

從信號與系統到控制

單元：連續摺積-1

利用單位脈衝函數表示連續時間信號

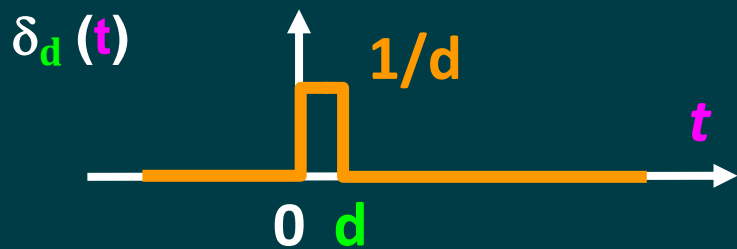
授課老師：連 豐 力

單元學習目標與大綱

- 定義 連續時間 的 單位 脈衝函數
- 利用 脈衝函數 表示 連續時間信號

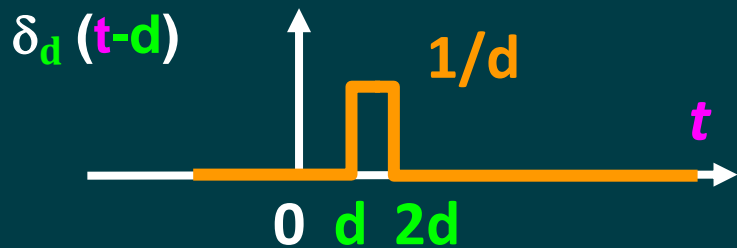
連續單位脈衝函數 (unit impulse)

- 近似的脈衝函數



$$\delta_d(t) = \begin{cases} \frac{1}{d}, & 0 \leq t < d \\ 0, & \text{otherwise} \end{cases}$$

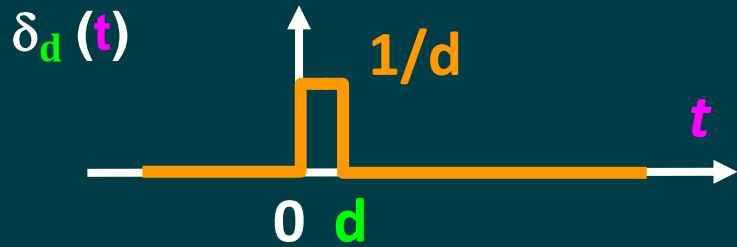
- 時間軸上，平移 d



$$\delta_d(t-d) = \begin{cases} \frac{1}{d}, & d \leq t < 2d \\ 0, & \text{otherwise} \end{cases}$$

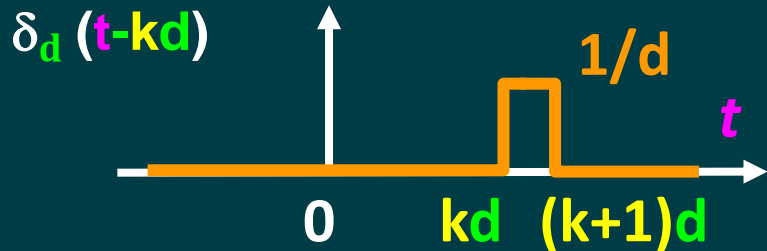
連續單位脈衝函數 (unit impulse)

- 近似的脈衝函數



$$\delta_d(t) = \begin{cases} \frac{1}{d}, & 0 \leq t < d \\ 0, & \text{otherwise} \end{cases}$$

