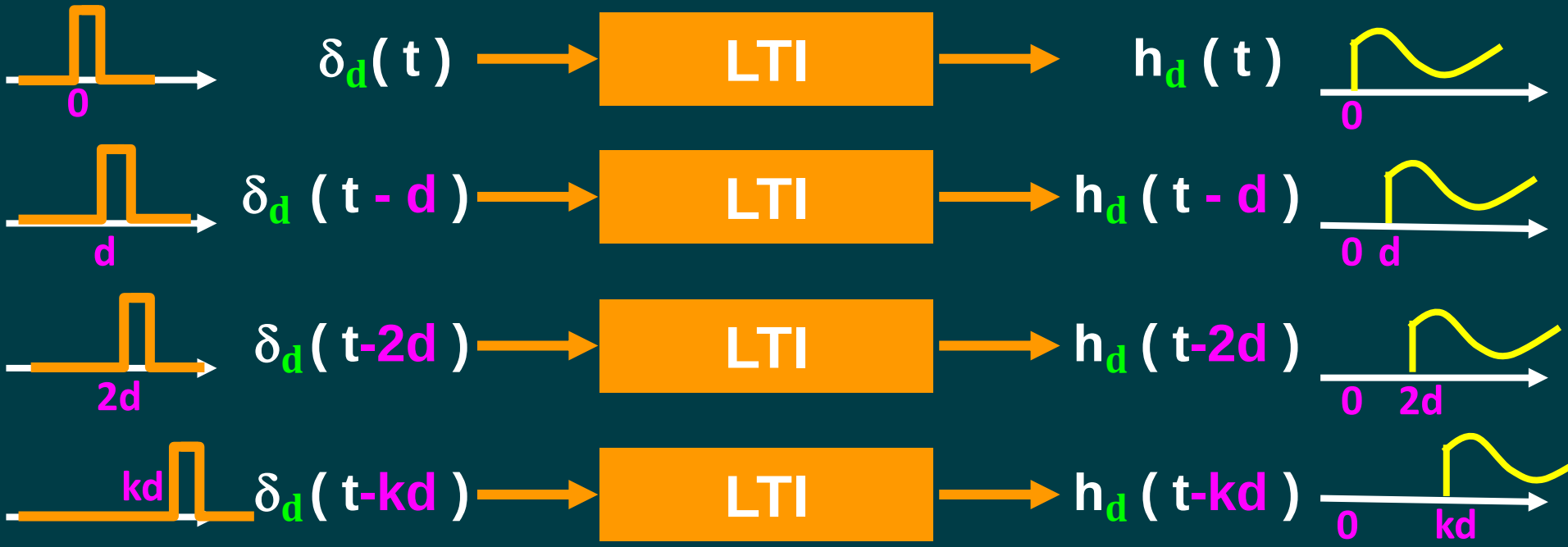


線性非時變系統 (LTI system)

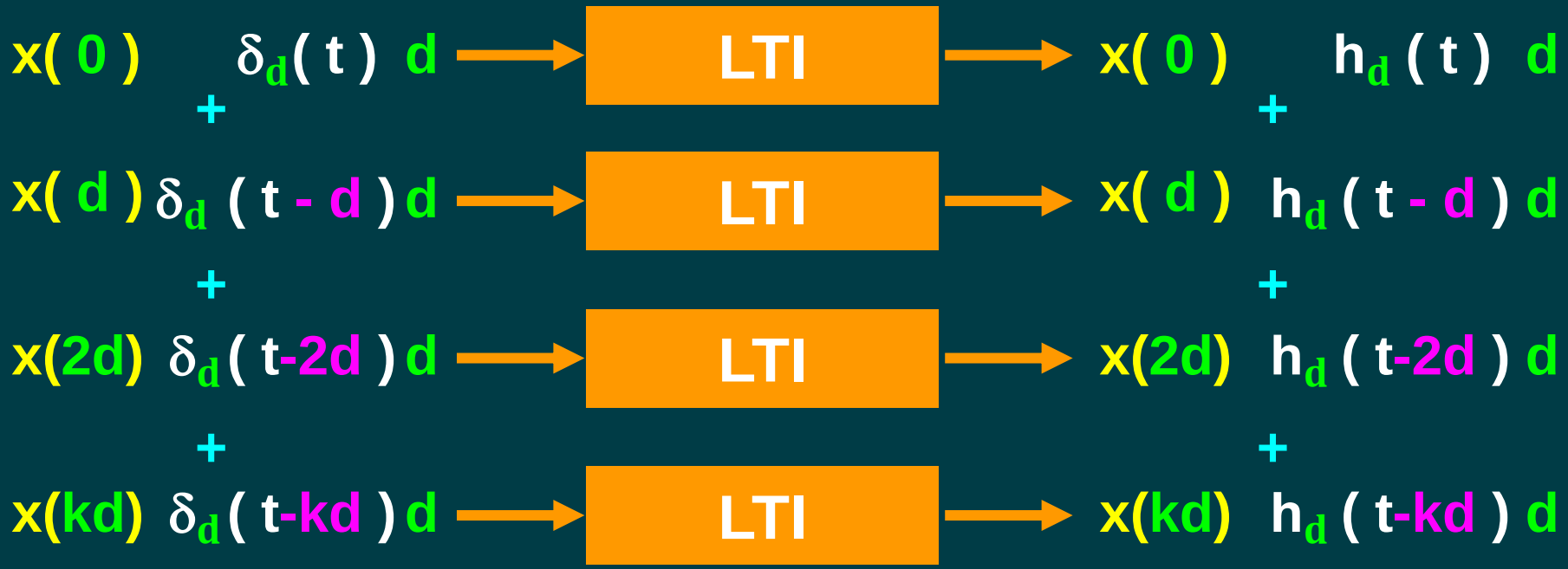
- 非時變 (Time-Invariant)