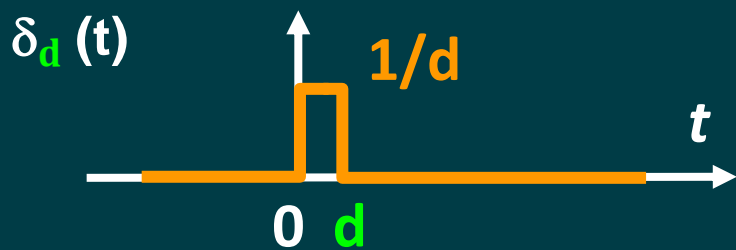
- 時間軸上，平移 kd



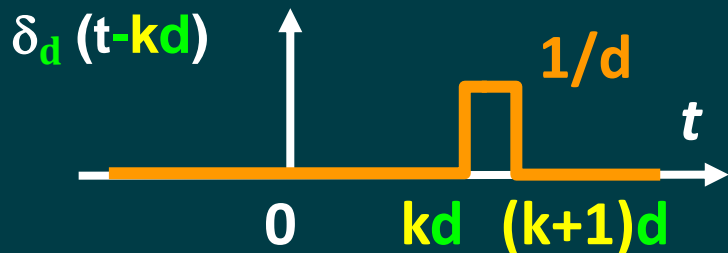
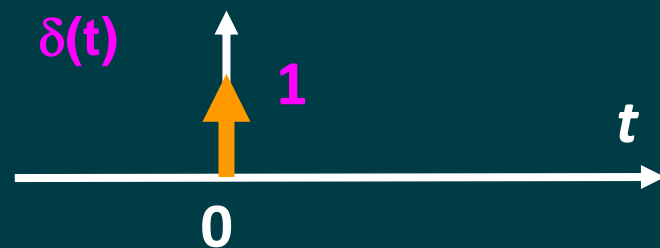
$$\delta_d(t - kd) = \begin{cases} \frac{1}{d}, & kd \leq t < (k+1)d \\ 0, & \text{otherwise} \end{cases}$$

連續單位脈衝函數 (unit impulse)

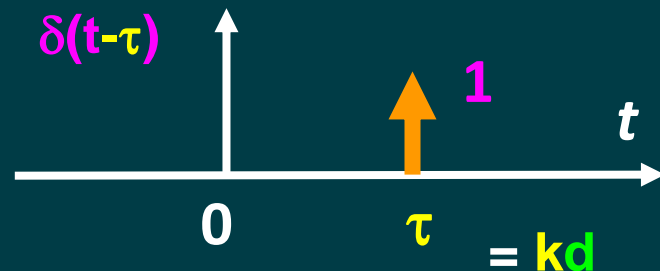
- 連續脈衝函數



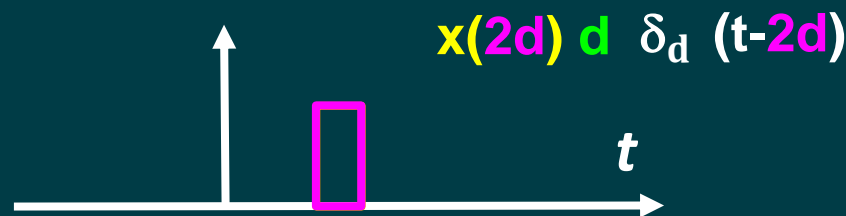
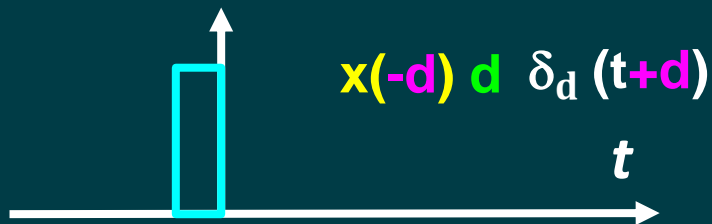
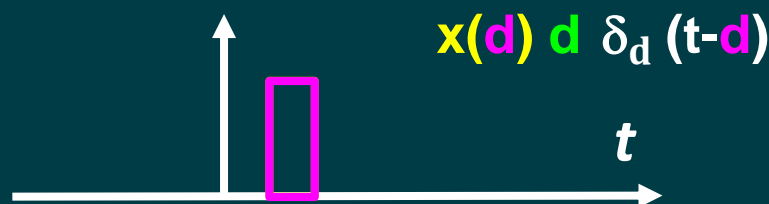
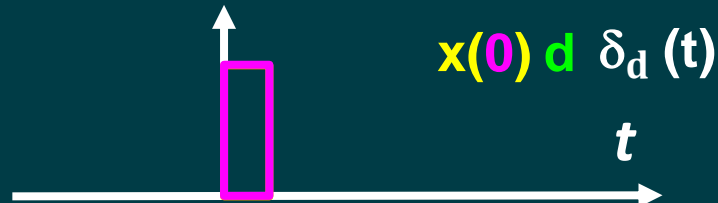
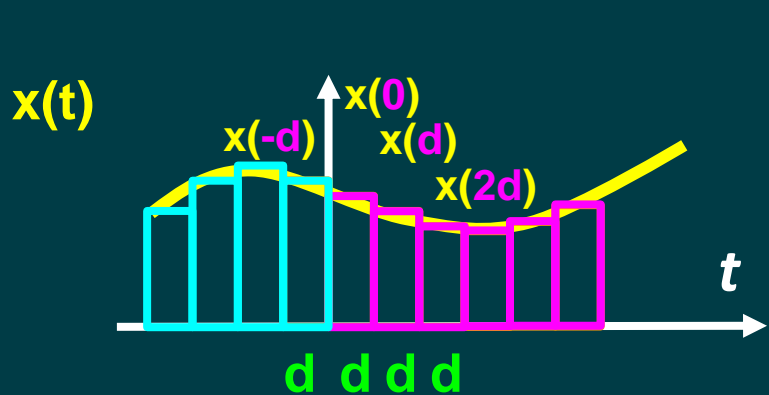
$d \rightarrow 0$



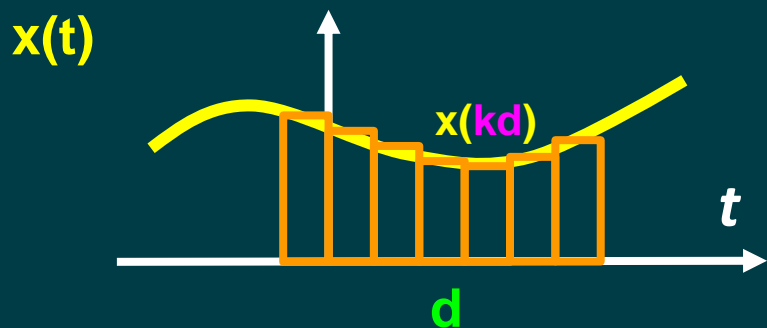
$d \rightarrow 0$



利用單位脈衝函數表示連續時間信號



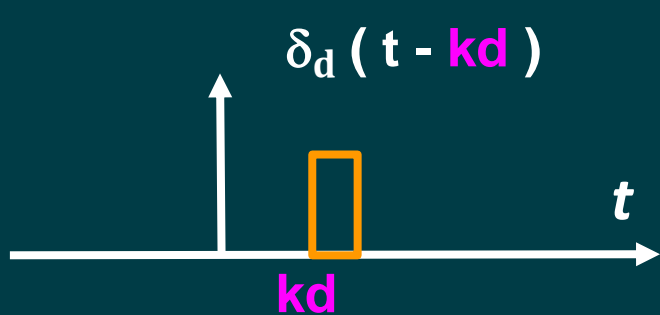
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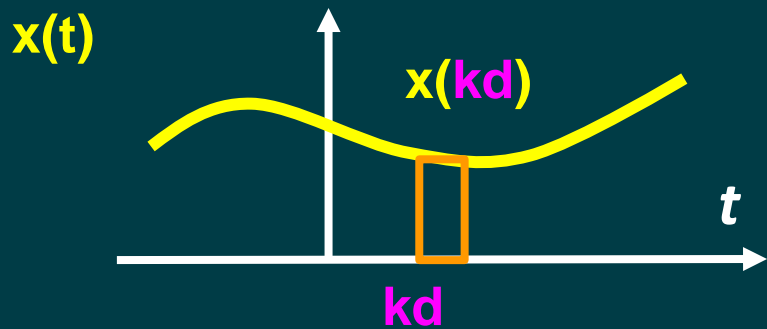
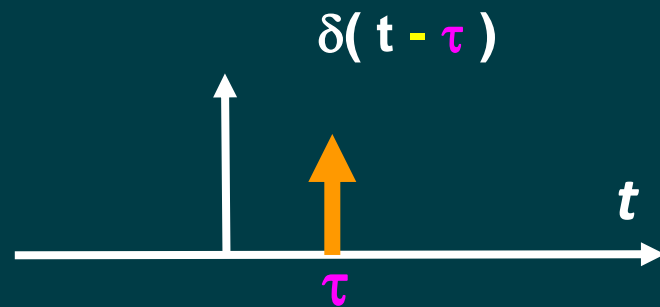
$$= \sum_{k=-\infty}^{+\infty} x(kd) \delta_d(t-kd) d$$

$$\begin{aligned} x_d(t) &= \dots + x(-d) \delta_d(t+d) d + x(0) \delta_d(t) d \\ &+ x(d) \delta_d(t-d) d + x(2d) \delta_d(t-2d) d \\ &+ \dots + x(kd) \delta_d(t-kd) d + \dots \end{aligned}$$