連續脈衝響應 (Impulse Response)



連續脈衝響應 (Impulse Response)



連續脈衝響應 (Impulse Response)

$$\ldots + x(0)\,\delta_d(t)\,d + x(d)\,\delta_d(t-d)\,d + x(2d)\,\delta_d(t-2d)\,d + \ldots + x(kd)\,\delta_d(t-kd)\,d + \ldots$$

$$= \sum_{k=-\infty}^{+\infty} x(kd)\,\delta_d(t-kd)\,d$$

$$= x_d(t)$$

$$\ldots + x(0)\,h_d(t)\,d + x(d)\,h_d(t-d)\,d + x(2d)\,h_d(t-2d)\,d + \ldots + x(kd)\,h_d(t-kd)\,d + \ldots$$

$$= \sum_{k=-\infty}^{+\infty} x(kd)\,h_d(t-kd)\,d$$

$$= y_d(t)$$

連續脈衝響應 (Impulse Response)

$$= \sum_{k=-\infty}^{+\infty} x(kd)\,\delta_d(t-kd)\,d \qquad = \sum_{k=-\infty}^{+\infty} x(kd)\,h_d(t-kd)\,d$$

$d \to 0$

$x(t)$

$\tau = kd$

$d = d\tau$

$$= \int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

$d \to 0$

$y(t)$

$\tau = kd$

$d = d\tau$

$$= \int_{-\infty}^{+\infty} x(\tau)\,h(t-\tau)\,d\tau$$

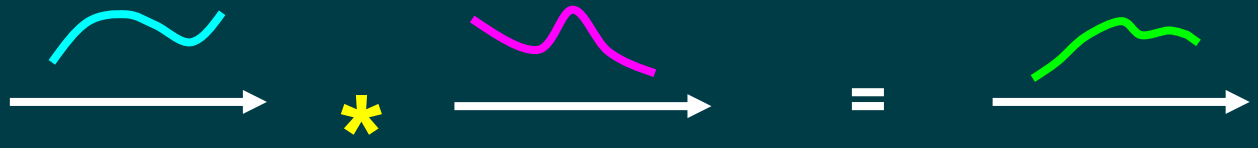
摺積積分 (Convolution Integral)

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau = x(t) * h(t)$$

- 連續摺積計算 - 摺積積分 (Convolution Integral)

摺積積分 (Convolution Integral)

x (t) * h(t) = y(t)



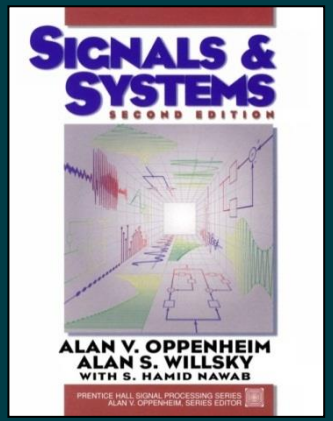
h(t)

x(t) → LTI → y(t) = x(t) * h(t)

δ(t) → LTI → h(t) • 連續脈衝響應

參考文獻

- Alan V. Oppenheim, Alan S. Willsky, S. Hamid,
  Signals & Systems,
  Prentice Hall, 2nd Edition, 1997



- **SciLab**:
  Open source software for numerical computation
  http://www.scilab.org/