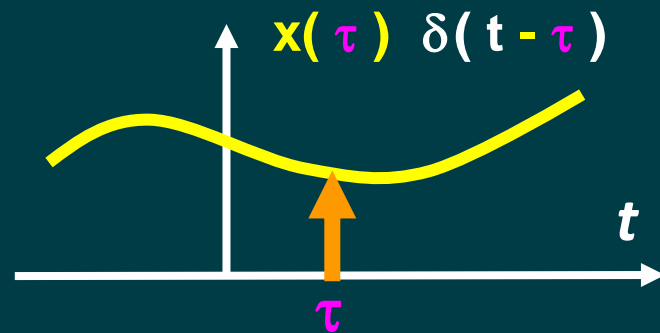
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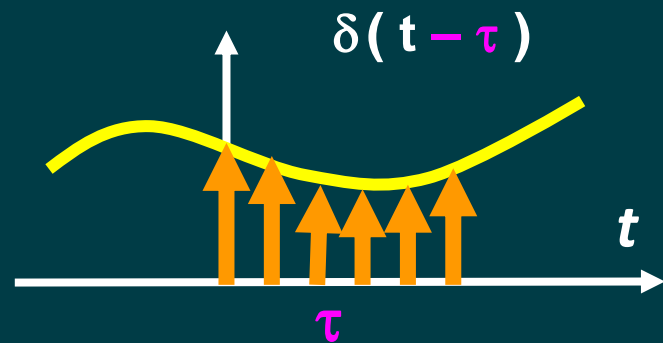
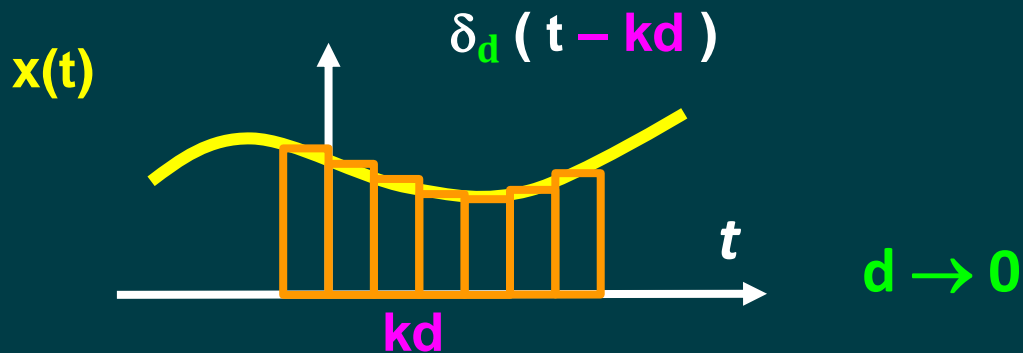
$d \rightarrow 0$



$d \rightarrow 0$



利用單位脈衝函數表示連續時間信號



$$x_d(t) = \sum_{k=-\infty}^{+\infty} x(kd) \delta_d(t - kd) d$$

$d \rightarrow 0$

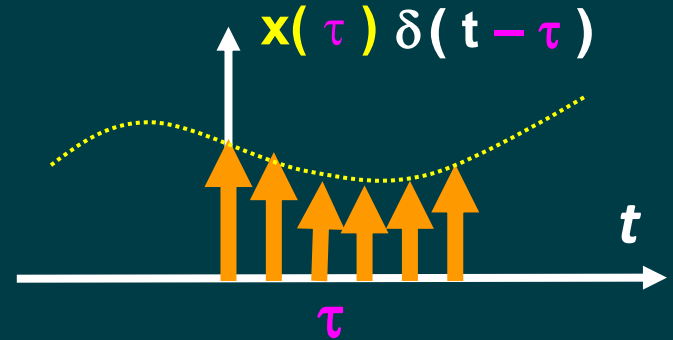
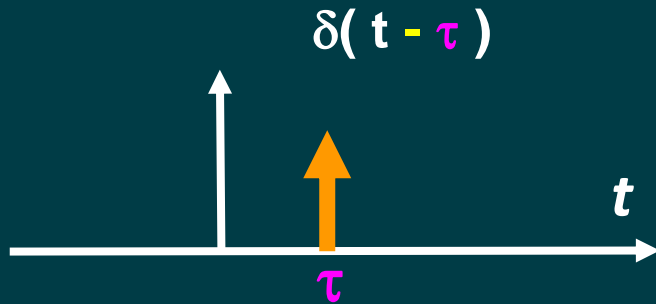
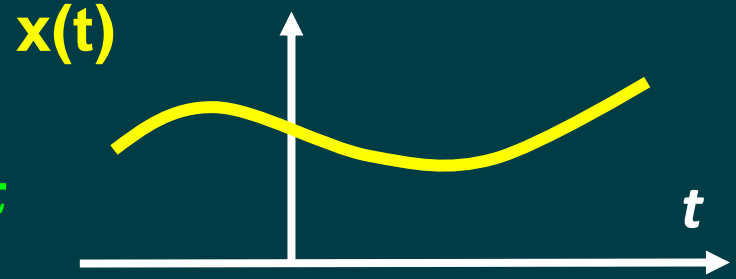
$$\tau = kd$$

$$d = d\tau$$

$$x(t) = \int_{-\infty}^{+\infty} x(\tau) \delta(t - \tau) d\tau$$

利用單位脈衝函數表示連續時間信號

$$x(t) = \int_{-\infty}^{+\infty} x(\tau) \delta(t - \tau) d\tau$$



參考文獻

- Alan V. Oppenheim, Alan S. Willsky, S. Hamid, **Signals & Systems**, Prentice Hall, 2nd Edition, 1997
- **SciLab:**
Open source software for numerical computation
<http://www.scilab.org/>

